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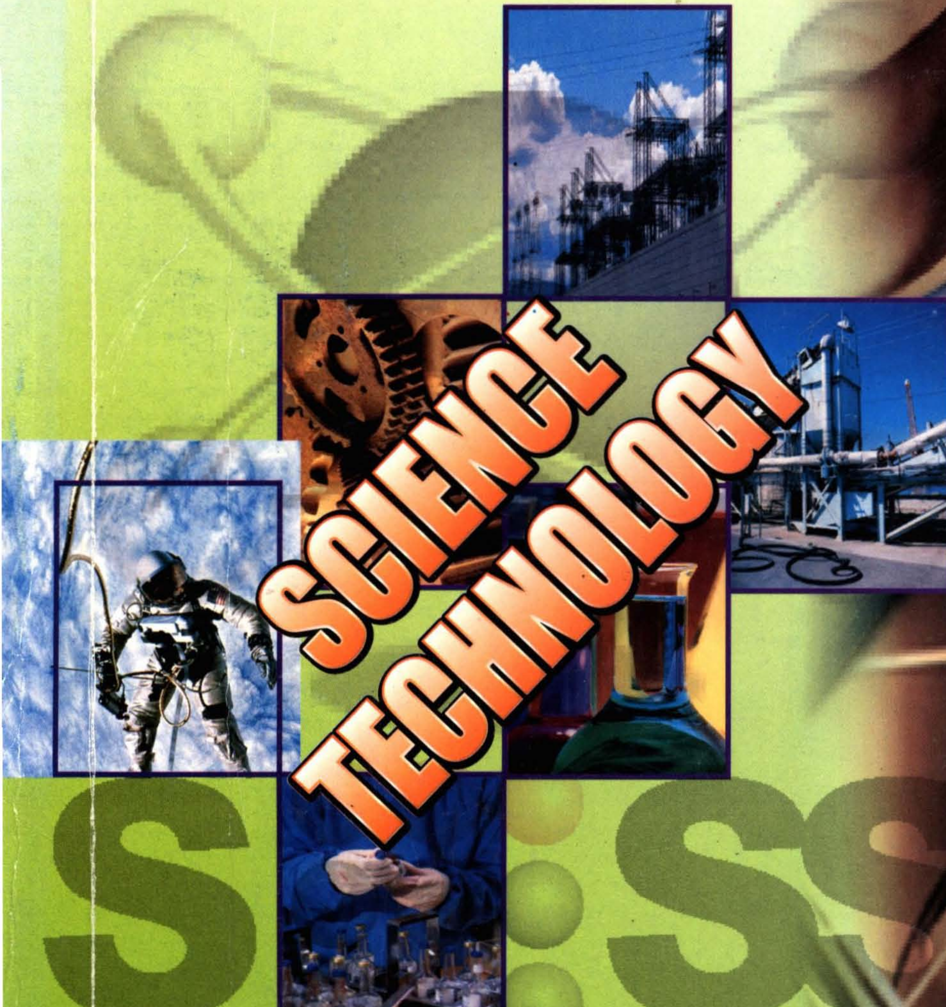

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Development of Computer Program for Analyzing Thick and Laminated Plate –Shell Assemblages by using Composite

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ABSTRACT

A new formulation on general shell theory has been proposed to accommodate the shear deformation effect and inertia effects are included. The analysis capability of the formulation is the vibration analysis of a thick and laminated plate-shell assemblages using composite. The proposed general shell formulation can be used for thick shell and can be converted to thin shell by neglecting the thickness-to-radius ratio.

Keywords: *Plate, shells, thick shell, shear deformation, rotary inertia, laminated composite*

Introduction

Thick prismatic composite plates and shells have been occupied extensive applications as structural elements. Due to its high strength-to-weight and stiffness-to-weight ratios, they are selected as the suitable material for engineering applications in aircraft and submarine structures, automobile components, building construction, sport equipments and so on. Therefore, an accurate analysis of thick prismatic shells, i.e. vibration analysis is a need to engineers for predicting the correct behavior of the thick prismatic shells.

Materials and Methods

Formulation of Finite Strip Equations

Displacement Field Functions
Finite strip method has been chosen as the numerical analysis tools for approximating the solution of the vibration of thick laminated shell. The selection of this method is due to the high number reduction of degree of freedom. This is important when the prismatic shell has a constant cross-section over the length. The prismatic shell is then divided into an equal length of strips that running parallels along the whole length, l, which in the x-axis direction. The two end longitudinal length can be rigidly connected to the other prismatic plate or shell structure or left to be 'free' if it is not connected.

The displacement function for the $u, v, w, \varphi_x, \varphi_y$ can be expressed as the multiplication of analytical, continuously-differentiable series function in the longitudinal x-direction and continuous polynomial functions in the crosswise direction. The cubic order polynomial function is used for interpolating those fundamentals displacement quantities. Timoshenko beam function is used as the longitudinal displacement function.

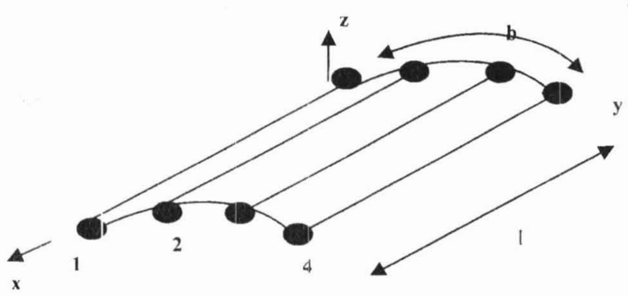

Fig. 1: Nodal Degrees of Freedom of a Shell Using

By applying the selected function to the corresponding direction,

$$u = \sum_{m=1}^{r} Y_m(x)\left(a_0 + a_1 y + a_2 y^2 + a_3 y^3\right) \qquad (1)$$

$$v = \sum_{m=1}^{r} Y_m(x)\left(b_0 + b_1 y + b_2 y^2 + b_3 y^3\right) \qquad (2)$$

$$w = \sum_{m=1}^{r} Y_m(x)\left(c_0 + c_1 y + c_2 y^2 + c_3 y^3\right) \qquad (3)$$

$$\varphi_y = \sum_{m=1}^{r} Y_m(x)\left(e_0 + e_1 y + e_2 y^2 + e_3 y^3\right) \qquad (4)$$

$$\varphi_x = \sum_{m=1}^{r} Y_m(x)\left(f_0 + f_1 y + f_2 y^2 + f_3 y^3\right) \qquad (5)$$

Here

r = m-th term of the longitudinal beam function
Ym(x) = the Timoshenko Beam Function

In the matrix form, equation (1-5) can be expressed as

$$\begin{Bmatrix} u \\ v \\ w \\ \varphi_y \\ \varphi_x \end{Bmatrix} = \sum_{m=1}^{r} \begin{bmatrix} Y_m & 0 & 0 & 0 & 0 \\ 0 & Y_m & 0 & 0 & 0 \\ 0 & 0 & Y_m & 0 & 0 \\ 0 & 0 & 0 & Y_m & 0 \\ 0 & 0 & 0 & 0 & Y_m \end{bmatrix} \begin{bmatrix} [\beta(y)] & 0 & 0 & 0 & 0 \\ 0 & [\beta(y)] & 0 & 0 & 0 \\ 0 & 0 & [\beta(y)] & 0 & 0 \\ 0 & 0 & 0 & [\beta(y)] & 0 \\ 0 & 0 & 0 & 0 & [\beta(y)] \end{bmatrix} \{a\}_m \qquad (6)$$

Or in more compacted form,

$$\{u\} = \sum_{m=1}^{r} \lceil S \rceil \lceil P \rceil \{a\}_m \qquad (7)$$

Note that in the equation (6) the symbol $\lceil \; \rceil$ denotes the matrix has its function diagnolised. The submatrices $[\beta(y)]$ is defined as $\lfloor 1 \quad y \quad y^2 \quad y^3 \rfloor$ and submatrice $\lceil S \rceil$ has diagnolised Y_m function.

The matrix $\{a\}_m$ is the undetermined coefficient matrix and it has a relationship with the nodal degree of freedom matrix {d}m as follows,

$$\{a\}_m = [C]\{d\}_m \qquad (8)$$

And the column matrix $\{a\}_m$ and $\{d\}_m$ is defined as

$$\{a\}_m = \{a_0 \quad a_1 \quad a_2 \quad a_3 \quad b_0 \quad b_1 \quad b_2 \quad b_3 \quad c_0 \quad c_1 \quad c_2 \quad c_3 \quad e_0 \quad e_1 \quad e_2 \quad e_3 \quad f_0 \quad f_1 \quad f_2 \quad f_3\}$$

$$\{d\}_m = \{u_1 \quad v_1 \quad w_1 \quad \varphi_{y1} \quad \varphi_{x1} \quad u_2 \quad \ldots\ldots\ldots \quad \varphi_{x4}\}$$

After the substitution, equation 7 becomes

$$\{u\} = \sum_{m=1}^{r} [S][P][C]\{d\}_m \ldots\ldots (9)$$

Stiffness Matrix Development

Using equation (9) leads to the following relationship,

$$\{E\} = \sum_{m=1}^{r} [S]_e [P]_e [C]\{d\}_m \ldots\ldots (10)$$

where subscript e indicates the differentiated terms.

General expression for the strain energy, U, for thick laminated shell by neglecting ε_{zz} terms and since there is no applied stress in the z –direction, the term σ_z becomes zero.

$$U = \frac{1}{2}\int_V \sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{zx}\gamma_{zx} + \tau_{xy}\gamma_{xy}\, dV \ldots\ldots (11)$$

Or in the compacted form

$$U = \frac{1}{2}\int [E]^T [Q][E] dV \ldots\ldots (12)$$

where matrix [Q] is the material properties matrix.

The strain energy given by equation (12) can be expressed in terms of nodal displacements as follows:

$$U = \frac{1}{2}\sum_{m=1}^{r}\sum_{n=1}^{r}\int_0^b\int_0^d\int_{-\frac{h}{2}}^{\frac{h}{2}} \{d\}_m^T [C]^T \left([P]_e^T [S]_{em}^T [Q][S]_{en}^T [Q][S]_{en} [P]_e\right)[C]\{d_n\} dzdydx \ldots\ldots (13)$$

By applying the Principle of Minimum Potential Energy, we thus obtain the stiffness matrix as

$$[K]=\begin{bmatrix}[K]_{11} & \dots & [K]_{1n} & \dots & [K]_{1r}\\ [K]_{m1} & \dots & [K]_{mn} & \dots & [K]_{mr}\\ [K]_{r1} & \dots & [K]_{rn} & \dots & [K]_{rr}\end{bmatrix}$$

with the term [K]

$$[K]=[C]^{T}[J]_{mn}[C] \qquad (14)$$

and

$$[J]_{mn}=\int_0^b\int_0^d\int_{-\frac{h}{2}}^{\frac{h}{2}}[P]_e^T[S]_{em}^T[Q][S]_{en}[P]_e\,dzdxdy \qquad (15)$$

Mass (Inertia) Matrix Development

The potential energy expression for the thick rigid body is given as

$$V_{pv}=\frac{1}{2}\omega^2\int_0^d\int_0^b I_1\left(u_0^2+v_0^2+w_0^2\right)+2I_2\left(u_0\varphi_x+v_0\varphi_y\right)+I_3\left(\varphi_x^2+\varphi_y^2\right)$$
$$+\frac{1}{R}\left[I_2\left(u_0^2+v_0^2+w_0^2\right)+2I_3\left(u_0\varphi_x+v_0\varphi_y\right)+I_4\left(\varphi_x^2+\varphi_y^2\right)\right]dxdy \qquad (16)$$

Where by the parameter of I_1, I_2, I_3, I_4 is defined as

$$\left(I_1,I_2,I_3,I_4\right)=\sum_{k=1}^{N}\rho^{(k)}\left(1,z,z^2,z^3\right)dz$$
$$=\int_{k=0}^{N}\rho^{(k)}\left(1,z,z^2,z^3\right)dz \qquad (17)$$

In the equation (17), the parameter $\rho^{(k)}$ denotes the homogenous mass per unit volume of thick rigid body for each of k-th layer of laminae. Substitution of equation (17) into equation (16) leads to the following final potential energy equation;

$$V_{pv}=\frac{1}{2}\omega^2\int_0^d\int_0^b\left(u_0^2+v_0^2+w_0^2\right)\left(\bar{\rho}+\frac{I_2}{R}\right)+\left(\varphi_x^2+\varphi_y^2\right)\left(I_3+\frac{I_4}{R}\right)$$
$$+\left[\left(u_0\varphi_x+v_0\varphi_y\right)\left(2I_2+2I_3\right)\right]dxdy \qquad (18)$$

By substituting the nodal degree freedoms in equation xxx and multiplying with

$\left(\bar{\rho}+\frac{I_2}{R}\right),\left(I_3+\frac{I_4}{R}\right)$ and $\left(I_2+I_3\right)$ as indicated in equation (18), the potential energy of a vibrating mass is

$$V_{pv}=\frac{1}{2}\omega^2\int_0^b\int_0\left[\sum_{m=1}^{r}\sum_{n=1}^{r}\{d\}_m^T[C]^T\left([P]^T[\perp][P]\right)[C]\{d\}_n dydx\right.\qquad(19)$$

In equation (19), the matrix $[\perp]$ is defined as follows;

$$\begin{bmatrix} I_1+\frac{I_2}{R} & 0 & 0 & 0 & 2I_2+\frac{2I_3}{R} \\ 0 & I_1+\frac{I_2}{R} & 0 & 2I_2+\frac{2I_3}{R} & 0 \\ 0 & 0 & I_1+\frac{I_2}{R} & 0 & 0 \\ 0 & 2I_2+\frac{2I_3}{R} & 0 & I_3+\frac{I_4}{R} & 0 \\ 2I_2+\frac{2I_3}{R} & 0 & 0 & 0 & I_3+\frac{I_4}{R} \end{bmatrix}$$

Applying the Principle of Minimum Potential Energy, the mass matrix is shown as

$$[M]=\begin{bmatrix} [M]_{11} & \cdots & [M]_{1n} & \cdots & [M]_{1r} \\ [M]_{m1} & \cdots & [M]_{mn} & \cdots & [M]_{mr} \\ [M]_{r1} & \cdots & [M]_{rn} & \cdots & [M]_{rr} \end{bmatrix}$$

with

$$[M]_{mn}=[C]^T[\Re]_{mn}[C]$$

Formulation for Free Vibration of Thick and Laminated Shell Analysis

By dropping all of suffixes in the stifness and mass matrix will lead to a familiar free vibration formulation, as follows;

$$\left([K]-\omega^2[M]\right)\{d\}=0$$

The order of matrix K and M are (r x r), which then represents to a set of r linear of homogenous equations. The determinant of this matrix $|K-\omega^2 M|$ must be equated to zero and the eigenvalues of the problem can be obtained numerically using Sturm sequence method.

Results and Discussion

A genuine formulation based on Finite Strip Method has been developed for a free vibration analysis of thick and laminated plate-shell structures. Shear deformation and rotary inertia has been included in the formulation. The computer program of the new formulation for the verification purposes is under effort of compiling and debugging process of the author.

The expected outcome of this new formulation will be the eigenvalues of a self vibrating thick plate and shell structures that are under the author's interest. The results will be compared with the other reseachers findings and the accuracy of the formulation can be evaluated.

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