

**UNIVERSITI TEKNOLOGI MARA**

**TECHNICAL REPORT**

**MATHEMATICAL MODELLING OF RSA CRYPTOSYSTEM  
BASED ON MODIFICATION OF MODULAR  
EXPONENTIATION WITH HESSIAN CURVE**

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## ABSTRACT

This study proposed mathematical modelling of the RSA cryptosystem based on the modification of modular exponentiation with Hessian curves. Due to the attacker's potential to attack the system, the lack of cryptosystems motivates the study to improve the mathematical modeling of the RSA cryptosystem. If the attackers develop a reverse procedure, the small number of private keys may lead to a lack of resistance against factorization, and the private key ' $d$ ' may also be exposed in the decryption phase. The study aims to modify the previous research articles by Dubey and Yadav (2015) from three prime numbers into four prime numbers to compute the composite of  $n$ . Besides, the study applied the point addition of Hessian curves in the key generation process and modified the public key  $e$  inspired by Intila et al. (2019) based on modular exponentiation in the encryption. The concepts of modular exponentiation, modular inverse and linear congruence were involved in the calculation of the study. Through the study, new mathematical modelling was derived on the key generation, encryption, and decryption process. Then, the numerical example is given with the implementation of the Desmos and Python software to test the proposed system. Consequently, the proposed system successfully satisfies the principle of the RSA cryptosystem motivated by Intila et al. (2019) and Dubey and Yadav (2015). Henceforth, all Hessian curve articles inspired by Fouotsa (2019), Smart (2001), and Naveen et al. (2019) are applicable to the proposed mathematical modelling of the RSA cryptosystem. This study can further extend by modifying with some logical developments in mathematical calculation and the Chinese Remainder Theorem to hide many texts into a single plaintext (Intila et al., 2019). Furthermore, this research may extend to develop the mathematical modelling of RSA using large prime numbers and the point doubling of the Hessian curve.