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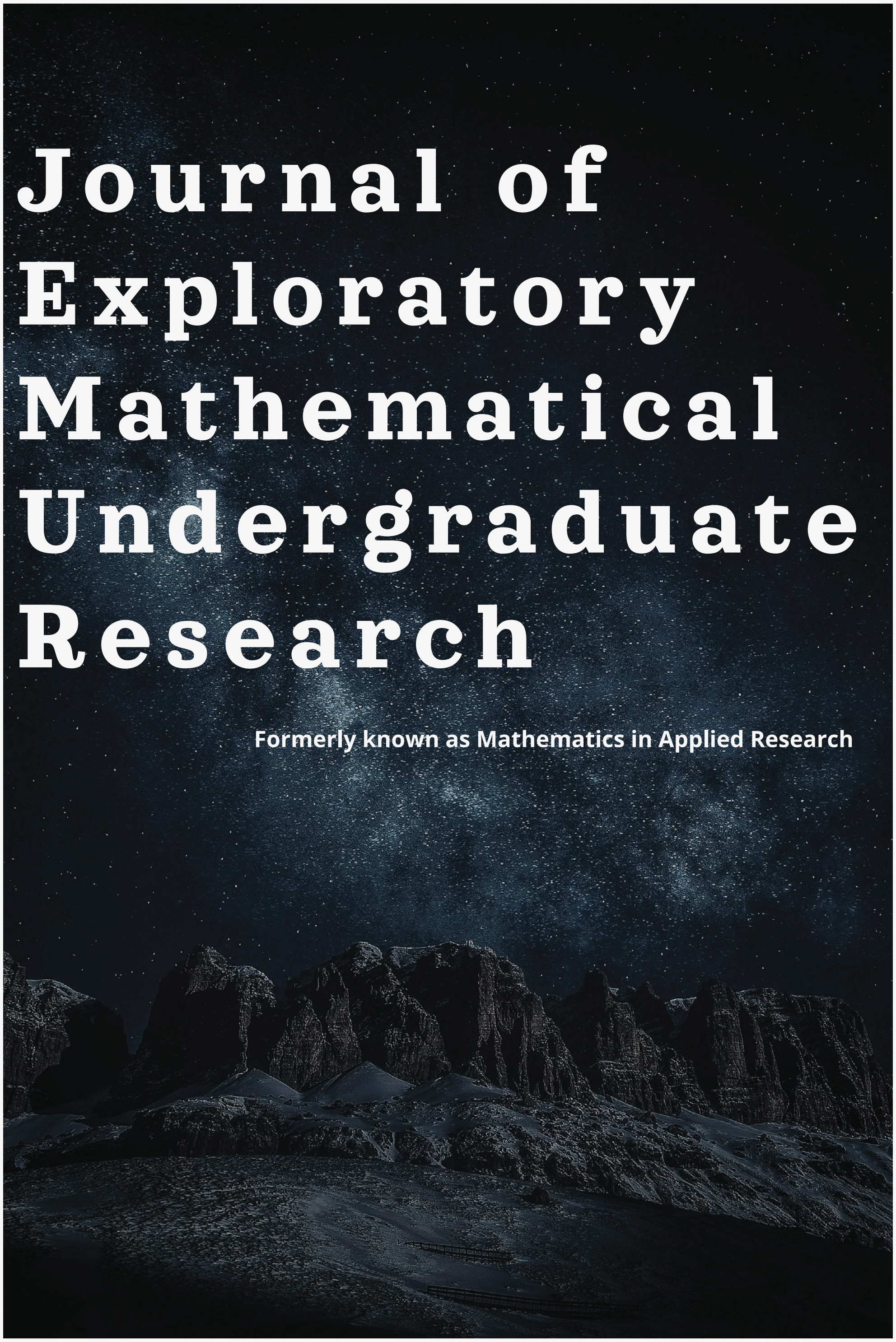

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PORTFOLIO OPTIMIZATION OF MALAYSIAN ASSETS WITH ADDITIONAL CARDINALITY CONSTRAINT

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Abstract

The main goal of this study is to identify the most effective approach for distributing funds, considering the influence of limitations on the number of assets (cardinality constraints) on reducing risk within a portfolio. We utilise the mean-variance model to evaluate the link between the number of assets in a portfolio and the expected risk-return trade-offs. In addition, we analyse the cardinality-constrained mean-variance (CCMV) model to evaluate its influence on risk for different numbers of investments. We utilise mean-variance analysis to determine the most optimal distribution of investments, with the CCMV model imposing limitations on the number of assets we can invest in and the size of each investment. The Sharpe ratio is employed to evaluate the performance of these models under different constraints, with a higher ratio indicating improved performance. In conclusion, this study emphasises the importance of maintaining a balanced relationship between the potential risks and rewards of investment.

Keywords: Cardinality constraint, investment, mean-variance model portfolio optimization, return, risk, Sharpe ratio

1. Introduction

Investment is defined as assets that have been acquired or invested in to develop wealth and save money from hard-earned income or appreciation. Stocks, bonds, and mutual funds are the several types of assets that can be invested in, and the investment goal is to generate the highest possible profit while minimizing the risk of loss. To provide investors with detailed information on profitable investments, an investment portfolio was introduced. The seminal paper of Markowitz (1952) introduced one of the most important problems in finance and investment, portfolio optimization. Should any asset in the portfolio be associated with an extremely high or low historical return, the optimization often finds a corner solution regarding the assets (Auh and Cho, 2023). A portfolio selection problem based on Markowitz (1952) is as follows:

$$X = \left\{(x_1, \cdots, x_n) | \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j = 1, \cdots, n\right\}$$

A portfolio choice, $x = (x_1, \cdots, x_n)$ represents the percentage of capital invested in the different asset j of the total number of the assets, n where the portfolio weights must be nonnegative numbers and the total equals to 1. Let x_j be the percentage of capital invested in asset j, where asset $j \in \{1, \cdots, n\}$ having a return R_j at the end of the investment period. As the future price of the asset is unknown, R_j is a random variable. Therefore, the random variable of portfolio's return is $R_x = x_1R_1 + \cdots + x_nR_n$. To determine the portfolio selection, another portfolio is considered as $R_y = y_1R_1 + \cdots + y_nR_n$ with different returns. The problem arises in choosing between two portfolios x and y in terms of a random variable R_x and R_y. Therefore, this is where the models are required to choose the best portfolio. The portfolio optimization model's aim will briefly define the preference relation between random variables.

Markowitz (1952) identified the trade-off faced by the investors as risk versus expected return and proposed variance as a measure of risk. The mean-risk model is a method used to select among random variables. An efficient portfolio is identified by solving an optimization problem where the risk is minimized, given a constraint on the expected return. By adjusting the level of expected return, we can derive various efficient portfolios. The primary goal of this study is to focus on the cardinality restrictions in order to limit the number of assets included in the portfolio optimization issue. This study employs the quadratic programming approach to address the problem of portfolio optimization with cardinality constraints.

A closely related area of research concerns financial scenario generation, this is about simulating future values for asset prices or returns to serve as parameters in optimization models (Maasar et al., 2022). The investment aims to have maximum returns with minimum risk. Therefore, the trade-off between risk and return is a vital issue for the investor (Yaman and Dalkılıç, 2021). Most recent studies have focused on the provision of portfolio optimization with cardinality constraints. For instance, according to Jimbo et al. (2017)) and Kobayashi et al. (2023), they mainly focused on cardinality constraint as the main method for portfolio optimization. It is said that the cardinality constraint is used to limit the number of invested assets when there are a large number of assets in the portfolio. According to Jimbo et al. (2017), the problem arises when using traditional numerical approaches in portfolio optimization. Thus, genetic algorithms as one of the populations of the stochastic search algorithms are required to solve the complex optimization problems. To sum up, diversifying the number of assets in a portfolio potentially increases the expected return but with varying levels of reliability.

Kobayashi et al. (2023) addressed the issue of large asset numbers by employing the mixed-integer second-order optimization (MISDO) approach to handle cardinality constraints. The cutting plane algorithm was utilized to solve the mixed-integer convex optimization problem while considering logical constraints. Furthermore, with cardinality constraints, a moment-based distributionally robust portfolio optimization model was presented, and a matrix completion-based problem reduction technique greatly sped up the cutting plane algorithm. For the distributionally robust portfolio optimization, this strategy produced better out-of-sample Sharpe ratios.

The studies for portfolio optimization have been done by Fu et al. (2023) and Khodamoradi et al. (2021). Commonly used method of metaheuristics to solve cardinality-constrained mean-variance (CCMV) in portfolio optimization problems because it offers a simpler and easier-to-implement approach that requires less computational time but these methods, though not guaranteed to find the optimal solutions can often find good solutions within a reasonable timeframe (Khodamoradi et al., 2021).

According to a case study by De Melo et al. (2022), to handle the cardinality constraint the genetic algorithms are adapted to control the number of assets in each portfolio and respect the prediction horizon perspective. The use of genetic algorithm operators within the prediction horizon perspective is an innovative approach that considers the prediction and constraints at each time step. In the studies that have been done by Yaman and Dalkılıç (2021), heuristic algorithms offer approximate solutions but cannot guarantee optimality within a reasonable time frame. To address these problems, it developed genetic algorithms. In the context of portfolio optimization, genetic algorithms applied to the unconstrained portfolio optimization problem, when downside risk was used as a measure. Overall, genetic algorithms are a powerful heuristic approach commonly used for portfolio optimization problems, providing solutions within a reasonable computational time.

In conclusion, a variety of models have been used to solve different kinds of case studies involving portfolio optimization with cardinality constraints. The main model used in this paper to solve the cardinality constraint and produce the optimal portfolio for risk-averse investors is the quadratic programming approach. Thus this study will embark on attaining the three objectives: 1) To analyze the correlation between the diversification of risk through the number of assets in the portfolio and the level of expected return for the mean-variance model; 2) To analyze the diversification of risk of portfolio based on different numbers of assets in cardinality constrained mean-variance model; and 3) To determine the best portfolio optimization model in terms of Sharpe ratio between mean-variance and cardinality-constrained mean-variance model.

2. Methodology

2.1. *Data Collection*

The data on the Malaysia stock market index is taken from the Financial Time Stock Bursa Malaysia Kuala Lumpur Composite Index (FBMKLCI). The data consists of the stock market index of 30 Malaysian companies with an investment period of one-month closing prices from 06/12/2010 to 06/10/2023.

2.2. *Portfolio Optimization of Mean Variance Model*

In portfolio optimization problems, a model that has been widely used to determine the best portfolio is the mean-risk model. The model considers two scalars that is expected value (mean) and risk measure value. In this model, the portfolio is optimized by minimizing risk with subjet to some specified target returns (Maasar et al., 2022). The efficient portfolios are obtained by solving optimization problems that minimize risk while aiming for a desired expected return that can be formulated as

$$\begin{aligned} \min \quad & \rho(R_x) \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i = 1 \\ & E(R_x) \geq d \\ & x_i \geq 0 \\ & i = 1, 2, \cdots, n \end{aligned} \tag{1}$$

where ρ represents the risk measure (variance), x_i's are the the components of vector X as stated in Section 1, R_x are the portfolio returns, and d represents the specified target returns (decided by the investors).

Based on Equation 1 where the objective is to minimize risk, the constraints were added to keep assets in the portfolio well-distributed with non-negative values. The desired expected return is added to the constraint in $E(R_x) \geq d$. The expected return of the portfolio $X = (x_1, x_2, \cdots, x_n)$ is given by

$$mean, E(R_x) = x^T R = \sum_{i=1}^{n} r_i x_i$$

The risk measure that will be used in this study is the variance measure. The variance was the first risk measure that was introduced by Markowitz (1952) and created a mean-variance approach (Maasar et al., 2022). The variance of random variable R_x denoted by $\sigma^2(R_x)$. The model of portfolio optimization by Markowitz is to minimize the variance by aiming for the lowest correlation between random variables. By calculating the variance of a linear combination of random variable

$$\sigma^2(aR_1 + bR_2) = a^2\sigma^2(R_1) + b^2\sigma^2(R_2) + 2abCov(R_1, R_2)$$

Where R_1 and R_2 are random variable, a and b are real numbers, and $Cov(R_1, R_2)$ is the covariance of R_1 and R_2 (Chou et al., 2017).

The variance of portfolio return is given by;

$$risk = \sigma^2(R_x) = \sum_{i=0}^{n} \sum_{j=0}^{n} x_i x_j \sigma_{ij}$$

Where, σ_{ij} is covariance between R_i and R_j and $i, j = 1, 2, \cdots, n$.

The covariance matrix of stock return is $S = (\sigma(R_i, R_j))_{n\times n}$. Therefore, the mean-risk model is given by;

$$
\begin{aligned}
\min \quad & risk = x^T S x = \sum_{i=0}^{n} \sum_{j=0}^{n} x_i x_j \sigma_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_i = 1 \\
& \sum_{i=1}^{n} r_i x_i \geq d \\
& x_i \geq 0 \\
& i, j = 1, 2, \cdots, n
\end{aligned}
$$

Where d is the target or desired level of expected return that will be determined by the decision maker.

2.3. *Portfolio Optimization of Cardinality Constraint Mean-Variance Model*

By incorporating the cardinality restriction into the portfolio, the portfolio optimisation model transforms into a mixed-integer quadratic programming problem. Therefore, it is necessary to employ quadratic programming in order to solve the problem using MATLAB.

A function of quadratic programming is created using the 'quadprog' optimization tool that takes the value of average expected return, assets covariance, target expected return, and cardinality size of the portfolio. In the function, the objective function is set to minimize the value of the standard deviation of the portfolio subject to constraints available. The constraints added to the objective function are budget constraint, return constraint, cardinality constraint, and non-negative constraint. The function then returns for weight of the portfolio after the objective function is minimized to each constraint.

Using the weight of the portfolio obtained from the function created, the expected return and standard deviation of cardinality constrained portfolio are calculated. To observe for the performance of the cardinality-constrained portfolio, the Sharpe ratio is calculated. The portfolio Sharpe ratio determines the percentage of return of the portfolio over the risk after the risk-free rate. The rate of risk-free rate is set to be 0% based on additional risk-neutral investors as the target of this study. The optimal portfolio with the highest value Sharpe ratio will be determined as the most optimal portfolio with the maximum return with the lowest risk. The Sharpe ratio is given by

$$\text{Sharpe Ratio} = \frac{return - rf}{risk}$$

where rf is the risk-free rate

The code will loop for $i = 1, 2, 3$ to obtain the value of standard deviation for cardinality constrained portfolio for every level of target return assigned that is 1 for low target, 2 for medium target, and 3 for high target. For every iteration, the value of portfolio weight, expected return, standard deviation, Sharpe ratio, and bar plot of portfolio weight is stored in the array. These values and plots will finally be displayed as output before the program ends.

3. Results and Discussions

The dataset for this analysis is derived from the FBMKLCI, which includes the stock market indices of 30 Malaysian companies. The investment period spans 13 years, from December 6, 2010, to October 6, 2023, and includes monthly closing prices.

For the analysis, the dataset comprises 130 time periods, beginning on December 6, 2012, and concluding on September 6, 2023. Each in-sample dataset is constructed by appending data for the subsequent month, ensuring a consistent total of 100 time periods across all in-sample datasets.

The portfolio is evaluated over the in-sample data using both the mean-variance and cardinality constrained mean-variance models. This approach allows for a comprehensive analysis of the portfolio's performance and risk characteristics.

3.1. *Analysis of Mean-Variance Portfolio*

Table 1: The number of assets and risk level (standard deviation) for every level of return for Mean-Variance model

Sample No.	No. of Assets			Standard Deviation		
	Low	**Medium**	**High**	**Low**	**Medium**	**High**
Sample 1	9	9	7	0.0206	0.0225	0.0255
Sample 2	11	9	8	0.0202	0.0219	0.0244
Sample 3	12	11	8	0.0210	0.0225	0.0252
Sample 4	12	11	9	0.0219	0.0226	0.0253
Sample 5	12	11	9	0.0210	0.0226	0.0253
Sample 6	12	12	9	0.0210	0.0223	0.0247
Sample 7	13	13	10	0.0213	0.0228	0.0252
Sample 8	12	12	9	0.0211	0.0221	0.0242
Sample 9	12	12	10	0.0210	0.0226	0.0249
Sample 10	12	12	10	0.0209	0.0224	0.0248

Based on Table 1, we conduct an optimization on 10 datasets using the mean-variance model approach. The performance of each portfolio is examined for low, medium, and high in-sample returns using the standard deviation. Additionally, we analyze the behavior of the number of assets to be included for each in-sample return.

The results of the portfolios for each in-sample return demonstrate the behavior of the number of assets to be included in the portfolio. For instance, Sample 1 revealed a decreasing trend in the number of assets to be included in the portfolio, with counts of 9, 9, and 7 as the target expected return increased from low (0.0053) to medium (0.0077) and high (0.010), respectively. A similar trend of decreasing asset count with increasing targeted expected return was observed across all samples.

Moreover, the analysis also revealed an increase in the standard deviation as the targeted expected return increased. For example, in Sample 1, the standard deviation of the portfolio increased from 0.00206 to 0.0225 and 0.0255 as the targeted expected return increased. This pattern of increasing standard deviation with increasing targeted expected return was consistently observed across all samples. This significant pattern suggests a trade-off between the expected return and the risk of the portfolio.

The analysis of Table 1 shows the correlation between the standard deviation and the quantity of assets included in the portfolio for each sample, considering different targeted expected returns. As the desired return for each sample increases, the variability of the portfolio's returns also increases, but the number of assets in the portfolio reduces.

The primary reasons for these observations are twofold: As the targeted expected return increases, the number of assets decreases; and when the targeted expected return increases, the standard deviation of the portfolio also increases.

Diversification plays a pivotal role in managing the overall risk of portfolios by investing in a variety of businesses, thereby minimizing financial risk. For instance, as the targeted expected return escalated from low (0.0053) to high (0.010) in Sample 3, the analysis exhibited a decrease in the number of assets to be included. Consequently, the behavior of the standard deviation (risk) was observed to increase, indicating that risk is diversified to manage the overall risk of the portfolio. A key strategy to accomplish this goal is through diversification, which involves carefully selecting assets with desirable long-term returns while ensuring a relatively low correlation among them. Therefore, it is consistent with the in-sample portfolio.

This realization underscores the dynamic relationship between expected return and associated risk (standard deviation), providing a critical perspective for risk management and decision-making. A well-diversified portfolio can manage risk more effectively than an individual asset portfolio, even though the risks associated with individual assets may increase with higher expected returns. This crucial insight from current portfolio theory offers a vital perspective for decision-making in portfolio management and risk optimization.

3.2. *In-Sample Analysis of Cardinality Constraint Mean-Variance Portfolio*

This section elucidates the computational results of the Cardinality Constrained Mean-Variance (CCMV) portfolio optimization, achieved through the application of the quadratic programming approach. The same in-sample return values, namely low (0.0053), medium (0.0077), and high (0.01), were utilized to ascertain the CCMV portfolio.

Three cardinality constraint setups are available with $K = 3, 5, 7$. These cardinality constraints have been implemented to restrict the number of assets included in the portfolio, thereby providing a more focused and manageable portfolio structure. This approach ensures that the portfolio is not overly diversified, which can lead to dilution of returns and increased complexity in portfolio management.

Table 2: Risk level (Standard Deviation) for Cardinality Constraints Mean-Variance model

Sample No.	$K = 3$			$K = 5$			$K = 7$		
	Low	**Medium**	**High**	**Low**	**Medium**	**High**	**Low**	**Medium**	**High**
Sample 1	0.0253	0.0254	0.0260	0.0216	0.0234	0.0265	0.0208	0.0230	0.0256
Sample 2	0.0249	0.0250	0.0256	0.0214	0.0220	0.0256	0.0206	0.0220	0.0245
Sample 3	0.0246	0.0267	0.0273	0.0227	0.0237	0.0268	0.0216	0.0224	0.0257
Sample 4	0.0248	0.0268	0.0274	0.0229	0.0240	0.0269	0.0217	0.0226	0.0256
Sample 5	0.0247	0.0267	0.0272	0.0223	0.0239	0.0268	0.0217	0.0226	0.0256
Sample 6	0.0245	0.0270	0.0272	0.0223	0.0241	0.0265	0.0214	0.0226	0.0252
Sample 7	0.0249	0.0268	0.0271	0.0224	0.0242	0.0267	0.0218	0.0234	0.0260
Sample 8	0.0253	0.0272	0.0294	0.0226	0.0250	0.0274	0.0219	0.0249	0.0265
Sample 9	0.0254	0.0270	0.0290	0.0222	0.0247	0.0271	0.0216	0.0233	0.0257
Sample 10	0.0252	0.0266	0.0289	0.0221	0.0241	0.0266	0.0215	0.0231	0.0256

Based on Table 2, the analysis reveals the behavior of the standard deviation in the portfolio optimization model under the implementation of cardinality constraints. The analysis indicates that the standard deviation of every portfolio, from Sample 1 to Sample 10, decreases as the number of assets included increases for a low target return. This pattern of decreasing risk is consistent across all samples, as indicated in columns $K = 3, 5, 7$.

This trend of decreasing risk with an increasing number of assets is also maintained for other target returns, such as medium and high targets. The decrease in risk can be attributed to the spread of risk across multiple assets. Diversification of the portfolio, achieved by investing in a variety of assets, mitigates risk by spreading investments across different companies or sectors.

In this approach, even if a single investment or a few investments underperform, the remaining ones in the portfolio can potentially compensate for the losses. This strategy underscores the importance of diversification in portfolio management, where the goal is not only to maximize returns but also to manage and minimize risk effectively.

3.3. *Performance Analysis*

Table 3: Performance Analysis of Portfolio Optimization

Target Return	**Performance Sharpe Ratio**			
	M-V	$K = 3$	$K = 5$	$K = 7$
Low Target Return (0.0053)	0.2532	**0.2999**	0.2750	0.2680
Mid Target Return (0.0077)	0.3439	0.3307	0.3259	**0.3523**
High Target Return (0.010)	0.4036	0.4203	0.4205	**0.4152**

The Sharpe ratio analysis is a tool used for evaluating portfolio performance across various levels of return. Table 3 presents the Sharpe ratio for different models, including the mean-variance model and the Cardinality Constraints Mean-Variance (CCMV) model with $K = 3, 5, 7$.

A high Sharpe Ratio is generally considered desirable as it indicates a larger return-to-risk differential (Cogneau and Hübner, 2009). However, it's important to note that a higher Sharpe ratio does not necessarily mean a model is superior. It simply provides one perspective on the trade-off between risk and return.

For the low target return (where $d \geq 0.0053$), the CCMV model with $K = 3$ has a Sharpe value of 0.2999. For the medium ($d \geq 0.0077$) and high target return ($d \geq 0.0100$), the CCMV model with $K = 7$ has the highest Sharpe values of 0.3523 and 0.4152, respectively.

These results suggest that the CCMV model tends to have a higher Sharpe ratio than the mean-variance model at low and high target returns for the given levels of K. However, this does not imply that the CCMV model is universally better or worse than the mean-variance model. The choice of model should be based on the specific needs and risk tolerance of the investor.

This analysis underscores the importance of considering both return and risk in portfolio optimization, and the role of the Sharpe ratio as a key metric in this process. It's crucial to remember that these results are specific to the data and period analyzed, and may not hold true under different market conditions or with different datasets.

4. Conclusion

In this research, the Cardinality-Constrained Mean-Variance (CCMV) portfolio optimization model is formulated and solved using a quadratic programming approach. The return analysis establishes a spectrum of low, medium, and high returns to mitigate the portfolio investment risk. Cardinality constraints are set for $K = 3, 5$, and 7 to limit the number of assets included in the portfolio. The Sharpe ratio is employed to evaluate the performance of each portfolio at various return levels, aiding in the determination of the optimized portfolio. The study reveals that limiting the number of assets in the portfolio leads to significant variations in both the expected return and the investment risk. Furthermore, refining the decision-making process allows investors to allocate their wealth and resources for investment purposes more effectively.

In conclusion, this study provides valuable insights into portfolio management and offers guidance for investors in the Malaysian market. It systematically investigates three primary objectives. Firstly, it examines the correlation between risk diversification, as determined by the number of assets in the portfolio, and expected returns in a mean-variance model. The correlation analysis concludes that an efficient tool for determining the optimal portfolio yields the highest expected return for a given level of risk. Secondly, it analyzes the risk diversification of a portfolio based on different numbers of assets in a cardinality-constrained mean-variance model. The number of assets in the portfolio is a crucial factor in risk diversification in this model. The more diversified the portfolio, the lower the risk. Lastly, it identifies the superior portfolio optimization model in terms of the Sharpe ratio between the mean-variance and the mean-variance cardinality constraint model. Based on the comparison of the Sharpe ratio of the mean-variance and the mean-variance cardinality constraint model, the model with the highest Sharpe ratio is deemed the optimal portfolio optimization model. These findings offer valuable insights into the relationship between risk, diversification, and portfolio optimization, contributing to a more comprehensive understanding of effective investment strategies.

This comprehensive study enhances investors' understanding of portfolio behavior, enabling them to make informed decisions to optimize their portfolios for maximum returns while effectively managing risk. By employing a CCMV model and focusing on Malaysian assets, the most advantageous portfolios were identified.

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