

**UNIVERSITI TEKNOLOGI MARA**

**QUANTUM STATES GENERATION  
IN TWO-CHANNEL KERR  
NONLINEAR COUPLER WITH  
FREQUENCY MISMATCH**

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**MSc**

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## ABSTRACT

The present study investigates squeezed light phenomena in two, three and four optical modes Kerr nonlinear directional coupler with frequency mismatch composed of two-channel waveguides using phase-space representation. In these devices, the propagation of light is described using the Hamiltonian together with the Von-Neumann equation. Following a standard procedure, the quantum mechanical master equation describes the time evolution of the density matrix is obtained and converted to its equivalent classical Fokker-Planck equation in phase space. A set of noisy stochastic differential equations is then obtained from the Fokker-Planck equation via Ito calculus. These linearly-coupled stochastic equations are solved numerically over numerous trajectories to calculate the electric field quadratures. The effect of the frequency mismatch on the dynamics of the electric field quadratures of the propagating modes in each channel is investigated. A comparison between the generated squeezed states from these couplers at different values of frequency mismatching  $\Delta\omega$ . It has been found that the Kerr nonlinear coupler with frequency mismatch, can exhibit enhanced squeezing under certain conditions. The use of symmetrical coherent initialization may lead to the generation of maximal squeezing, and this strongly depends on the value of  $\Delta\omega$ . The amplitude of squeezing seems to increase with  $\Delta\omega$ . The generated squeezed light has different patterns such as normal squeezing, leaf-like squeezed states, and collapses and revivals. Moreover compared with the case of frequency matching, the two-mode Kerr coupler exhibit a better squeezing in each channel. For a three-mode coupler with frequency mismatch and symmetrical coherent excitation, several patterns of squeezed light such as leaf-like squeezing and collapses-and-revivals phenomena are obtained. In addition, the fundamental mode in the second channel initially with higher frequency exhibits squeezed light with greater amplitude and rapid oscillations. On the other side, when the first channel is prepared in a coherent state while the second channel is prepared in vacuum state, both channels show collapses and revivals-like squeezing. The maximal level of squeezing appears when  $\Delta\omega$  is minimal. In particular, the squeezing amplitude in the second channel decreases with the increase in  $\Delta\omega$ . This could be due to the destructive interference between the fundamental mode and the first-order mode in the second channel. In addition, a higher frequency mismatch between the modes seems to decrease the periodic exchange of energy between the waveguides. Furthermore, the squeezed states of light generated in a four-mode Kerr coupler where each channel contains two modes (fundamental and the first-order modes) are investigated using both positive and Wigner functions. At lower values of frequency mismatch, the results of numerical simulation from both representations agree very well. However, for higher frequency mismatch, discrepancies between both representations emerge. The squeezing pattern of the fundamental mode in each channel is different and a better-squeezed light is detected in the second channel with higher frequency. In addition, several patterns of squeezing are obtained including leaf-like squeezed states and collapses-and-revivals phenomena.

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