UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

COUNTING ROOTS OF THE POLYNOMIAL SYSTEMS BY USING MIXED VOLUME OF THE NEWTON POLYTOPES

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ABSTRACT

Linear and nonlinear polynomials play crucial roles in many scientific disciplines including mathematics, physics, chemistry, and engineering. Root counting is a fundamental mathematics problem which is open research. In this project, systems of sparse polynomials (many zero coefficients) are being studied with the real solutions. The study is limited to a system of three polynomial equations in two variables. Moreover, the components of algebraic geometry like Newton polytope, Minkowski sums of two-fold, and mixed volume that can be used for root counting are discussed via implemented these components to some sparse polynomial systems. The results from the Maple program are compared to confirm the root counting result using the mixed volume method. Since the study dealing with the sparse polynomial systems with two variables, to derive the sparse matrices, the developed Maple program, "multires.mpl" is used. These sparse matrices are then used to generate the determinant by using Maple 2015. The determinant derived from the matrix is called resultant homogenous polynomial. It is found that the result from Maple 2015 shows the existence of extraneous factors besides the determinant for some sparse polynomial systems. Meanwhile, by mixed volume method the number of roots is the same as the degree of the determinant (resultant homogenous polynomial). This study reveals that by using mixed volume the number of roots of the polynomial systems is exact. The results could provide theoretical advances and practical answers in areas such as robotics, computer graphics, cryptography, and scientific simulations. An important suggestion arising from this study is to further investigate the mixed volume method to systems with higher dimensions and greater complexity. In addition, it is important to focus on addressing any potential factors that may affect the accuracy and reliability of root counting algorithms in computational programs like Maple. Collaboration among experts in the same field is essential for advancing computational tools in this domain. Through the exploration of these avenues, the study aims to offer valuable insights and practical solutions that have broad interdisciplinary applications. This will contribute to enhancing both theoretical understanding and problem-solving in real-world scenarios.