UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

FEKETE-SZEGÖ PROBLEM OF CERTAIN CLASS OF UNIVALENT FUNCTIONS

HOO SIEW LEE A/P HOO LEW AUN (2020878668)
SITI NOR ASIAH BINTI MUHAMAD ATAN (2020813646)
NUR ALYSHA DANIA BINTI ROSLAN (2020847396)
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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

The Fekete-Szego problem is a classical problem in complex analysis that concerns the properties of univalent functions. Specifically, there is limited method to find geometric properties of certain class of univalent function. This study is conducted to define new class of univalent functions and to find upper bound of Fekete-Szegö problem for this class of functions using Hankel determinant. We generalized class of univalent function from Yahya et al. (2013) that is a subclass of analytic univalent function, f defined in the unit disk, $E=\{z:|z|<1\}$, which satisfied the condition $\operatorname{Re}\left\{e^{i\alpha}\frac{zf^{'}(z)}{f(z)-f(-z)}\right\}>\delta$, for $(z\in E)$, $|\alpha|<\pi,\ \cos\alpha>\delta$ and $0\le\delta<1$. Then, the application of lemma by Pommerenke, and the methods use in Rathi (2015) been utilized with the purpose of acquiring the main results. The findings had also contributed new results under Fekete-Szegö determinants which the results obtained would act as a guidance and reference to the others mathematicians and subsequently would strengthen previous studies' result. It is suggested that to always opt for the simplest form of equations specifically for fractions. Since there are a lot of theorems that need to be proved and one requires a lot of steps, so using the simplest form of fractions really turns out to be useful.

CHAPTER 1: INTRODUCTION

1.1 Introduction

Mathematics is essential to the natural sciences, engineering, the medical field, computer science, and the social sciences. The majority of mathematical work involves identifying characteristics of abstract things and using reasoning to prove those characteristics. Multiple deductive rule applications to already established results make up a proof. These conclusions include previously proved theorems, axioms, and a few basic characteristics that are regarded as the true pillars of the theory under consideration. These topics are covered by four major modern mathematics subdisciplines, namely number theory, algebra, geometry, and analysis. Mathematics also includes the topic of complex analysis.

Complex analysis is commonly known as the theory of functions of one complex variable. It is the study of derivatives, operations, and other properties of complex numbers. It is also an extremely powerful tool with an unexpectedly enormous number of practical applications for solving physical problems. One of the subfields of complex analysis is geometric function theory. This field of study explores and discusses the geometric properties of analytic functions. The theory of univalent functions had been introduced by Koebe in 1907 as cited in Goodman (1983).

In 1916, the geometric function theory was then being established by a German mathematician that is Ludwig Bieberbach, (Zorn, 1986). Bieberbach worked on the second coefficient a_2 of a function $f \in N$ in the form of Taylor series expansion that is $|a_2| \le 2$.

$$f(z) = z + a_2 z^2 + a_3 z^3 + ... = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

Bieberbach had proven that $|a_2| \le 2$, if and only if a_2 is a rotation of the Koebe function with equality,

$$z(k) = \frac{1}{4}\left[\left(\frac{1+k}{1-k}\right)^2 - 1\right] = k + 2k^2 + 3k^3 + ... = \frac{k}{(1-k)^2} = \sum_{n=1}^{\infty} nk^n$$