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**VANDERMONDE MATRICES INVERSION VIA
SYNTHETIC DIVISION
(P41S22)**

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	ii
TABLE OF CONTENTS	iii
LIST OF TABLES	iv
LIST OF FIGURES	iv
ABSTRACT	v
CHAPTER 1	1
INTRODUCTION	1
1.1 Motivation.....	1
1.2 Problem Statement	2
1.3 Objectives	3
1.4 Significant and Benefit of Study.....	3
1.5 Scope and Limitation of Study.....	3
1.6 Definition of Terms.....	4
CHAPTER 2	6
BACKGROUND THEORY AND LITERATURE REVIEW	6
2.1 Background Theory	6
2.1.1 VDM model and its inverse.	6
2.1.2 The existing inverse of VDM.....	8
2.2 Literature Review.....	9
2.2.1 Vandermonde Matrix	9
2.2.2 Inversion of Vandermonde Matrix.....	10
CHAPTER 3	13
METHODOLOGY AND IMPLEMENTATION	13
3.1 Methodology	14
3.1.1 The Synthetic Division formulation of the inverse VDM.....	14
3.1.2 The application of the inverse VDM in solving the interpolation polynomial equation.....	15
3.1.3 Development of the algorithm of Synthetic Division in Phyton programming language.	16
3.1.4 Analysis of results.....	16
3.2 Implementations.....	17
CHAPTER 4	35
RESULTS AND DISCUSSION	35
4.1 Results.....	35
4.1.1 Pseudo-code for Synthetic Division.....	35
4.1.2 Results for inverse of VDM	38
4.1.3. Comparison on computing time.....	41
4.1.4. Application of the inverse of VDM	42
CHAPTER 5	43
CONCLUSIONS AND RECOMMENDATIONS	43
5.1 Conclusions.....	43
5.2 Recommendations.....	43
REFERENCES	45
APPENDIX A	48

Python Code For Synthetic Division	48
APPENDIX B	51
Python Code For Numpy Function	51
APPENDIX C	52

LIST OF TABLES

Table 1: Results for inverse of VDM.....	38
Table 2: Comparison on computing time.....	41
Table 3: Application of the inverse of VDM	42

LIST OF FIGURES

Figure 1: Research methodology	13
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ABSTRACT

Vandermonde matrix (VDM) research has covered a wide range of topics, including polynomial interpolation, numerical analysis, signal processing, and many others. Vandermonde matrices are a fundamental concept in applied mathematics, natural science, and engineering. The VDM's inverse form is one of its most important features. Methods for inverting VDM involved multiplying two matrices, such as a product of triangular matrices, computing the determinant and cofactor matrices, and elementary row operations to determine the elements of the inverse of VDM. These techniques, however, have a high computational cost. As a result, this project investigates another approach known as the Synthetic Division method. In this study, the objective is to develop the algorithm of the inverse of VDM by Synthetic Division method in Python programming language. Furthermore, is to analyze the effectiveness and time computation by Synthetic Division method and function `numpy.linalg.inv()` in the Python NumPy module for the computation of the inverse of VDM and last is to demonstrate the application of the inverse of VDM. To compute the elements of the inverse of VDM, the method uses arithmetic operations, multiplications, and additions. To conduct the investigation, the Synthetic Division formula is applied to VDM of sizes 2×2 and 3×3 in order to understand the pattern of the results. The investigation continues with the development of a Synthetic Division algorithm in Python programming for matrices of up to size 11. Furthermore, the time computation of computing the inverse of VDM is compared between Synthetic Division and function `numpy.linalg.inv()` in the Python NumPy module. The NumPy function works well for all matrices, except for the size 9. On the other hand, the Python Synthetic Division algorithm appears to be consistent in terms of time computation regardless of matrix size. As a result, the Synthetic Division method is an efficient way of calculating the inverse of VDM. Finally, one application of the inverse of VDM in polynomial interpolation was demonstrated in this project. The Python code is currently being used to generate the inverse of VDM for $n = 11$. Hence, the Synthetic Division algorithm developed in Python will be generalised in the future for n -dimensional matrices.