

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**NUMERICAL SOLUTIONS OF NONLINEAR ORDINARY
DIFFERENTIAL EQUATION BY MODIFIED EULER METHOD
AND MODIFIED FEHLBERG METHOD**

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ABSTRACT

In many areas of mathematics and in applications in engineering, the first order nonlinear ordinary differential equation (ODE), which comprises the Bernoulli equation and the Riccati equation, is crucial. Many methods, including Newton's interpolation, Aitken's method, and Differential Transform method, have been used to solve these two forms of equations. Due to their nonlinear behaviour, the Bernoulli and Riccati equations are challenging to solve analytically, hence not all numerical methods can provide accurate solutions. Therefore, the goal of these study is to solve these equations using the Modified Euler Method and Modified Fehlberg Method. The accuracy and efficiency for both methods are determined based on absolute error. Several examples from previous research are used to solve the Bernoulli and Riccati equations using the proposed methods. As a result, this study may lead to the conclusion that the findings show that RKF5(6) is more efficient and precise approach for solving the Riccati equations and Example 1 of the Bernoulli equation since its absolute error are smaller than MEM. On the other hand, MEM is accurate among those two approaches in Example 2 of the Bernoulli Equation because its absolute error is smaller than that of RKF5(6). Then, it is suggested that other researchers use the Daftardan-Geiji and Jafari method (DJM) and the Elzaki Iterative Method (EIM) to solve Bernoulli and Riccati Equations in order to assess the method's accuracy and effectiveness and to determine whether the proposed approaches would yield the best results.