UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

COEFFICIENT INEQUALITY FOR CERTAIN CLASS OF CLOSE-TO-CONVEX FUNCTIONS DEPENDENT ON A REAL PARAMETER

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ABSTRACT

The geometric functions theory of complex analysis is the basis of studying analytic univalent functions and investigating the geometric properties inherent in specific classes of analytic functions. However, this area of study faces significant challenges since only a small percentage of researchers are interested in discovering the properties. It might involve complex mathematical concepts, complex proofs, or abstract logic, which makes it difficult for many researchers to work on because of its level of difficulty. In addition, many researchers deal with difficulties in selecting suitable methods to manage mathematical analysis of their study. Thus, this study conducted to define a new generalised class of close-to-convex functions and determine the upper bound of the Fekete-Szegö inequality, $\left|a_3-\mu a_2^{\,2}\right|$ of close-to-convex functions using Hankel Determinant method. The generalisation of the class $S(\beta,\delta)$ was based on Rathi's work by using Hankel Determinants and Pommerenke's lemma. In this research, a class of close-to-convex functions, $S(\beta,\delta)$ defined in the unit disc, $E=\{z\in\mathbb{C}:|z|<1\}$ which satisfied the condition $\mathrm{Re}\left\{e^{i\beta}\dfrac{zf'(z)}{g(z)}\right\}>\delta,$ for $z\in D,\ |a|<\pi,\ \cos\beta>\delta, 0\le\delta<1$ and $g(z)=\dfrac{2z-z^2}{2}$ was studied. The central finding of this study involves the derivation of an upper bound of the Fekete-Szegö inequality and a new theorem is obtained under Fekete-Szegö determinant. The findings contribute to a more comprehensive understanding and knowledge of the geometric function theory. The conducted project gave opportunity to use different classes in the search of the Fekete-Szegö and other methods also can be applied to find the properties of Fekete-Szegö.