

**NUMERICAL SOLUTION OF WILLIAMSON HYBRID  
FERROFLUID OVER A STRETCHING SHEET**

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## ABSTRACT

Non-Newtonian Williamson hybrid ferrofluid has become the topic of interest among researchers due to the shear thinning properties that can represent real life application such as medical industry. Hybrid ferrofluid are also considered because of higher heat transfer rather than single domain ferrofluid. This research aims to discover the flow of non-Newtonian Williamson hybrid ferrofluid along the stretching sheet. Partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs) by using similarity transform variable. The transformed ODEs are solved numerically by using mathematical algorithm encoded in Maple software. The effect of velocity and temperature profiles are discussed for several parameters such as magnetic parameter, stretching parameter, non-Newtonian Williamson fluid parameter, Prandtl number, and local Eckert number are also discussed. The values of skin friction coefficient are summarized and compared with earlier published research articles to authenticate the obtained numerical algorithm. It is discovered that the velocity profile increases as the value of magnetic and stretching parameter are increased. On the other hand, the temperature profile increases because of the increase in magnetite and copper nanoparticles volume fraction, stretching parameter, non-Newtonian fluid parameter and Eckert number.

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