

**UNIVERSITI TEKNOLOGI MARA**

**TECHNICAL REPORT**

**EXTREMAL PROPERTIES OF CERTAIN CLASS  
OF CLOSE-TO-CONVEX FUNCTIONS**

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## ABSTRACT

This study is concerned with the theory of geometric functions represents a subdivision within the broad field of complex analysis. Every class of functions has extremal properties which are coefficient bound and growth theorem. This is a study of classical mathematics which involves many definitions and theorems that must be proved. In addition, it is also hard to introduce a unique class of univalent functions because some of class the function may not univalent. In this study, identifying the extremal properties presents challenges as suitable calculation methods are scarce and can prove to be intricate. The new generalized class of functions that are close-to-convex function  $M(\lambda, \delta, t)$  and  $f(0) = f'(0) - 1 = 0$ , that satisfies  $\operatorname{Re} \left\{ \frac{e^{i\lambda} z f'(z)}{g(z)} \right\} > \delta$  where

$|\lambda| < \frac{\pi}{2}, \cos \lambda > \delta, 0 \leq \delta < 1, g(z) = \frac{z}{1 - (1-t)z}$  and  $-1 \leq t \leq 1$ . The objectives for this study

are as to determine a new class of close – to – convex function and to find extremal properties of this class of function by using Herglotz and representation theorem. The new result of the coefficient bound that obtain is

$$|a_n| \leq \frac{2}{n} \left[ \frac{1}{2} + (1-t)(n-1) B_{\lambda\delta} \right].$$

When  $M(\lambda, \delta, -1)$ , then found that

$$\begin{aligned} |a_n| &\leq \frac{2}{n} \left[ \frac{1}{2} + (1-(-1))(n-1) B_{\lambda\delta} \right] \\ &\leq \frac{2}{n} \left[ \frac{1}{2} + 2(n-1) B_{\lambda\delta} \right], \end{aligned}$$

Therefore, the result will become the new result of coefficient bound. Based on the result, this might help other researchers to find other new extremal properties of this class of function as their references.