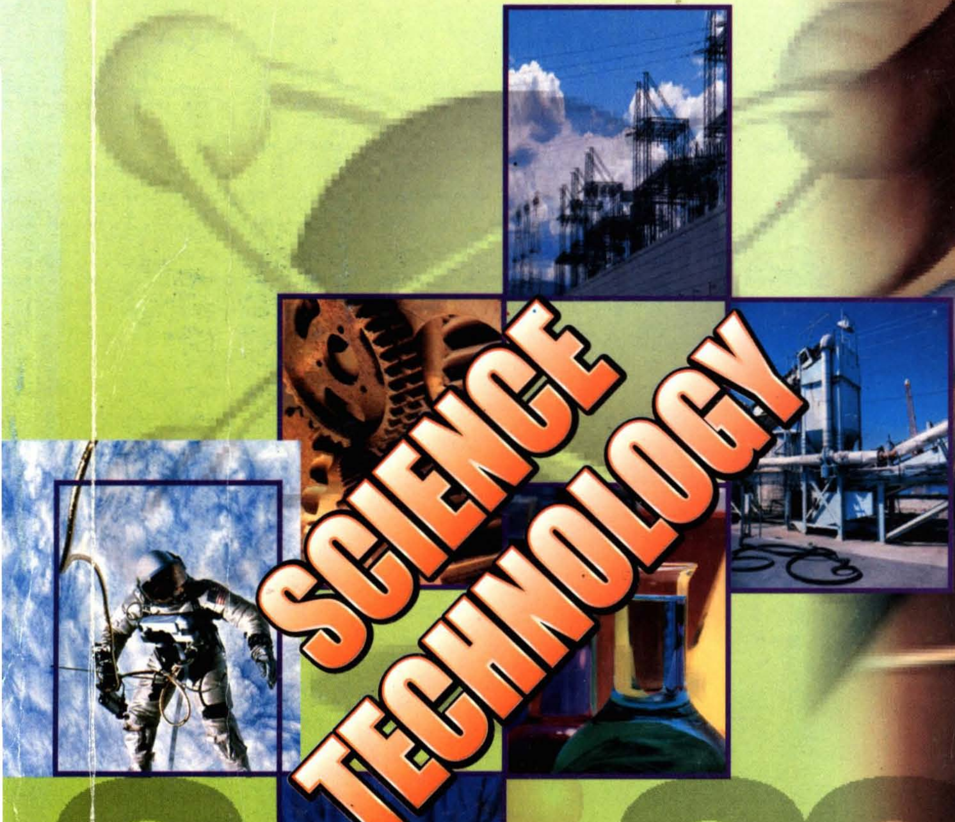
UNIVERSITI TEKNOLOGI MARA

ISO 9001:2000 No Sijil :0500132

Globalising Knowledge and Information

NATIONAL SEMINAR ON SCIENCE TECHNOLOGY & SOCIAL SCIENCES 2006



30-31 May 2006

Swiss Garden Resort & Spa

Kuantan, Pahang

Numerical Modeling of Temperature Distribution in 2D Model Using Transmission Line Modeling

N.N Abdul Razak
S.M Wah

ABSTRACT

This paper introduces the application of the Transmission Line Modeling (TLM) method for simulating heat flow or thermal distribution in 2-dimensional model comprising of conducting materials. The developed model is able to predict the temperature distribution in a material device and the first geometry considered was a 2D slab with a square heat generation region of 100°C in the center of the slab. The other edges of the slab were assumed to have a zero heat flow condition. The slab is assumed to be of Aluminum 2702 kg/m^3, 903 J/kg.K, 237 W/m.k with a 50x50 mesh size. Coding and simulation are done by using the Salford Plato Software. A high degree of certainty is offered by numerical modeling and the numerical method which is adopted by this paper is the TLM method. TLM has become a powerful numerical tool in solving problems of heat and mass diffusion through a medium. It is based on using transmission line elements to describe all energy storage elements. It is systematic, simple, explicit and unconditional stability has been established. The basic approach of the TLM method is to obtain a discrete model which is then solved exactly by numerical means. The method works in term of incident and scattered voltages and is different from conventional methods. In a 2D TLM model, waves propagate on a mesh of transmission lines interconnected at nodes. A voltage pulse will be incident at the node and this pulse will be partially reflected and transmitted according to transmission line theory. Energy is thus conserved and spreads isotropically from the excited node. The results prove that TLM is a perfectly compatible method to employ and can be used to solve real engineering problems, involving complex geometries, boundary conditions and material properties. Furthermore, the method is expressed in terms of circuit concept which is familiar to engineers and accuracy can be increased by reason of meshing.

Keywords: *TLM, heat distribution, steady-state.*

Introduction

Transmission Line Modeling (TLM) is a general numerical technique suitable for solving field problems. Its main applications have been in electromagnetics, mainly investigation of wave propagation and later for other electronics problems. Recently, the technique has been applied to thermal and diffusion properties problems. The TLM method belongs to the general class of differential time domain numerical modeling methods. The basic approach is to obtain a discrete model which is then solved exactly by numerical means and approximation is only introduced at the discretisation stage. This is to be contrasted with the traditional approach in which an idealized continuous model is first obtained and then this model is solved approximately. For electromagnetics systems, the discrete model is formed by conceptually filling space with a network of transmission lines in such a way that the voltage and current give information on the electric and magnetic fields. The point at which the transmission lines intersect is referred to as a node. At each time step, the voltage is incident upon the node from each of the transmission lines. These pulses are then scattered to produce a new set of pulses which become incident on adjacent nodes at the next time step. Before TLM method for propagation and diffusion problems are developed, Finite Difference Time Domain method (FDTD) is often used for propagation problems. Both TLM and Finite Difference Time Domain methods are based on Maxwell's equation. The E and H field components provided by FDTD and TLM methods are transient fields, which are function of time. Because of this, nonlinear elements and dielectric can be incorporated with both methods. However, although FDTD is explicit just as TLM is, but its application is mainly limited to electromagnetics. The method works with more conventional voltages E field and currents H field. Although TLM was originally used for modeling electromagnetic wave propagation but since it is based on Huygens principle it could be used for modeling any phenomena which obeys this principle.

The principle of TLM modeling is modeled by sampling the space and representing it as a Cartesian mesh of passive transmission line components. Depending on the process being modeled, this can be in one, two or even three dimensions. But for the purpose of this paper, the method is applied only to the two dimensional cases. The transmission lines are joined when they cross, the junctions being termed the nodes. This mesh defines the space discretisation of the model. Assume that at a time zero, an impulse is incident to the middle node. This node then scatters the voltage to its 4 neighbouring nodes. At each time step, each node receives an incident wave from its neighbours and scatters it to its neighbours. By repeating the above, the voltage distribution on a particular medium can be calculated. An impulse incident on the middle node will see impedance one third on the characteristics impedance of the connecting transmission lines. This gives rise to a reflection coefficient of -0.25, resulting in a negative impulse of half the incident amplitude being reflected in the incident element. Positive impulses of the same amplitude are then transmitted into the other three directions. The four reflected impulses are then incident on the surrounding nodes, and themselves are scattered. The process is discretised in time and space so that impulses reflected from a node in one incident and on its neighbours in the next. Combining the calculations for impulses incident in all four branches connected to a node leads to a very simple calculation for the voltage at that node after each iteration. In the next iteration, the scattered pulses will be incident on four new nodes. At each node the process will be repeated and the pulses being injected will then disperse through the network.

Model Development

In contrast to an analytical solution, which allows for temperature determination at any point of interest in a medium, a numerical solution enables determination of the temperature at discrete points only. The first step in any numerical analysis must therefore be to select this point. This is done by subdividing the medium of interest into a number of small regions and assigning to each reference point that is at its centre.

The reference point is frequently termed a nodal point or simply a node on the aggregate of points is termed a nodal network, grid or mesh. Note that, each node represents a certain region and its temperature is a measure of the average temperature of the region. The numerical accuracy of calculations that are performed depends strongly on the number of designated nodal points. Accuracy will be limited for smaller mesh. However, a large number of points may be chosen to obtain a finer mesh and thus a higher accuracy.

The derivation for scattering and connection process procedure of the 2D Aluminum slab can be derived from the Thevenin equivalent circuit of the shunt TLM node. A circuit structure describing a block of space dimensions Δx, Δy, and Δz is referred to as a shunt node. Governing from the equation below is the wave equation for 2D propagation;

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} \tag{1}$$

We can intuitively set voltage V_Z and the current into each port related to E_Z, H_X and H_y. The relationship between field and circuit parameters may be established but a more intuitive approach is adopted here. Propagation along x and y are done separately where then the results are combined to produce an equation describing the spatial and temporal variation of Vz.

Along x-propagation;

$$\frac{\partial^2 Vz}{\partial x^2}\frac{(\Delta x)^2}{Lx} = Cz\frac{\partial^2 Vz}{\partial t^2} \tag{2}$$

Along y-propagation;

$$\frac{\partial^2}{\partial x^2}\left(\frac{Vz}{\Delta z}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{Vz}{\Delta z}\right) = (2\varepsilon)\mu\frac{\partial^2}{\partial t^2}\left(\frac{Vz}{\Delta z}\right) \tag{3}$$

where $\frac{-Vz}{\Delta z} \leftrightarrow Ez$, $\frac{Ix}{\Delta y} \leftrightarrow Hy$, $\frac{Iy}{\Delta x} \leftrightarrow Hx$ and $2\varepsilon, \mu$ are model of medium parameters.

The process of derivation starts from the centre of the node towards each port, which will give

$$Vz = 0.5(V_1^i + V_2^i + V_3^i + V_4^i) \quad (4)$$

$$V_1^r = 0.5(-V_1^1 + V_2^1 + V_3^1 + V_4^1) \quad (5)$$

$$V_2^r = 0.5(V_1^1 - V_2^1 + V_3^1 + V_4^1) \quad (6)$$

$$V_3^r = 0.5(V_1^1 + V_2^1 - V_3^1 + V_4^1) \quad (7)$$

$$V_4^r = 0.5(V_1^1 + V_2^1 + V_3^1 - V_4^1) \quad (8)$$

where Vz is the total voltage of V^i and V^r, $V_{1,2,3,4}{}^i$ are the incident voltage and $V_{1,2,3,4}{}^r$ are the reflected voltage.

A similar expression for the scattering equation of the shunt node would be represented as a matrix below:

$$S = 0.5 * \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \quad (9)$$

Pulses incident on ports of node at axis x and y at time step $k+1$ are pulses reflected from adjacent nodes at the previous time step k. The new incident voltages on node (x,y) depend entirely on the nodes connected to it. This is known as the connection process and it is given as below:

V_1^i [k +1] [x] [y] = V_3^r [k] [x] [y -1] (10)
V_2^i [k +1] [x] [y] = V_4^r [k] [x -1] [y] (11)
V_3^i [k +1] [x] [y] = V_1^r [k] [x] [y + 1] (12)
V_4^i [k +1] [x] [y] = V_2^r [k] [x + 1] [y] (13)

Material Properties

Several important properties are required for the analysis of conduction heat transfer with the developed model. Thermophysical properties need to be considered for they govern either the rates of energy transfer or diffusion rate of momentum transfer. For this case, the three important properties that were took into account for model development are specific heat, cp (J/kg.K), density ρ (kg/m^3) and thermal conductivity, (W/m.k) This three properties describe the diffusion equation that can be used to model thermal conduction in a material. The diffusion equation is as below;

$$\frac{\partial^2\theta}{\partial x^2} = \frac{S}{k_{th}}\frac{\partial\theta}{\partial t} \quad (14)$$

However, to compensate with the developed model the equation is suited with an applied heat source resulting in the equation below;

$$\frac{\partial\theta}{\partial t} = \frac{kth}{S}\Delta^2\theta + \frac{Q}{S} \quad (15)$$

where S = specific heat in J/Km^3, k_{th} = thermal heat conductivity in W/Km, θ = temperature and Q = heat source

(W/m^3).

The equivalence of circuit parameters to thermal are;

$$R = \frac{\Delta l / 2}{kth * A} \tag{16}$$

$$C = S * A * \Delta l \tag{17}$$

$$I = Q * A * \Delta l \tag{18}$$

$$Z = 2\Delta t / C \tag{19}$$

where R= resistance, C=capacitance, I= current, Z= impedance and A= area of model.

Results

A 3D graph plot of the temperature profile for the Aluminum slab with mesh 50x50 with a square excitation heat source region of 100° in the middle of the node can be seen in Figure 1. The other edges of the slab were assumed to have zero heat flow conduction. This initial setup is important as temperature fields in any body, depend not only on the properties of the material, but also on the initial condition and the condition imposed on its boundaries. The initial condition is given by specifying the temperature T at time t=0 at points on the solid body. The computational results for different timesteps are shown as below:

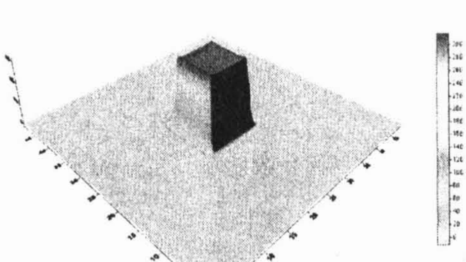


Fig. 1: Timestep 1 of 1000

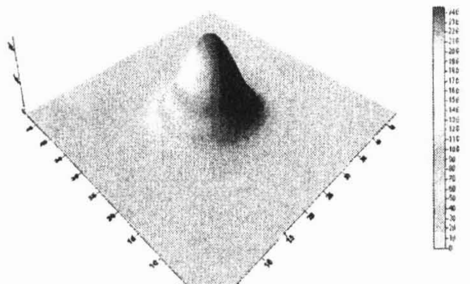


Fig. 2: Timestep 50 of 1000

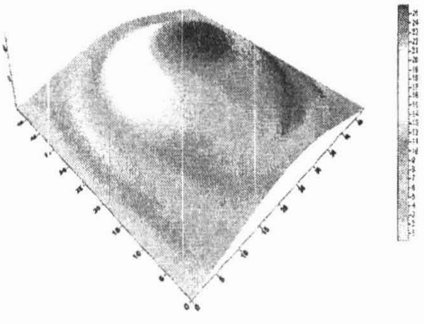


Fig. 3: Timestep 700 of 1000

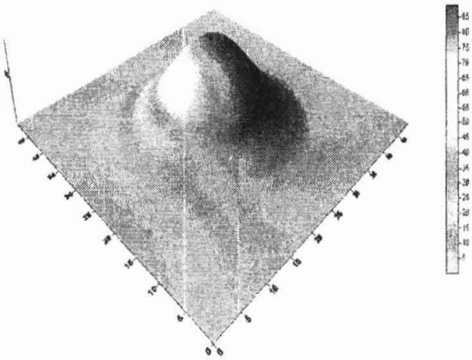


Fig. 4: Timestep 1000 of 1000

The temperature contours in Figure 1, 2, 3 and 4 clearly indicate patterns of conduction from the initial temperature at t = 1 until the last timestep of t = 1000. The highest temperature hits almost 320° at initial point but as conduction begins heat begins to distribute across the whole 2D solid and as can be seen in Figure 4, at timestep t = 1000 steady state is almost reached. This confirms the theoretical aspect that at a certain point, temperature across a solid would remain relatively constant, where there is no temporal change in temperatures.

The model is validated and run again for different timesteps and for each case the results of validation are plotted below where the temperature profiles show that point of steady state conduction is achieved at t = 50 000. Figure 6 shows the heat source shaded relief of the Aluminum plate after total heat distribution.

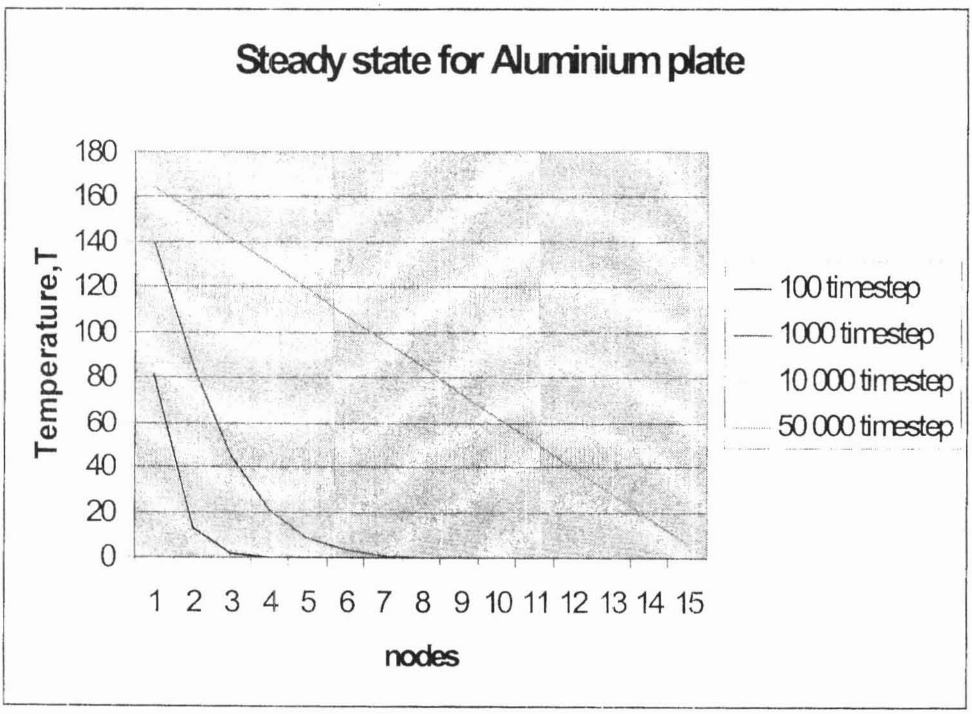


Fig. 5: Steady State Profiles of Different Timesteps Group

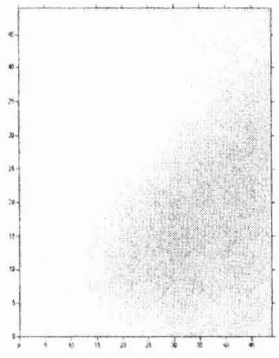


Figure 6: Shaded Relief of Aluminum Plate

Conclusion

Engineering problems which involve real materials and real boundary conditions can be solved analytically provided a large number of simplifications are made. Hence, analytical solutions thus obtained will only be an approximate. Often the cases were, we deal with problems which involve complicated geometries or nonlinear properties, which exact analytical solutions usually become difficult or impossible. Numerical solutions is usually the best if not the only approximate solution and this paper has shown that the Transmission Line Modeling method has been proved easy and it is perfectly compatible to show temperature distribution in a solid 2D model. Therefore, this numerical technique, the TLM method can be used to solve real engineering problems involving complex geometries, boundary conditions and material properties.

References

Christopolous, C. (1993). *The Transmission Line Modeling (TLM) Method*. IEEE Press, Piscataway

Johns, D.P, Mallik, A. & Wlodarczyk, A.J. (1992). *TLM Enhancement for EMC Studies*. IEEE Proc. Regional Symp. On EMC.

Singleton, P. (1985). *Wave velocity of Transmission Line using Stud TLM*. Msc

N.N ABDUL RAZAK & S.M WAH, Department of Electrical Engineering, College of Engineering, University Tenaga Nasional.

Department of Electrical Engineering, College of Engineering, University Tenaga Nasional.