

**NUMERICAL SOLUTION TO SOLVE INITIAL VALUE
PROBLEMS (IVPs) OF FIRST-ORDER ORDINARY
DIFFERENTIAL EQUATION USING SINGLE-STEP AND
MULTISTEP METHODS**

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ABSTRACT

The numerical solution can be used to estimate the solution to several forms of ordinary differential equations (ODEs) through repeated iterations. Numerical solutions are also extremely crucial in scientific computation since they are broadly utilized to depict actual or real-world issues. Consequently, it is sufficient to determine the numerical approximation, which allows us to implement the various techniques to solve the differential equation numerically. The aim of applying the numerical solution is the desire to have the exact solution as possible. Moreover, the numerical solution with a high level of precision is commonly used to obtain the exact solution for the problem. Therefore, this project implements the single-step methods and the multistep methods for solving initial value problems of first-order ordinary differential equations. For the single-step method, the methods are EM, MEM, and RK4. Meanwhile, AB2, AB3, and AB4 explicit methods are used for the multistep method. Thereby in this project, the single-step RK4 has the highest level of accuracy for approximating the solutions of every IVPs. In the meantime, for each method used, the approximated solution approaches to the exact solution as the step size decreases. The smaller step size, $h=0.0125$ resulted in more preciseness for approximated solutions. Lastly, for the multistep method, a better approximation is obtained at a higher step of Adams-Bashforth, which is AB4.

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