The Impact of Predator Attack Behaviour on Interactions in Two-Prey and One-Predator Systems

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Article Info

ABSTRACT

This research explores a three-species model of prey and predator interactions, which includes primary and secondary prey and a predator. The main concern is the potential disturbance caused by uncontrolled predator attacks, which can harm the coexistence of species and disrupt the entire food chain system. Thus, this research aims to analyse the impact of predator attacks on primary and secondary prey, focusing on coexistence dynamics or potential extinction within the system. The approach involves formulating a simplified ordinary differential equation for a two-prey and one-predator system, calculating equilibrium points, and performing theoretical stability analysis. Numerical simulations are executed using Maple and MATLAB software to validate the theoretical findings, providing a strong foundation for the research outcomes. The main findings indicate that high-intensity predator attacks can result in the extinction of prey populations, while moderate-intensity attacks facilitate the coexistence of all species. In summary, this research emphasises the significant role of comprehending the interactions between predators and prey within ecological environments. Through clarifying the consequences of predator attacks on prey populations and their survival or disappearance trends, this research provides essential perspectives for initiatives aimed at ecological preservation.

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1. Introduction

Interactions between prey and predators constitute a fundamental aspect of ecological dynamics that illustrates the complex relationships between species in an ecosystem. These interactions play a crucial role in shaping population dynamics and influencing the overall stability of the ecosystem [1-3]. When predators and prey engage, the predator typically hunts, captures, and consumes the prey, thereby exerting substantial pressure on prey populations [4-7]. This interaction is essential for comprehending ecosystem dynamics as it directly affects species composition, abundance, and the overall health of the ecosystem.

Traditional ecological models often represent prey-predator interactions using two-species or three-species models [8-12]. In a simple two-species model, the interaction involves a prey species and its predator, such as the interaction between rabbits (prey) and foxes (predators). In

more intricate scenarios, three-species models may involve either a two-prey and one-predator system or a one-prey and two-predator system. For instance, a two-prey and one-predator system might include rabbits and mice as prey, with foxes as the predator. Conversely, in a one-prey and two-predator system, the interaction could involve rabbits as prey, with both foxes and hawks as predators.

Some researchers, including [13], have suggested models that incorporate cooperative behaviour among prey species during predation events. Elettreby's system [13] involves two teams of prey with different densities collaborating to evade predation by a single team of predators. Expanding on this concept, Tripathi *et al.* [14] extended the analysis to include a three-team system comprising two prey species and one predator, where both teams of prey collaborate during predation events and the predation rates of the two prey teams are assumed to differ.

Building on the contributions of [13] and [14], this study aims to investigate prey-predator interactions by excluding team cooperation and competition factors among prey populations. Instead, the emphasis is on examining variations in the predator's attack rate on primary and secondary prey species. The predator's attack behaviour is recognised as a critical factor influencing ecosystem dynamics, with the potential to disrupt food chains and threaten species' survival. Therefore, this study has a threefold objective: to establish a simple two-prey one-predator system, to calculate equilibrium points and explore stability analysis, and to analyse the impact of the predator's attack behaviour on species interactions through numerical examples.

This research has important implications for grasping and controlling ecosystems. By elucidating the effects of predator attack rates on prey populations, it provides insights into ecosystem resilience and stability. Understanding how predator behaviour influences species interactions may assist with guiding ecological preservation and extinction prevention strategies. Ultimately, this research contributes to a deeper understanding of the complex dynamics within predator-prey systems, highlighting the importance of considering multiple factors in ecological studies and management practices.

2. Literature Review

The early development of mathematical models that illustrate the dynamics between prey and predators can be attributed to the groundbreaking contributions of Lotka and Volterra during the early 20th century. In 1920, Alfred Lotka presented a mathematical model of species interactions, focusing on prey-predator relationships [15]. Subsequently, in 1926, Vito Volterra proposed an independent model similar to Lotka's, which became known as the Lotka-Volterra model. This classical model is grounded in the interplay of predator and prey populations, wherein the prey population exhibits logistic growth in the absence of predators, and the predator population is directly tied to the abundance of prey [16]. Describing the interaction between these populations, the Lotka-Volterra model utilises nonlinear ordinary differential equations and has undergone extensive examination in terms of stability analysis, chaos control, and synchronisation [17-20].

Furthermore, the model has been improved to incorporate factors such as prey diseases, temporal delays, and impulsive effects, which reflect the complexities of real ecological systems [21-23]. Additionally, scholarly investigations have explored the utilisation of the Lotka-Volterra model across various domains, including ecoepidemiology, evolutionary dynamics, and technology evolution forecasting [19], [24-26]. The examination of predator-prey model dynamics has also delved into bifurcation, centre manifold, and stability analyses, contributing valuable insights into the qualitative behaviours of these ecological systems [17], [27-29]. Furthermore, the exploration of multi-species interactions and nonautonomous dynamics in predator-prey systems has broadened the realm of ecological modelling, enabling the consideration of intricate ecological relationships [10], [30].

Transitioning to more complex ecological scenarios, the development of three-species models has allowed for a more nuanced understanding of ecological interactions. The three-species models extend the scope of analysis, capturing the interdependencies and dynamics involving multiple prey and predator species. Elettreby [13] proposed a unique system featuring two teams of prey interacting with one team of predators, where the prey teams collaboratively help each other. Despite conducting equilibrium point calculations, stability analysis, and numerical simulations, the study did not specifically focus on individual parameters, leaving room for further investigations. Tripathi *et al.* [31] extended Elettreby's team approach model [13], studying the local stability behaviour with varying predation rates for two prey teams. Their findings highlighted the impact of predation rates and interactions between prey teams on equilibrium points. However, similar to [13],

the study did not delve into specific parameter analyses. Sharma and Samanta [32] introduced a two-prey one-predator model with Holling type II functional response and predator handling time, revealing that the ratio of predator handling times significantly influences system stability. Above a threshold value, the population becomes unstable. Tripathi *et al.* [14] proposed a three-team prey-predator model, emphasising the importance of the team approach. The study demonstrated that team collaboration has a stabilising effect, potentially preventing species extinction. Notably, the team approach revealed differences in long-term evolution between systems with and without help. Despite the valuable findings from prior studies, there is still a need for more exploration, particularly in understanding how predators behave when attacking their prey. Thus, these knowledge gaps motivate additional research into predator-prey interactions, which improves our understanding of ecological systems.

3. Methodology

This section describes the formulation of the predator-prey model, the calculation of equilibrium points, the employment of stability analysis, and the execution of numerical simulations. Figure 1 illustrates the methodology flow for the research.
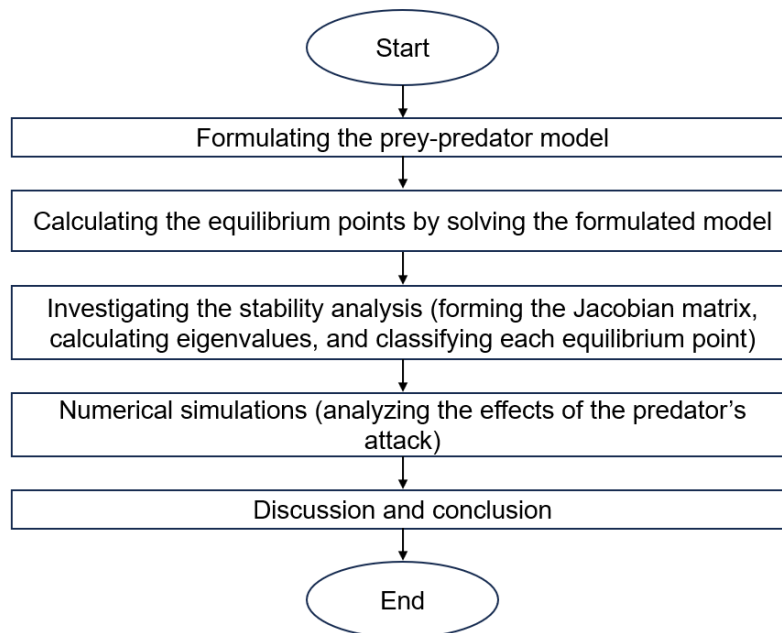

Figure 1. The flow diagram of the research method

3.1 Prey-predator System

In this study, we examine a three-species model that incorporates primary and secondary prey, along with a predator. Our model is influenced by the investigations conducted by [13] and [14]. Notably, we have made modifications by excluding two specific ecological factors: competition for prey population density and collaborative behaviour between the primary and secondary prey. To illustrate, consider the scenario of gazelles and zebras sharing a common predator, the lion. Despite this shared threat, gazelles and zebras do not cooperate or compete in the presence of the lion. There is no direct assistance or resource-based rivalry between the two prey species. The lion preys independently on both gazelles and zebras, and there are no collaborative or competitive dynamics observed between the two prey species. These instances exemplify situations in which multiple prey species coexist with a shared predator. Yet, there is an absence of cooperative or competitive interactions among the prey species in the predator's presence. This results in a dynamic two-prey one-predator system characterised by independent predation pressures on each prey species. In this research, we only focused on the influence of predator attacks on both prey populations. The following assumptions are considered for the model:

i. When predation is absent, each group of prey experiences logistic growth, with the potential for competition among individual prey members, as indicated by the terms $ax(1-x)$ and $by(1-y)$.

ii. The impact of predation is evident in the decline of growth rates for both primary and secondary prey, proportional to the populations of both prey and predator. This is represented by the terms $-cxz$ and $-dyz$.

iii. In the absence of any prey, the predator faces mortality, leading to the inclusion of the term $-hz$.

iv. In situations where sufficient prey is lacking, competition arises among individual predators, introducing the term $-ez^2$, where e measures the intensity of intra-predator competition.

v. The terms fxz and gyz denote that the predator's growth rate depends on the availability of prey and the size of the predator population, highlighting the interconnectedness of these variables.

Based on these assumptions, the model compartment of two-prey and one-predator interactions is illustrated in Figure 2.
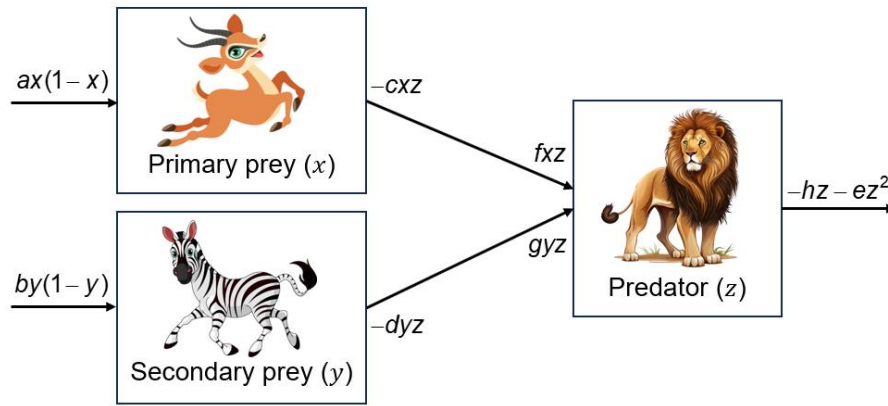

Figure 2. Model compartment of two-prey and one-predator interactions

As a result, the mathematical framework used in this study leads to the following system of ordinary differential equations:

$$\frac{dx}{dt} = ax(1-x) - cxz,$$
$$\frac{dy}{dt} = by(1-y) - dyz, \quad (1)$$
$$\frac{dz}{dt} = -hz - ez^2 + fxz + gyz,$$

where x, y, z are the primary prey, secondary prey, and predator, respectively. All parameters are assumed to be positive constant, with initial conditions $x(0) > 0$, $y(0) > 0$, $z(0) > 0$.

3.2 Positivity

In the following theorem, we demonstrate the positive nature of solutions in relation to the given system (1). This validation is crucial to substantiate the biological significance of the derived solutions.

Theorem 1. *All solutions of system (1) with $x(0) > 0$, $y(0) > 0$, $z(0) > 0$ are non-negative.*

Proof. Using the same approach as [33], the non-negative of the solution can be proved by the equation as follows:

$$x(t) = x(0)\, e^{\int_0^t \left[a(1-x(\theta))-cz(\theta) \right] d\theta} \geq 0,$$

$$y(t) = y(0)\, e^{\int_0^t \left[b(1-y(\theta))-dz(\theta) \right] d\theta} \geq 0,$$

$$z(t) = z(0)\, e^{\int_0^t \left[-h-ez(\theta)+fx(\theta)+gy(\theta) \right] d\theta} \geq 0.$$

Given the well-known property that exponential functions yield non-negative values for any real number and considering the initial conditions $x(0) > 0,\, y(0) > 0,\, z(0) > 0$, it follows that all solutions of system (1) maintain non-negativity throughout any time t.

3.3 Equilibrium Points and Stability Analysis

In this section, we calculated the equilibrium points and investigated their stability analysis. By solving $\dfrac{dx}{dt} = \dfrac{dy}{dt} = \dfrac{dz}{dt} = 0$ using Maple software, there were seven biological significant equilibrium points as follows:

i. Extinction of all species, $E_1 = (0,0,0)$.

ii. Survival of secondary prey, $E_2 = (0,1,0)$.

iii. Survival of primary prey, $E_3 = (1,0,0)$.

iv. Survival of all prey, $E_4 = (1,1,0)$.

v. Survival of secondary prey and predator, $E_5 = \left(0, \dfrac{be+dh}{be+dg}, \dfrac{b(g-h)}{be+dg} \right)$.

vi. Survival of primary prey and predator, $E_6 = \left(\dfrac{ae+ch}{ae+cf}, 0, \dfrac{a(f-h)}{ae+cf} \right)$.

vii. Coexistence of all species, $E_7 = \left(x^*, y^*, z^* \right)$, where $x^* = \dfrac{abe+adg-bcg+bch}{abe+adg+bcf}$,

$y^* = \dfrac{abe-adf+bcf+adh}{abe+adg+bcf}$, $z^* = \dfrac{ab(f+g-h)}{abe+adg+bcf}$.

To examine the local stability of each equilibrium point, it is necessary to compute the Jacobian matrix and determine the eigenvalues associated with each equilibrium point [33], [34], [35], [36]. Hence, the Jacobian matrix for system (1) is expressed as:

$$J(x,y,z) = \begin{bmatrix} a(1-x)-ax-cz & 0 & -cx \\ 0 & b(1-y)-by-dz & -dy \\ fz & gz & -2ez+fx+gy-h \end{bmatrix}. \tag{2}$$

Subsequently, the equilibrium point can be classified according to its eigenvalues, with further details available in [37], [38]. The following theorems are presented to aid in the analysis of local stability at each equilibrium point.

Theorem 2. *The point* $E_1 = (0,0,0)$ *is always an unstable saddle.*

Proof. By calculating the Jacobian matrix (2) at the point E_1, the obtained eigenvalues are $\lambda_1 = -h, \lambda_2 = b$ and $\lambda_3 = a$. Since all parameters are confirmed to be positive constants, the eigenvalues satisfy the conditions $\lambda_1 < 0$ and $\lambda_{1,2} > 0$. Thus, E_1 exhibits a consistently unstable saddle node.

Theorem 3. *The point $E_2 = (0,1,0)$ is always an unstable saddle.*

Proof. The eigenvalues of $J(E_2)$ are $\lambda_1 = g - h, \lambda_2 = -b$ and $\lambda_3 = a$. Notably, considering the condition $\lambda_2 < 0$ and $\lambda_3 > 0$, it can be observed that E_2 consistently signifies an unstable saddle-node, regardless of the particular value of λ_1.

Theorem 4. *The point $E_3 = (1,0,0)$ is always an unstable saddle.*

Proof. The eigenvalues of $J(E_3)$ are $\lambda_1 = f - h, \lambda_2 = b$ and $\lambda_3 = -a$. Considering that $\lambda_2 > 0$ and $\lambda_3 < 0$, regardless of the value of λ_1, it can be observed that E_3 consistently denotes an unstable saddle node.

Theorem 5. *The equilibrium point $E_4 = (1,1,0)$ is locally asymptotically stable if $f + g < h$.*

Proof. The eigenvalues of $J(E_4)$ are $\lambda_1 = f + g - h, \lambda_2 = -b$ and $\lambda_3 = -a$. If $\lambda_1 < 0$, then the condition for local asymptotic stability is $f + g < h$. Therefore, E_4 is locally asymptotically stable if $f + g < h$.

Theorem 6. *The equilibrium point $E_5 = \left(0, \dfrac{be + dh}{be + dg}, \dfrac{b(g - h)}{be + dg}\right)$ is locally asymptotically stable provided the following conditions are satisfied: $d > \dfrac{be}{h}$ and $a < \dfrac{bc(g + h)}{bc + dg}$.*

Proof. The eigenvalues of $J(E_5)$ are given by

$$\lambda_{1,2} = \frac{1}{2(be + dg)}\left(bh(d - e) + be(b + g) \pm \sqrt{\left(b^3 e^3 + 2e((h - g) + dh)eb^2 + \left((g - h)^2 e^2 - 4d\left(g + \frac{h}{2}\right)(g - h)e + d^2 h^2\right)b - 4d^2 gh(g - h)\right)b} \right), \text{ and}$$

$\lambda_3 = a - \dfrac{bc(g + h)}{bc + dg}$. Considering $\lambda_{1,2} < 0$ and $\lambda_3 < 0$, we obtain two conditions: $\lambda_{1,2} < 0$ holds if $d > \dfrac{be}{h}$, and $\lambda_3 < 0$ holds if $a < \dfrac{bc(g + h)}{bc + dg}$. Consequently, E_5 achieves local asymptotic stability when these conditions are satisfied.

Theorem 7. *The equilibrium point $E_6 = \left(\dfrac{ae + ch}{ae + cf}, 0, \dfrac{a(f - h)}{ae + cf}\right)$ is locally asymptotically stable provided the following conditions are satisfied: $c > \dfrac{ae}{h}$ and $b < \dfrac{ad(f - h)}{ae + cf}$.*

Proof. The eigenvalues of $J(E_6)$ are given by

$$\lambda_{1,2} = \frac{1}{2(ae + cf)}\left(-a(c - e)h - a(a + f)e \pm \sqrt{\left(a^3 e^2 + 2((h - f)e + ch)ea^2 + \left((f - h)^2 e^2 - 4c\left(f + \frac{h}{2}\right)(f - h)e + c^2 h^2\right)a - 4c^2 fh(f - h)\right)a} \right) \text{ and}$$

$\lambda_3 = b - \dfrac{ad(f - h)}{ae + cf}$.

We consider $\lambda_{1,2} < 0$ and $\lambda_3 < 0$. This yields two conditions: $\lambda_{1,2} < 0$ holds if $c > \dfrac{ae}{h}$ and $\lambda_3 < 0$ holds if $b < \dfrac{ad(f - h)}{ae + cf}$. Thus, E_6 is locally asymptotically stable if all these conditions are satisfied.

Theorem 8. *The equilibrium point* $E_7 = \left(x^*, y^*, z^*\right)$ *is locally asymptotically stable provided the following conditions are satisfied:*

i. $a_{11} + a_{22} + a_{33} < 0$

ii. $a_{22}a_{33} + a_{11}a_{33} + a_{11}a_{22} > a_{32}a_{23} + a_{13}a_{31}$

iii. $a_{11}a_{23}a_{32} + a_{13}a_{31}a_{22} > a_{11}a_{22}a_{33}$

iv. $\left(-a_{11} - a_{22} - a_{33}\right)\left(a_{22}a_{33} + a_{11}a_{33} + a_{11}a_{22} - a_{32}a_{23} - a_{13}a_{31}\right) > a_{11}a_{23}a_{32} + a_{13}a_{31}a_{22} - a_{11}a_{22}a_{33}$

Proof. The Jacobian at the equilibrium point E_7 is given by

$$J(E_7) = \begin{bmatrix} a_{11} & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

where $a_{11} = a(1 - x^*) - ax^* - cz^*$, $a_{13} = -cx^*$, $a_{22} = b(1 - y^*) - by^* - dz^*$, $a_{23} = -dy^*$, $a_{31} = fz^*$, $a_{32} = gz^*$, $a_{33} = -2ez^* + fx^* + gy^* - h$.

The corresponding characteristic polynomial is $\lambda^3 + Q_1\lambda^2 + Q_2\lambda + Q_3 = 0$. In accordance with the Routh–Hurwitz criteria, the local asymptotic stability of E_7 must satisfy the following conditions:

i. $Q_1 > 0$ where $Q_1 = -\left(a_{11} + a_{22} + a_{33}\right)$

ii. $Q_2 > 0$ where $Q_2 = a_{22}a_{33} + a_{11}a_{33} + a_{11}a_{22} - a_{32}a_{23} - a_{13}a_{31}$

iii. $Q_3 > 0$ where $Q_3 = a_{11}a_{23}a_{32} + a_{13}a_{31}a_{22} - a_{11}a_{22}a_{33}$

iv. $Q_1Q_2 > Q_3$.

Due to the complexity of the systems, we established the stability of E_7 through numerical simulation in the subsequent section.

4. Results and Discussion

This section discusses the validation of analytical findings and presents numerical simulation results.

4.1 Validation of Analytical Findings

We employed numerical simulations to validate the analytical proofs provided in Theorems 2 – 8. Throughout these simulations, we specifically varied the parameters associated with the predator's attack rate on primary prey, c and secondary prey, d while keeping other parameter values constant. Our methodology draws inspiration from a previously established approach [33] for determining these parameter values and setting initial conditions. It's important to note that our model doesn't perfectly mirror real-world scenarios involving interactions among three species. Thus, we tailored the selection of parameter values to match stability conditions derived from prior findings.

There were four specific scenarios for numerical simulations were chosen, as detailed in Table 1. In Simulations 1 and 2, the primary prey experiences greater predator attack intensity compared to the secondary prey. Simulations 3 and 4 demonstrate a scenario where the predator's attack intensity on primary prey is reduced compared to the secondary prey. All simulations were carried out using Maple software.

Theorems 2, 3, and 4 imply that the equilibrium points $E_1 = (0,0,0)$, $E_2 = (0,1,0)$ and $E_3 = (1,0,0)$ always exhibit unstable saddle nodes, regardless of any positive parameter values.

Through all numerical simulations in Table 1, it was observed that all the eigenvalues are associated with E_1, E_2 and E_3 possess an opposite sign. Consequently, these empirical results support Theorems $2-4$, allowing us to confidently conclude that the equilibrium points E_1, E_2 and E_3 indeed represent unstable saddle nodes.

Table 1. Parameter values for each numerical simulation scenarios

Simulations	Parameters							
	a	b	c	d	e	f	g	h
Simulation 1	0.5	0.2	0.5	0.1	0.01	1.4	1.5	1
Simulation 2	0.5	0.2	0.8	0.1	0.01	1.4	1.5	1
Simulation 3	0.5	0.2	0.5	0.65	0.01	1.4	1.5	1
Simulation 4	0.5	0.2	0.5	0.75	0.01	1.4	1.5	1

Conversely, Theorem 5 indicates that the equilibrium point $E_4 = (1,1,0)$ is asymptotically stable if the condition of $f + g < h$ is satisfied. However, by employing all the simulation examples, it becomes evident that this condition consistently contradicts ($f + g > h$), and the eigenvalues consistently exhibit an opposite sign. Consequently, it is concluded that E_4 represents an unstable saddle node.

Next, we verify Theorems 6, 7, and 8 and provide a summary of the stability and numerical analysis results in Table 2. The stability result shows that E_5, E_6 and E_7 exhibit an asymptotically stable spiral due to the negativity of their respective eigenvalues. Therefore, this numerical verification is consistent with the claims made in Theorems $6-8$.

Table 2. Stability and numerical analysis results

Simulations	Critical points	Eigenvalues	Stability result
Simulation 1 (All species coexists)	$E_7 = (0.1204, 0.5602, 0.8796)$	$\lambda_1 = -0.0473 + 0.3819i$ $\lambda_2 = -0.0473 - 0.3819i$ $\lambda_3 = -0.0865$	
Simulation 2 (Extinction of primary prey)	$E_5 = (0, 0.6711, 0.6579)$	$\lambda_1 = -0.0704 + 0.2493i$ $\lambda_2 = -0.0704 - 0.2493i$ $\lambda_3 = -0.0263$	Asymptotically stable spiral
Simulation 3 (All species coexist)	$E_7 = (0.6977, 0.0175, 0.3023)$	$\lambda_1 = -0.1699 + 0.3478i$ $\lambda_2 = -0.1699 - 3478i$ $\lambda_3 = -0.0155$	
Simulation 4 (Extinction of secondary prey)	$E_6 = (0.7163, 0, 0.2837)$	$\lambda_1 = -0.1805 + 0.3327i$ $\lambda_2 = -0.1805 - 0.3327i$ $\lambda_3 = -0.0128$	

4.2 Numerical Simulations

In this section, we conduct an extensive analysis to determine how predator attacks affect both primary and secondary prey populations. To facilitate this analysis, we employed MATLAB software to generate the phase plane and time series plots. This software proves to be a valuable tool for creating insightful diagrams, which aid in visualising the dynamics of the system. The phase plane plot visually depicts the system's dynamics in three-dimensional space, revealing the relationships and interactions among the three variables. This visualisation is crucial for identifying

patterns and trends in the system's behaviour. Conversely, time series plots offer a chronological view, illustrating how the variables evolve over time and enabling the observation of trends, oscillations, or other dynamic features. The initial conditions for all simulation scenarios were set to $(0.1, 0.1, 0.1)$. This value represents the proportion of the initial population size of primary prey, secondary prey, and predators at time zero. Accordingly, Figures 3 – 6 depict the dynamic behaviours of prey-predator interactions as described under the simulation scenarios in Table 2.

In the initial scenario (Simulation 1), where the intensity of predator attacks on primary prey $(c = 0.5)$ is higher than that on secondary prey $(d = 0.1)$, the phase plane in Figure 3 illustrates a solution curve moving from the initial point $(0.1, 0.1, 0.1)$ toward the equilibrium point $E_7 = (0.1204, 0.5602, 0.8796)$. These findings are in accordance with the theoretical analysis, which concluded that the equilibrium solution exhibits an asymptotically stable spiral. Additionally, the time series plot demonstrates coexistence among all species, with populations initially experiencing fluctuations before gradually stabilising over time. In ecological terms, these fluctuations may be attributed to natural oscillations in population sizes. The observed trend indicates a harmonious coexistence between prey and predator populations, wherein the initial fluctuations are reflective of the intrinsic variability within ecological systems. This state of dynamic equilibrium implies that the interactions between predator and prey populations are favourable for prolonged coexistence, mitigating the risk of immediate extinction and thereby enhancing the overall stability of the ecosystem.
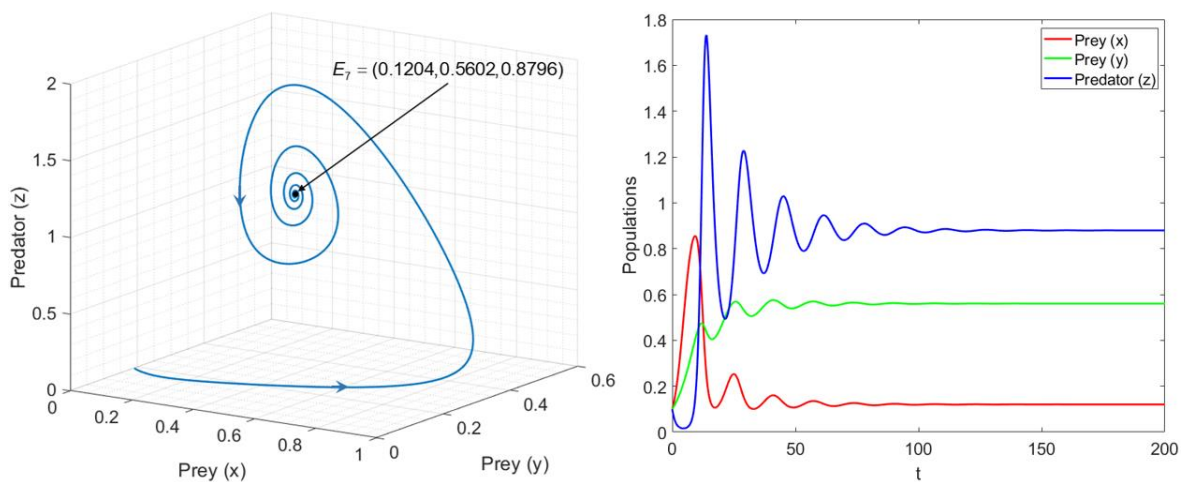

Figure 3. The phase plane and time series plot (Simulation 1) when the intensity of predator attacks on primary prey $(c = 0.5)$ are greater than secondary prey $(d = 0.1)$.

In Simulation 2, the intensity of predator attacks on primary prey is further increased from $c = 0.5$ to $c = 0.8$, while predator attacks on secondary prey remain constant at $d = 0.1$. The phase plane in Figure 4 reveals an asymptotically stable spiral behaviour, with the solution curve converging towards $E_5 = (0, 0.6711, 0.6579)$. However, the time series plot underscores a potential challenge: the heightened intensity of predator attacks on the primary prey leads to their prolonged extinction, while the secondary prey and predators maintain a stable state. This scenario demonstrates the ecological consequences of an imbalance in predator attacks, which disproportionately affect the primary prey due to a lack of mutual support.
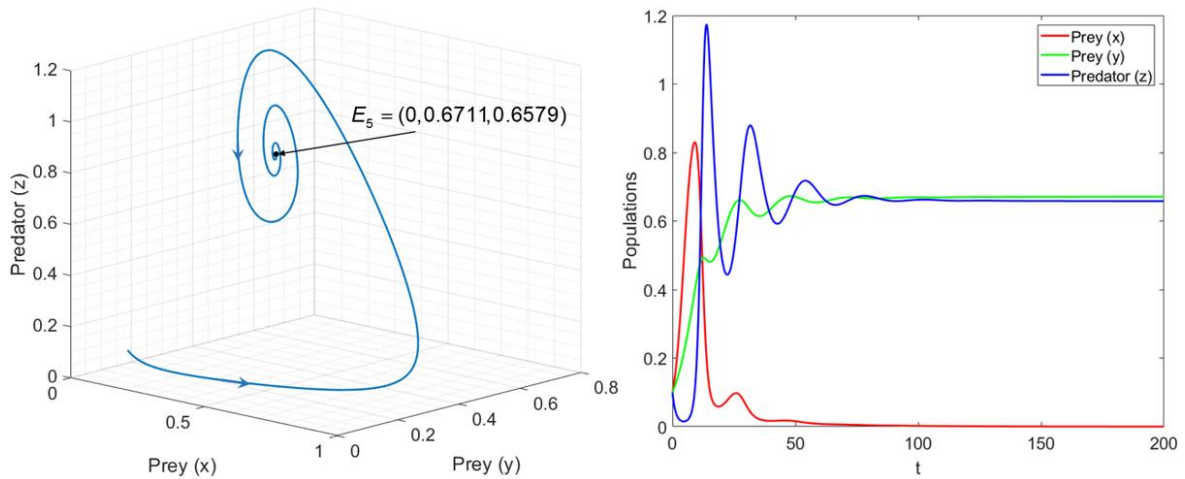
Figure 4. The phase plane and time series plot (Simulation 2) when the intensity of predator attacks on primary prey $(c = 0.8)$ are greater than secondary prey $(d = 0.1)$.

In the third scenario (Simulation 3), the intensity of predator attacks on primary prey $(c = 0.5)$ is lower than that on secondary prey $(d = 0.65)$. Similar to Simulation 1, the phase plane in Figure 5 illustrates the solution curve moving towards the asymptotically stable spiral equilibrium point $E_7 = (0.6977, 0.0175, 0.3023)$. The time-series plot indicates coexistence among all species, with populations initially exhibiting fluctuations before stabilising over time. This ecological resemblance to Simulation 1 highlights the system's resilience in maintaining stability in an environment of varying predator attack intensities.
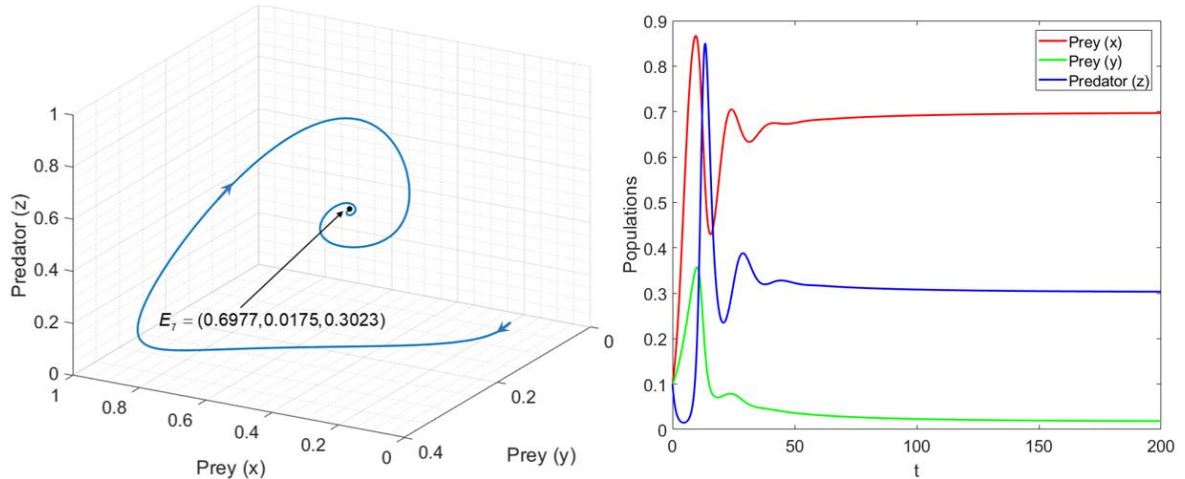

Figure 5. The phase plane and time series plot (Simulation 3) when the intensity of predator attacks on primary prey $(c = 0.5)$ are lower than secondary prey $(d = 0.65)$.

In the final scenario (Simulation 4), predator attacks on the secondary prey are increased from $d = 0.65$ to $d = 0.75$, while predator attacks on the main prey remain at $c = 0.5$. The phase plane in Figure 6 displays an asymptotically stable spiral, directing the trajectory towards $E_6 = (0.7163, 0, 0.2837)$. However, the time series plot reveals that heightened predator attacks on secondary prey may lead to their potential extinction, while primary prey and predator populations persist in a stable state. This outcome emphasises the ecological consequence that excessive targeting of secondary prey may result in their decline or extinction without significantly affecting the primary prey or predator populations.
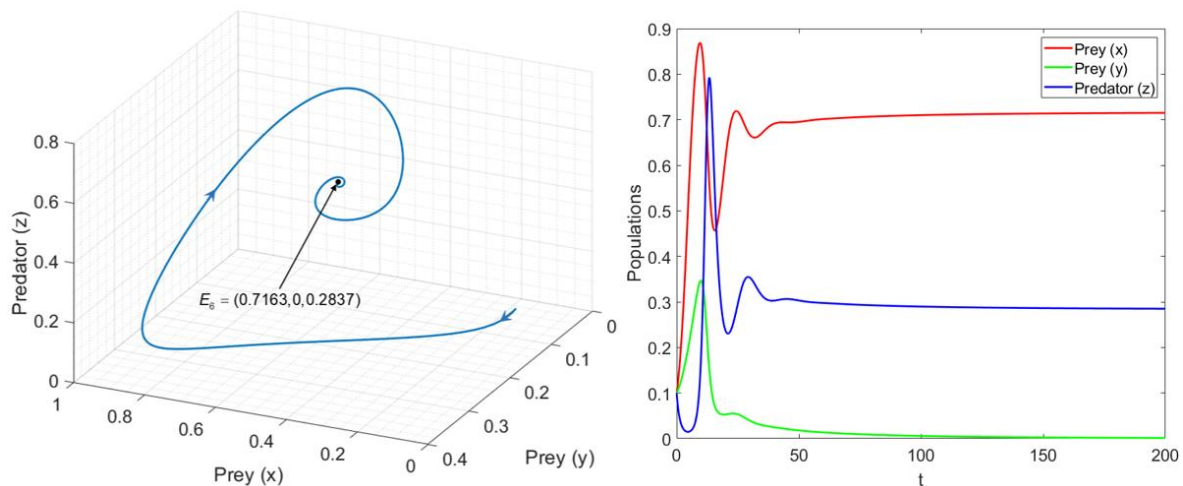
Figure 6. The phase plane and time series plot (Simulation 4) when the intensity of predator attacks on primary prey $(c = 0.5)$ are lower than secondary prey $(d = 0.75)$.

In conclusion, our investigation of the dynamics of prey-predator interactions under varying predator attack intensities on primary and secondary prey revealed significant ecological patterns. Simulation scenarios demonstrated that a balance of predator attack rates on different prey species is crucial for coexistence. Our findings highlight the challenging nature of ecological relationships as well as the importance of a balanced predator-prey dynamic for species stability and coexistence within ecosystems.

5. Conclusion

In summary, our extensive exploration of interactions between prey and predators, encompassing diverse scenarios of predator attacks on primary and secondary prey, has yielded valuable insights into the dynamics of ecological systems. Throughout the research, we formulated a simple three-species prey-predator system, conducted stability analyses, and validated our analytical findings through numerical simulations. Our results underscore the pivotal role played by the intensity and distribution of predator attacks in influencing the coexistence or extinction patterns of prey populations. Excessive predation on primary prey may lead to potential extinction, while a more balanced approach, involving moderate attacks on primary prey and heightened attacks on secondary prey, supports a sustainable coexistence among all species. In contrast to the work of [14], which adopts a collaborative approach involving both primary and secondary prey and emphasises its stabilising effect, our research contributes to the understanding of the intricate interdependencies within predator-prey systems, focusing on scenarios where prey do not engage in collaborative efforts. This highlights the importance of considering multiple factors for a comprehensive understanding of ecological stability. Thus, future investigations could enhance the model's realism by incorporating elements such as intraspecific and interspecific competition in all species, providing a more accurate representation of real-world ecological scenarios.

Conflict of Interest

The authors declare no conflict of interest in the subject matter or materials discussed in this manuscript.

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