UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

UPPER BOUND OF THE SECOND HANKEL OF CERTAIN CLASS OF UNIVALENT FUNCTIONS

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

Geometric function theory of a complex functions makes a study of analytic univalent functions and various geometric properties of certain classes of analytic functions. Furthermore, only a few researchers are interested in discovering these function features. Next, the difficulties and problems arise when researcher selecting an accurate method. Although it gives a same result, the solution can become complicated and make researcher take time to understand whether the method use is accurate or not. Thus, the purpose of this study is to define theorem and definition of geometry functions theory and a new subclass of analytics functions and to determine the coefficient inequality of second Hankel determinant for the class of functions using method of proving by Janteng et al. (2007), Kaharudin et al. (2011) and Yahya et al. (2013).

In this study, we introduced new subclasses of analytic functions $S(\alpha,\delta,t)$, defined in the unit disk, $\mathcal{U}=\{x\in\mathbb{C}:|x|<1\}$, which satisfies the condition $\mathrm{Re}\left\{e^{i\alpha}\dfrac{(1-t)zf'(z)}{f(z)-f'(tz)}\right\}>\delta,$ where $|a|<\pi,\cos\alpha>\delta,0\le\delta<1,-1\le t<1$ and $t\in\mathbb{C}$. In our methodology steps, we use the Lemma of Pommerenke (1975), Lemma of Toeplitz determinants and Lemma of Libera (1983) and Zlotkiewicz (1982). For this study, we obtained the upper bound for the second Hankel in the form of α,δ and t. The final result can be reduced to several types of new class of functions for other researcher. For example, when $t=0$ we can reduce to Janteng et al. (2007) and we can reduced to Kaharudin et al (2011) when $t=1$. The result of second Hankel determinant obtained is a keen result and this property led to development of second Hankel determinant.

LIST OF SYMBOLS

Symbol

$\mathbb{C}$	Set of complex numbers
D	Domain
S	Class of normalized analytic function $f(x) = x + \sum_{n=2}^{\infty} a_n x^n$
$\mathcal{U}$	Open unit disk $\{x \in \mathbb{C} : \lvert x \rvert < 1\}$
A	Class of normalized analytic functions in the open unit disk, $\mathcal{U}$ of the form
P	Class of all function in the form of $P(z) = 1 + p_1 z + p_2 z^2 + \ldots + p_n z^n = 1 + \sum_{n=1}^{\infty} p_n z^n.$
K	Class of convex function
St	Class of starlike function
C	Class of close-to-convex function