IMPROVED FAULT ANALYSIS TOOLS IN UNBALANCED DISTRIBUTION NETWORKS

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Abstract- This paper addresses a new approach for a short-circuit analysis algorithm for radial three-phase distribution networks, based on two relationship matrices method, the bus-current-injection-to branchcurrent matrix (BIBC) and branch-current-to busvoltage matrix (BCBV). Both matrices developed from the topological structures of distribution systems are used to analyze the variations of bus voltages, bus-current injections and branch currents under fault conditions. A short-circuit-analysis method can then be developed from these two matrices and be used to solve the various types of single or simultaneous unsymmetrical faults by using only one model. This model consists of four impedances that their values can be varying from zero to extreme so that each kind of unsymmetrical faults can be simply modeled. Since the proposed method does not use the traditional admittance matrix, the time-consuming procedures such as trifactorization or inverse of admittance matrix are not necessary. Simulation of IEEE34-bus network and the results obtained such as phase fault currents and voltage profiles are presented and discussed.

Keywords— Short circuit analysis method, Unsymmetrical fault, Unbalanced Distribution networks, Four impedances model, BIBC, BCBV.

1 INTRODUCTION

Overcurrent protective devices, such as circuit breakers and fuses, should isolate faults at a given location safely with minimal circuit and equipment damage and minimal disruption of the plant's operation. Other parts of the system, such as cables, busways, and disconnecting switches, shall be able to withstand the mechanical and thermal stresses resulting from maximum flow of short-circuit current through them. The magnitudes of short-circuit currents are usually estimated by calculation, and equipment is selected using these results. Also as the distribution system becomes more heavily loaded and the need an ability to reconfigure the system for service restoration, load balancing and loss reduction grows; the network configuration will be changed more frequently. With each change protection device settings in the system may need updating. Therefore, there is a need for fast and more accurate short circuit calculations [1]. For this reasons, this paper come out with the mitigation technique on short-circuit analysis for radial three phase distribution network, based on two relationship matrices method i.e. the bus-current-injection-to-branch current matrix and the bus-current-to-bus-voltage matrix. These two matrices can be accomplished by simple building algorithms and easily implemented. Basically both matrices can be employed to solve the various types of unsymmetrical faults analysis include single line (SLG), double line-to-ground (DLG), line-to-line (L-L) and three line-to-ground by using only one model. This model is consists of four impedances which can be set from zero to extreme value in order to suit with each kind of unsymmetrical faults. Previous method, admittance matrix is being applied traditionally in one model for fault calculations. Also compensation method had been introduced before which short circuit currents are calculated by omitted the loads [2].

In three phase power, an unbalanced fault is a fault which does not affect each of the three phases equally. This is in contrast to a symmetrical fault, where each of the phases is affected equally. In practice, most faults in power systems are unbalanced; however, as unsymmetrical faults are difficult to analyze, analysis of this faults is built up from a thorough understanding of symmetrical components. The main computational advantage of the conventional symmetrical component method is that the three sequence matrices are treated separately. For this reason, fault analysis based on this method has remained popular for several decades. However, since distribution feeder circuit configurations are basically unbalanced, three-phase short circuit analysis in its actual a-b-c phase representation has become more accurate. Thus, this method is not well suited anymore [3].

From that aspect, several short-circuit analysis methods based on phase co-ordinates, hybrid compensation method and three-phase impedance matrix have been proposed[4]-[6]. In this paper a short circuit analysis algorithm for radial three-phase distribution networks is presented. The three-phase short circuit analysis, in its actual a-b-c phase representation, can be applied to balanced or unbalanced distribution systems.

2 THEORETICAL BACKGROUND

2.1 Unbalanced Three-Phase Line Model

Figure 1 shows a three-phase line section between bus i and j. A 4x4 matrix, which takes into account the self and mutual coupling effects of the unbalanced three-phase line section, can be expressed as shown in (1)

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix}$$
(1)



Figure 1 Three Phase line system

After Kron's reduction is applied, the effects of the neutral or ground wire are still included in this model and (1) can then be rewritten as equation (2)

$$[Z_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix}$$
(2)

For single-phase and two-phase line sections, the 3×3 impedance matrix will be characterized by a row and column of zeros occupying the position of the missing phase or phases. The relationship between bus voltages and branch currents in Fig. 1 can be expressed as shown in (3)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_c \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \\ I_{Cc} \end{bmatrix}$$
(3)

2.2 Algorithm Development

According to radial structures of distribution feeders, two relationship matrices, the bus-current-injection-to branch-current matrix (**BIBC**) and branch-current-to bus-voltage matrix (**BCBV**) were derived and used to observe the network relationship [7]. In this section, the development procedure will be described in detail [8]. For distribution networks, the equivalent-current-injection based model is more practical. For bus *i*, the complex load S_i is expressed by

$$S_i = (P_i + Q_i) \quad i = 1 \dots N$$
 (4)

And the corresponding equivalent current injection at the k-th iteration of solution is

$$I_i^k = I_i^r (V_i^k) + j I_i^i (V_i^k) = \left(\frac{P_i + Q_i}{V_i^k}\right)^*$$
(5)

where V_i^k and I_i^k are the bus voltage and equivalent current injection of bus at the *k*-th iteration, respectively. I_i^r and I_i^i are the real and imaginary parts of the equivalent current injection of bus *i* at the *k*-th iteration, respectively.



2.2.1 Relationship Matrix Development

A simple distribution system shown in Fig. 2 is used as an example. The power injections can be converted to the equivalent current injections by (5), and the relationship between the bus current injections and branch currents can be obtained by applying Kirchhoff's Current Law (KCL) to the distribution network. The branch currents can then be formulated as functions of equivalent current injections. For example, the branch currents B_1, B_3, B_5 can be expressed by equivalent current injections as

$$B_1 = I_2 + I_3 + I_4 + I_5 + I_6$$

$$B_3 = I_4 + I_5$$

$$B_5 = I_6$$
(6)

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Therefore, the relationship between the bus current injections and branch currents can be expressed as

$$\begin{bmatrix} B_1\\ B_2\\ B_3\\ B_4\\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2\\ I_3\\ I_4\\ I_5\\ I_6 \end{bmatrix}$$
(7a)

Equation (7a) can be expressed in general form as

$$[B] = [BIBC][I] \tag{7b}$$

where BIBC is the bus-injection to branch-current (*BIBC*) matrix. The constant *BIBC* matrix is an upper triangular matrix and contains values of 0 and +1 only.

The relationship between branch currents and bus voltages as shown in Fig. 2 can be obtained by (3). For example, the voltages of bus 2, 3, and 4 are

$$V_2 = V_1 - B_1 Z_{12} \tag{8a}$$

$$V_3 = V_2 - B_2 Z_{23} \tag{8b}$$

$$V_4 = V_4 - B_3 Z_{34} \tag{8c}$$

where V_i is the voltage of bus i, and Z_{ij} is the line impedance between i bus and j bus. Substituting (8a) and (8b) into (8c), (8c) can be rewritten as

$$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34} \tag{9}$$

From (9), it can be seen that the bus voltage can be expressed as a function of branch currents, line parameters, and the substation voltage. Similar procedures can be performed on other buses; therefore, the relationship between branch currents and bus voltages can be expressed as

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix}$$
(10a)

Equation (10a) can be rewritten in general form as

$$[\Delta V] = [BCBV][B] \tag{10b}$$

where BCBV is the branch-current to bus-voltage (BCBV) matrix [8]. The detail building formulation development can be found in [9].

2.3 Four Impedance Model

Most of faults that occur on distribution system are unsymmetrical faults. Unsymmetrical faults include single line (SLG), double line-to-ground (DLG), lineto line (L-L) and three line-to-ground etc. Usually, short circuit analysis methods for calculation fault parameter use separated models for any type of faults that increase the volume of calculations and memory requirements. In this paper any types of unsymmetrical faults is simulated by only one model that called four impedance model [10]. As Figure 3 shown, this model consists of four variable impedances that three impedances are on three phases and another is located between phases and ground.



Figure 3 Four Impedance model

These impedances can be set from zero to extreme. Therefore, any type of unsymmetrical faults can be modeled by setting these impedances suitably and simulated easily.

Table1	Configuration	setting of	four	impedances

Types of Fault	Zn	Zn	Z _{f3}	Zra
SLG	0	1e12	1e12	0
L-L	le12	0	0	1e12
DLG	1e12	0	0	0
3-phase	0	0	0	0

3 METHODOLOGY

According to proposed four impedance model, when a fault occurs on bus k as shown in fig.3, the current boundary conditions can be written as:

$$I_k^a = I_{k,f}^a, \quad I_k^b = I_{k,f}^b, \quad I_k^c = I_{k,f}^c$$
(11)
where $I_{k,f}^a, \quad I_{k,f}^b, \quad I_{k,f}^c$ are the current phases a, b, c

for three phases connected to ground expressed as: $I_{k,f} = I_{k,f}^a + I_{k,f}^b + I_k^c$ (12) where $I_{k,f}$ is total current phases to ground at fault condition. Also the voltage boundary conditions on bus k can be written as:

$$V_{k,f}^{a} = Z_{f1}I_{k,f}^{a} + Z_{f4}I_{k,f}$$

$$V_{k,f}^{b} = Z_{f2}I_{k,f}^{b} + Z_{f4}I_{k,f}$$

$$V_{k,f}^{c} = Z_{f3}I_{k,f}^{c} + Z_{f4}I_{k,f}$$
(13)

where $V_{k,f}^{a}$, $V_{k,f}^{o}$, $V_{k,f}^{c}$ are the voltage phases a, b and c on fault condition

After the fault occurs at bas k, the fault-bus currents will flow and make the bus voltages of phases. Therefore, the voltages variation after the fault can be expressed as:

$$\Delta V_{k,f}^{b} = V_{k,0}^{b} - Z_{f1} I_{k,f}^{b} - Z_{f4} I_{k,f}$$

$$\Delta V_{k,f}^{b} = V_{k,0}^{b} - Z_{f2} I_{k,f}^{b} - Z_{f4} I_{k,f}$$

$$\Delta V_{k,f}^{c} = V_{k,0}^{c} - Z_{f3} I_{k,f}^{c} - Z_{f4} I_{k,f}$$
(14)

where $V_{k,0}^{abc}$ are the pre-fault voltage and $V_{k,f}^{abc}$ are after-fault voltage respectively.

Substituting (11) into (7b), the variation of the branch currents generated by the fault current can be expressed as:

$$\begin{bmatrix} B_f \end{bmatrix} = \begin{bmatrix} BIBC_k^a & BIBC_k^b & BIBC_k^c \end{bmatrix} \begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix}$$
(15)

where $BIBC_k^{abc}$ are the columns vector of BIBC matrix corresponding to three phases of bus k

By substituting (15) into (10b), the variation of the bus voltages by the fault branch expressed as:

$$\begin{bmatrix} \Delta V_{k,f}^{a} \\ \Delta V_{k,f}^{b} \end{bmatrix} = \begin{bmatrix} BCBV_{k}^{a} \\ BCBV_{k}^{b} \\ BCBV_{k}^{c} \end{bmatrix} \begin{bmatrix} BIBC_{k}^{a} \\ BIBC_{k}^{c} \\ BIBC_{k}^{c} \end{bmatrix}^{T} \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix}$$
(16)

where $BCBV_k^{abc}$ are the rows vector of BCBV matrix corresponding to three phases of bus k

Substitute (12),(14) into (16):

$$\begin{bmatrix} V_{k,0}^{a} - Z_{f1}I_{k,f}^{a} - (I_{k,f}^{a} + I_{k,f}^{b} + I_{k}^{c})Z_{f4} \\
V_{k,0}^{b} - Z_{f2}I_{k,f}^{b} - (I_{k,f}^{a} + I_{k,f}^{b} + I_{k}^{c})Z_{f4} \\
V_{k,0}^{c} - Z_{f3}I_{k,f}^{c} - (I_{k,f}^{a} + I_{k,f}^{b} + I_{k}^{c})Z_{f4} \end{bmatrix}$$

$$= \begin{bmatrix} BCBV_{k}^{a} \\
BCBV_{k}^{b} \\
BCBV_{k}^{c} \end{bmatrix} \begin{bmatrix} BIBC_{k}^{a} \\
BIBC_{k}^{c} \end{bmatrix}^{T} \begin{bmatrix} I_{k,f}^{a} \\
I_{k,f}^{c} \\
I_{k,f}^{c} \end{bmatrix}$$

$$= \begin{bmatrix} L_{k}^{aa} & L_{k}^{ab} & L_{k}^{ac} \\
L_{k}^{ba} & L_{k}^{bb} & L_{k}^{bc} \\
L_{k}^{ca} & L_{k}^{cb} & L_{k}^{cc} \end{bmatrix} \begin{bmatrix} I_{k,f}^{a} \\
I_{k,f}^{b} \\
I_{k,f}^{c} \\
I_{k}^{c} \\
I$$

Where as shown in [7],

$$\begin{bmatrix} L_k^{aa} & L_k^{ab} & L_k^{ac} \\ L_k^{ba} & L_k^{bb} & L_k^{bc} \\ L_k^{ca} & L_k^{cb} & L_k^{cc} \end{bmatrix}$$

$$= \begin{bmatrix} BCBV_k^aBIBC_k^a & BCBV_k^aBIBC_k^b & BCBV_k^aBCBV_k^c \\ BCBV_k^bBIBC_k^a & BCBV_k^bBIBC_k^b & BCBV_k^bBCBV_k^c \\ BCBV_k^cBIBC_k^a & BCBV_k^cBIBC_k^a & BCBV_k^cBCBV_k^c \end{bmatrix}$$

$$= \begin{bmatrix} \sum Z_{aa} & \sum Z_{ab} & \sum Z_{ac} \\ \sum Z_{ba} & \sum Z_{bb} & \sum Z_{bc} \\ \sum Z_{ca} & \sum Z_{cb} & \sum Z_{cc} \end{bmatrix}$$
(18)

Equation (17) can be written as:

$$\begin{bmatrix} V_{k,0}^{a} \\ V_{k,0}^{b} \\ V_{k,0}^{c} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} L_{k}^{aa} & L_{k}^{ab} & L_{k}^{ac} \\ L_{k}^{ba} & L_{k}^{bb} & L_{k}^{bc} \\ L_{k}^{ca} & L_{k}^{cb} & L_{k}^{cc} \end{bmatrix} \\ + \begin{bmatrix} Z_{f1} + Z_{f4} & Z_{f4} & Z_{f4} \\ Z_{f4} & Z_{f2} + Z_{f4} & Z_{f4} \end{bmatrix} \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} \\ = \begin{bmatrix} [L] + [Z_{f}] \end{bmatrix} \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix}$$
(19)

Therefore from (10), fault current can be calculated by

$$\begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} = \begin{bmatrix} [L] + [Z_{f}] \end{bmatrix}^{-1} \begin{bmatrix} V_{k,0}^{a} \\ V_{k,0}^{b} \\ V_{k,0}^{c} \end{bmatrix}$$
$$= [Z_{sc}] [V_{k,0}^{abc}]$$
(20)

where Z_{sc} is the matrix related to the fault-bus voltage and current at fault position. Various types of faults can be simulated by changing Z_{f1} to Z_{f4} that represent Z_f matrix and then Z_{sc} matrix respectively. After fault current is calculated, the branch currents and bus voltages caused by fault can be calculated directly using (15) and (16) [9]. The flowchart of programming is shown as Figure 5 in Appendix 1.

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4 RESULTS AND DISCUSSIONS

Different faults conditions have been studied by using MATLAB 7.6 and test network used in case studies is the modified IEEE 34 bus radial distribution network [11][12], Figure 4 as shown in Appendix 2.

Simplifying, the autotransformer 24.9/4.16 kV/kV in the original IEEE 34-bus test feeder is replaced with the line and the network is modeled with the single voltage level. However, the automatic voltage regulator and configuration phasing of each buses are presented here. The following cases are used to demonstrate the proposed short-circuit analysis program:

Case 1: The fault currents and the fault path in network. Single-phase to ground fault is the most popular and threephase fault generally results in maximum short-circuit current. Therefore, in this case, these types of faults on three buses are considered and the results are shown in Table 2.

Bus No.	Single Line (A) A	Three Phase-to –Ground (A)			
		8	b	c	
4	1556	1977	2628	2588	
13	562	718	812	781	
20	309	384	490	483	

 Table2
 Fault currents in three buses of IEEE 34bus network

Table 2 shows that the three phase fault currents are more than single-phase to ground currents in each bus. Also, fault currents for buses that are near to substation are higher. For example, the fault current on bus 4 is higher than buses 13, 20 and 33.

Case 2: The effect of fault on voltage profiles. The voltage profiles for IEEE 34-bus test before fault is shown in Figure 6.



Figure 6 Voltage profiles before fault

Figure 7 shows that this feeder is unbalanced; therefore, an exact three-phase short circuit method can provide more accurate solutions. In this case, the effects of two faults on the voltage profiles are analyzed. These faults are: single line to ground on phase b of bus 4 and a line to line between phase a and phase c that the effects of them are shown in Figure 7 and 8 respectively (referring IEEE 34-buses configuration phasing of bus 4 is BACN).



Figure 7 Voltage profiles of SLG fault on phase b.



Figure 8 Voltage profiles of L-L fault on phase a and c.

Figure 7 shows that the single line to ground faults in an increase in voltage magnitude on the other two phases, especially at the same bus. The amount of voltage rise depends upon the system's X/R ratio and mutual impedances between phases. In Figure 8, the voltages magnitudes of fault phases are decreased but exactly the same after fault due to

5 CONCLUSIONS

This paper introduced a method for performing short circuit analysis in unbalanced distribution networks. Two matrices and a simple model are developed to analyze the variations of bus voltages, bus current injections and branch currents under fault conditions. From the above test results, it can be seen that the algorithm can be used to handle proposed unsymmetrical faults efficiently. As the results have been shown, the fault current magnitudes and the effect of them on voltage profiles depend on type and position of fault to substation. For example, three phases to ground fault have the highest current magnitude, so it effects on whole of feeder more than another types. Also if the fault occurs near to substation, the fault current and its effect on voltage profiles will be more severe. Since the proposed method employs the actual three-phase models for short circuit analysis and does not require building the traditional admittance matrix, fault analysis on unbalanced distribution system will become more simple and efficient.

ACKNOWLEDGMENTS

In the name of Allah, The Most Gracious, Most Merciful and Him alone worthy of all praises. Alhamdulillah, with the permission of Allah S.W.T. this project was successfully done. I would like to express my sincere gratitude to all those people who provided a constant source of help and motivation throughout my work. First of all, I would like to thank to my final year project supervisor, Mr. Mohd Fuad bin Abdul Latip whose patience, inspiration, contribution of precious idea, support and constant guidance throughout a year to help me complete this project. I would also like to thank to my family and friends who provided a great assistance in terms of materials, effort and ideas, directly or indirectly, I would like to dedicate my special thanks for their blessings and encouragement. May all of efforts will be blessed with great rewards from the Almighty Allah.

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Flowchart of programming

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Figure 4 IEEE 34-BUSES