e-ISSN: 2289-6589

MODELLING OF OSTRICH EGG USING CUBIC HERMITE CURVE

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Submission date: 30 Sept 2018 *Accepted date: 30 Dec 2018* *Published date:30 Jan 2019*

Abstract

Cubic Hermite Curve is applied to calculate the volume of the ostrich eggs. Twelve different sizes of ostrich were used as samples to get the volume. Length, width and actual volume are the input measured to generate the calculated volume. Vernier caliper and measuring cylinder are used to get the measurement of the input. Archimedes Principal method has been used to measure an actual volume of eggs. In order to generate calculated volume, the best fitted curve of the ostrich eggs was obtained by using the best fitted point in Cubic Hermite Curve equation. Besides, another way to determine the best fitted curve is by comparing the relative error of the three different gradients between calculated volume and actual volume.

Keywords: Cubic Hermite Curve, volume of ostrich eggs, gradient

1.0 INTRODUCTION

Ostrich which is also known as the "camel bird" is the largest and heaviest living bird in the world. The ostrich is known as camel bird because of its long neck, sweeping eyes, and prominent eyes as well as its jolting walk. One fact about the ostrich is that their eyes are bigger than its brain.

For the purpose of this study which is to find the mathematical representation of ostrich egg's curve, the Cubic Hermite curve method is applied. Cubic Hermite curve is also known as parametric cubic spline that is represented by four degrees of freedom as illustrated in Figure 1. Two degrees of freedom are defined as the positions of the two end points of the curve and the other two degrees of freedom are defined as the tangents to the endpoints of the curve.

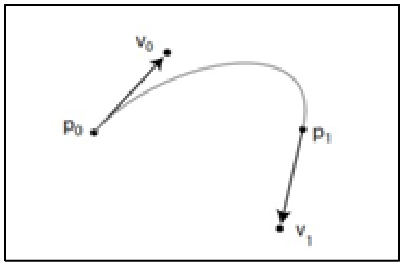

Figure 1 Construction of the curve by Hermite Interpolation

This Hermite curve is used to interpolate given data points and commonly used in a three-dimensional planar curve. This curve is the simplest polynomial curve and heavily used in geometric modeling because of the lowest degree polynomial curves that are not generically convex (Finn, 2004). The general equation of the Cubic Hermite curve is:

$$C(t)= a_0+ a_1 t + a_2 t^2 + a_3 t^3 \tag{1}$$

In the study by Yong & Cheng (2004), Equation (1) is naturally defined by four points and must meet with the following tangents and points:

$$C(0) = P_0 \qquad C(1) = P_1 \tag{2}$$
$$C'(0) = V_0 \qquad C'(1) = V_1 \tag{3}$$

After completing some arrangements, the basic equation of the curve is as follows:

$$C(t) = (1-3t^2 +2t^3)P_0 +(t-2t^2 +t^3)V_0 +(3t^2-2t^3)P_0 +(-t^2 +t^3)V_1 \tag{4}$$

This study of identifying the equation of ostrich egg by using Cubic Hermite curve is important to generate an equation that is simpler and easier for the users to refer to when they need it. For instance, it is important for the ostrich breeders to know the size of the egg to make it easier for them to grade the egg according to its size and quality, so that they can sell the egg at suitable and affordable price based on its grade.

Besides, this project on finding the ostrich egg's equation by using Cubic Hermite curve has never been done by other researcher before. In 2015, a research had previously conducted by Suziana et al. (2015), but it was only done on chicken egg. It is a hope that the result of this study will benefit everyone as well as to expose the people that this ostrich industry is actually existed in Malaysia since many people are unfamiliar and did not know its existence.

2.0 LITERATURE STUDY

2.1 Shape of Egg

Nedomova & Buchar (2013) stated that mathematical description of an egg profile allows us to calculate the egg volume, long circumference length, surface area, normal area of the egg, radius of curvature and angle between the tangent and the long axis to the shell at any point. It contradicted with the study done by Romanoff & Romanoff (1949) which stated that the contour of eggs was impossible to express in mathematical terms. However, many mathematics researchers express the egg profile by using mathematical terms.

Eggs is either oval or circular or elliptical depends on the type of the eggs. Nishiyama (2012) stated that Ostrich's eggs represented by ellipses. There are various size of eggs and Nishiyama found that the normal size of ostrich's eggs at 16cm x 12cm as shown in Figure 2.

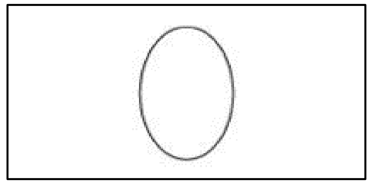

Figure 2 Elliptical shape of the egg

2.2 Equation of Egg Curve

Narushin (1997) developed his idea by considering the contour of half-projection of an egg. Narushin presented a profile of an egg mathematically by the given equation:

$$y = \frac{1.5396\mathrm{B}\sqrt{\mathrm{L}^{0.5}x^{1.5} - x^2}}{\mathrm{L}},\ y = -\frac{1.5396\mathrm{B}\sqrt{\mathrm{L}^{0.5}x^{1.5} - x^2}}{\mathrm{L}} \tag{5}$$

where x is the coordinate along the longitudinal axis and y is the transverse distance to the profile.

In another study, Narushin (2001b) found that the new equation for egg profile. Narushin substituted y= r sin θ and x= r cos θ, and he obtained the result as follows:

$$y = \sqrt{\mathrm{L}^{2/(n+1)}x^{2n/(n+1)} - x^2},\ y = -\sqrt{\mathrm{L}^{2/(n+1)}x^{2n/(n+1)} - x^2} \tag{6}$$

in which $n = 1.057(\mathrm{L/B})^{2.372}$.

Suziana et al. (2015) conducted a study on generating the egg profile using Cubic Hermite curve where the egg profile is divided into two parts and it is assumed that it is a combination of two symmetrical curves. The result of the study concluded that the equation of egg profile is given as follows:

Equation of the first curve:

$$\left.\begin{aligned} x_1(t) &= L\left(\frac{11}{16}t^2 - \frac{1}{8}t^3\right) \\ y_1(t) &= \frac{B}{2}\left(t + t^2 - t^3\right) \end{aligned}\right\} \quad 0 \le t \le 1 \tag{7}$$

Equation of the second curve:

$$\left.\begin{aligned} x_2(t) &= L\left(\frac{9}{16} + t - \frac{11}{16}t^2 + \frac{1}{8}t^3\right) \\ y_2(t) &= \frac{B}{2}\left(1 - 2t^2 + t^3\right) \end{aligned}\right\} \quad 0 \le t \le 1 \tag{8}$$

where L = the maximum length, and B = the maximum breadth of the egg.

2.3 Equation of Egg Volume

In order to complete the profile of an egg, the calculation of its volume is needed. Throughout his study, Narushin (2001) found an accurate and available formula for volume of eggs. From the measurement of L and B, the basic formula for calculating the volume of egg is given by:

$$V = k_v LB^2 \tag{9}$$

in which k_v is a coefficient for volume calculation, L is the egg length and B is the egg breadth.

Besides, the volume of an egg can be obtained from the general formula for volume of a shape which is V = B×L×W. According to Nedomova & Buchar (2013) the volume of egg can be written as:

$$V = \frac{\pi}{6} LB^2 \tag{10}$$

where L and B are the length and the breadth of the egg.

By using Cubic Hermite curve, Suziana et al. (2015) obtained the following egg volume formula:

$$V = \frac{271}{1680} \pi LB^2 \approx 0.50676880 \, LB^2 \tag{11}$$

where L is the maximum length and B is the maximum breadth of the egg.

3.0 METHODOLOGY

3.1 Representation of the Egg Curve in Cartesian Coordinate

Every egg has a different shape, whether it is oval, pyriform, circular or elliptical. Thus, the equation of the volume and curve of the egg also vary according to their size and shape. Although it is different, the general equation to calculate the volume of the egg is still the same; where the length and width are needed to generate the equation.

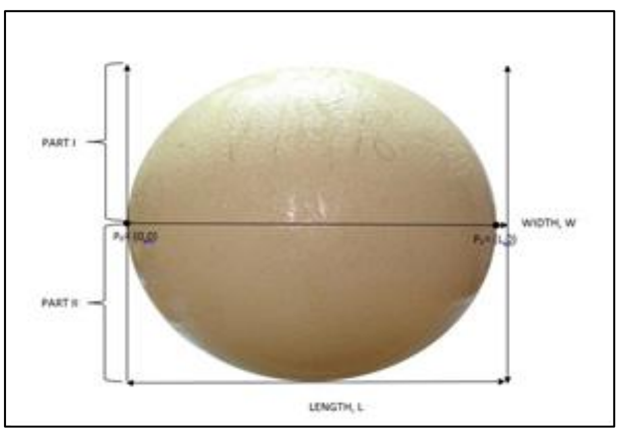

Figure 3 Ostrich egg in Cartesian Coordinate

Figure 3 illustrates the length and width of the egg in Cartesian coordinate. A research Nishiyama (2012) stated that the shape of ostrich egg is ellipse. Thus, the egg can be divided into two parts, Part I and Part II. So, the curve above and below is similar in size. Thus, this study can focus in finding the volume equation of the upper part only. The equation of volume for egg profile can be obtained by revolving the egg about x-axis.

3.2 Collection of Data

For this study, the primary data of ostrich eggs will be obtained by conducting an experiment using Archimedes Principle. Twelve ostrich eggs in various sizes were used in this experiment. The ostrich eggs were obtained from the Ostrich Department at INFOTERNAK Farm and the experiment was conducted at the ostrich department laboratory with the help from the director.

Table 1 Main characteristics of the eggs

Types of Egg	Length, L (cm^3)	Width, W (cm^3)	Actual Volume, V (cm^3)
Ostrich 1	15.8	12.2	1073
Ostrich 2	14.8	12.4	1000
Ostrich 3	14.35	12	980
Ostrich 4	14.1	12	953
Ostrich 5	14.2	11.85	965
Ostrich 6	15.25	12	1035
Ostrich 7	15.3	12.45	1033
Ostrich 8	13.2	11.55	900
Ostrich 9	13.8	11.5	928
Ostrich 10	16.45	12.15	1110
Ostrich 11	13.35	11.75	900
Ostrich 12	14.6	11.5	990

3.3 Simplification of the Equation of the Curve

Based on Figure 2, the curve of the egg profile can be divided into two parts which are upper curve and lower curve. Since the two curves are symmetrical, this study will focus on finding the equation of the curve for the upper part only and the equation of volume will derive by revolving the curve by x-axis. The value of y can be determined by using the relationship as follows:

$$y = \frac{W}{2}$$

Equation of the curve, P(t):

From (4), the equation of the curve can be written in parametric forms as below:

$$x(t) = f_0\, P_0(x) + f_1\, m_0(x) + f_2\, P_1(x) + f_3 m_1(x) \qquad (12)$$

$$y(t) = f_0\, P_0(y) + f_1\, m_0(x) + f_2\, P_1(y) + f_3\, m_1(x) \qquad (13)$$

where f_0, f_1, f_2 and f_3 are the basis functions such that $f_0 = 1 - 3t^2 + 2t^3$, $f_1 = t^3 - 2t^2 + t$, $f_2 = 3t^2 - 2t^3$, and $f_3 = t^3 - t^2$. Meanwhile $P_0(x,y)$ and $P_1(x,y)$ are the critical points where $P_0(x,y)$ is (x_0,y_0) and $P_1(x,y)$ is (x_1,y_1), and m_0 and m_1 are the tangents at each critical point, (x_0,y_0) and (x_1,y_1).

3.4 Equation of Volume for Egg Profile

The equation of volume for egg profile can be obtained by revolving the egg profile curve in Figure 3 about x-axis. In order to integrate (5) and (6), the equation of volume of a solid by using definite integral needs to be modified. After a slightly modification, the volume's formula is:

$$V = \pi \int_0^1 (y(t))^2 \, x'(t) dt \quad (14)$$

Thus, equation (14) can be used in calculating the volume of the ostrich egg profile.

4.0 RESULT AND DISCUSSION

Three observations with different values of critical points have been done in order to get the best fitted curve of egg profile. The graphs shown below are the complete graphical illustration of ostrich eggs revolving about the x-axis.

4.1 First Gradient Test when ($m_0(x)=m_1(x)=0.5$, $m_0(y)=22.5$, $m_1(y)=-22.5$)

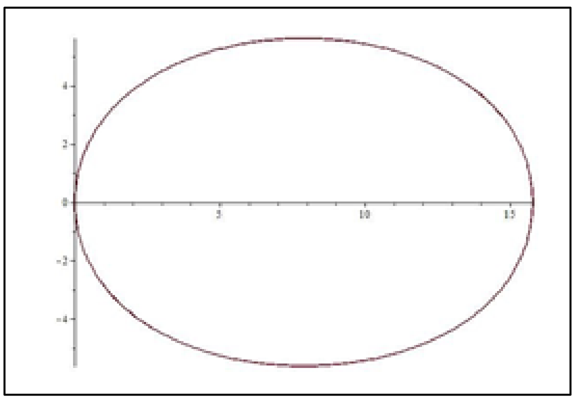

Figure 4 The Graphical Illustration of Ostrich Egg 1 (First Gradient Test)

4.2 Second Gradient Test when ($m_0(x)=m_1(x)=1$, $m_0(y)=22.5$, $m_1(y)=-22.5$)

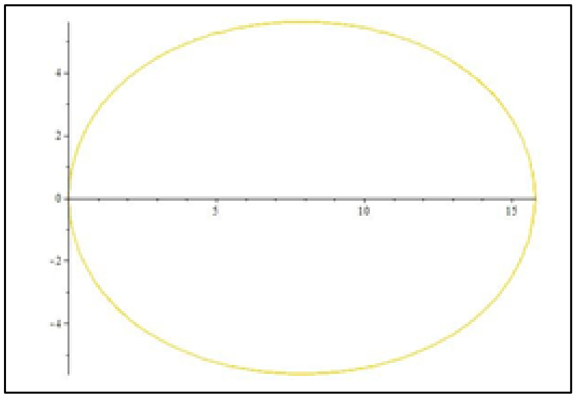

Figure 5 The Graphical Illustration of Ostrich Egg 1 (Second Gradient Test)

4.3 Third Gradient Test when ($m_0(x)=m_1(x)=3$, $m_0(y)=24$, $m_1(y)=-24$)

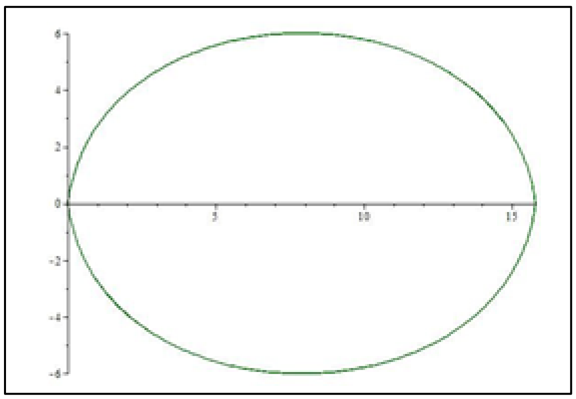

Figure 6 The Graphical Illustration of Ostrich Egg 1 (Third Gradient Test)

Based on the observations, the first and second gradient tests are quite similar to the actual ostrich egg but the shape of ostrich egg for the third gradient test is slightly different. Thus, this study concludes that the most fitted curves are from the first and second gradient test.

Besides, this study has also calculated the volume of the ostrich egg with the three different gradients. The volume for three different gradient tests is as shown in Table 2.

After the calculation of the volume of ostrich egg with different gradient tests, this study also computes that the absolute error of the actual volume and calculated volume obtain the best fitted curve among the three observations. The results obtained are as in Table 3 – 5.

Table 2 Volume of Ostrich egg with different values of gradient

Types of Egg	First Gradient Test (cm^3)	Second Gradient Test (cm^3)	Third Gradient Test (cm^3)
Ostrich 1	1069.375699	1061.802217	1173.627208
Ostrich 2	1001.214359	993.6408766	1096.074749
Ostrich 3	970.54175551	962.9682732	1061.176143
Ostrich 4	953.5014205	945.9279382	1041.788028
Ostrich 5	960.3175544	952.7440721	1049.543274
Ostrich 6	1031.886962	1024.31348	1130.973355
Ostrich 7	1035.295029	1027.721547	1134.850978
Ostrich 8	892.156214	884.5827317	971.9908151
Ostrich 9	933.0530182	925.4795359	1018.52229
Ostrich 10	1113.680571	1106.107088	1224.036306
Ostrich 11	902.3804148	894.8069328	983.6236841
Ostrich 12	987.5820909	980.0086085	1080.564257

Table 3 Comparison between actual volume and calculated volume (First Gradient Test)

Types of Egg	Actual Volume (cm^3)	Calculated Volume (cm^3)	Absolute Error
Ostrich 1	1073	1069.375699	3.624301
Ostrich 2	1000	1001.214359	1.214359
Ostrich 3	980	970.5417555	9.4582445
Ostrich 4	953	953.5014205	0.5014205
Ostrich 5	965	960.3175544	4.6824456
Ostrich 6	1035	1031.886962	3.113038
Ostrich 7	1033	1035.295029	2.295029
Ostrich 8	900	892.156214	7.843786
Ostrich 9	928	933.0530182	5.0530182
Ostrich 10	1110	1113.680571	3.680571
Ostrich 11	900	902.3804148	2.3804148
Ostrich 12	990	987.5820909	2.4179091

Table 4 Comparison between actual volume and calculated volume (Second Gradient Test)

Types of Egg	Actual Volume (cm^3)	Calculated Volume (cm^3)	Absolute Error
Ostrich 1	1073	1061.802217	11.197783
Ostrich 2	1000	993.6408766	6.3591234
Ostrich 3	980	962.9682732	17.0317268
Ostrich 4	953	945.9279382	7.0720618
Ostrich 5	965	952.7440721	12.2559279
Ostrich 6	1035	1024.31348	10.68652
Ostrich 7	1033	1027.721547	5.278453
Ostrich 8	900	884.5827317	15.4172683
Ostrich 9	928	925.4795359	2.5204641
Ostrich 10	1110	1106.107088	3.892912
Ostrich 11	900	894.8069328	5.1930672
Ostrich 12	990	980.0086085	9.9913915

Table 5 Comparison between actual volume and calculated volume (Third Gradient Test)

Types of Egg	Actual Volume (cm^3)	Calculated Volume (cm^3)	Absolute Error
Ostrich 1	1073	1173.627208	100.627208
Ostrich 2	1000	1096.074749	96.074749
Ostrich 3	980	1061.176143	81.176143
Ostrich 4	953	1041.788028	88.788028
Ostrich 5	965	1049.543274	84.543274
Ostrich 6	1035	1130.973355	95.973355
Ostrich 7	1033	1134.850978	101.850978
Ostrich 8	900	971.9908151	71.9908151
Ostrich 9	928	1018.52229	90.52229
Ostrich 10	1110	1224.036306	114.036306
Ostrich 11	900	983.6236841	83.6236841
Ostrich 12	990	1080.564257	90.564257

Based on the result above, the smallest range of error for each size of ostrich eggs obtained from the comparison of actual volume and calculated volume between three different gradient tests is the error for the first gradient test in which is between 0 and 10. In conclusion, this study shows that the best fitted curve for the ostrich egg profile is from the first gradient test where the gradient m_0 and m_1 at critical point x_0 and x_1 is 0.5 respectively and the gradient m_0 at critical point y_0 is 22.5 and m_1 at critical point y_1 is -22.5.

5.0 CONCLUSION AND FUTURE WORKS

In this study, Cubic Hermite Curve has been applied to get the best fitted curve of the ostrich eggs and to generate calculated volume by using Maple software. Hence, the general equation of Cubic Hermite Curve has been tested on the sample of twelve different sizes of ostrich eggs for the objectives to be achieved. Through our experiment, it shows that the smaller the gradient at the critical point, the best fitted curve obtained. Besides, the comparison between the actual volume and the calculated volume is done and the range of error is between 0 and 10 from the smallest gradient.

As a conclusion, all objectives stated for this study are achieved. However, there is still room to make some changes to improve the results. Other than measuring the volume, this study may also be extended further by studying other aspects such as surface area and cross sectional area of the egg through the egg length by using the egg profile obtained. In addition, the egg profile can also be studied using Hermite curve with a higher degree, for example Quintic Hermite curve. Besides, the measurement of the actual length, width and volume need to be accurately measured.

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