Effect of Displacement Vector in the Direction of Arrival Estimation

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Abstract—**There is a relationship between the direction of a signal and the associated received steering vector. Therefore, it should be possible to invert the relationship and estimate the direction of a signal from the received signals. An antenna array should be able to provide for direction of arrival (DOA) estimation. There is also Fourier relationship between the beam pattern and the excitation at the array. This allows the DOA estimation problem to be treated as equivalent to spectral estimation. This paper clarifies the effect of variation displacement vector in estimating the DOA in a smart antenna application. The objective is to find the optimum value of element spacing where it will give the best DOA estimation of signal impinging on a uniform linear array (ULA). The algorithms used in detecting the DOA are the Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational invariance Techniques (ESPRIT). The two algorithms produce an ambiguity in the estimated direction-of-arrival results, when the antenna element spacing on a linear array is more than half a wavelength.**

Keywords—**Direction of Arrival (DOA), Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational invariance Techniques (ESPRIT), Uniform Linear Array (ULA)**

I. INTRODUCTION

Smart antenna generally refers to antenna array with smart signal processing algorithm used to identify spatial signal signature such as the direction of arrival of the signal, and use it to calculate beamforming vectors, to track and locate the antenna beam on the mobile or target. Smart-antenna systems provide opportunities for higher system, capacity, improved quality of service (QoS), and improved power control (PC) and extended battery life in portable units [2].

In smart antenna system, digital signal processor plays an important role. It functions to estimate the DOA of all impinging signals from the time delays of each antenna element. Moreover, it is responsible to estimate the appropriate weights to scan the maximum radiation of the antenna pattern towards the signal of interest (SOI), and to place nulls toward the signal not of interest (SNOI) [3]. Hence, accurate estimation of DOA is needed for smart antenna system. There are many DOA algorithms found in the literature today. The most popular among those algorithms are MUSIC and ESPRIT which are both categorized as eigendecomposition-based methods. Although they are known to be super-resolution, they generate ambiguous error when applied to estimate the DOA of radio signal impinging on a ULA of element spacing more than half wavelength [4].

A. *Direction of Arrival Estimation*

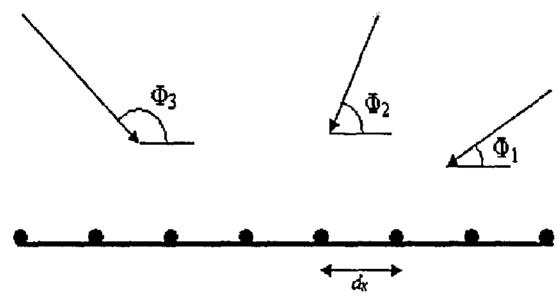

Figure 1. Linear Array configuration model

Direction of arrival is referred to the direction from which usually a propagating wave arrives at a point, where usually a set of sensor are located. This set of sensors form what is called a sensor array. Often, there is the associated technique of beamforming which is estimating the signal from a given direction.

Figure 1 shows the Linear Array configuration model where d_x is the element spacing and Φ_i is the element-to-element phase shift (phase gradient). The scan angle, θ can be written as in equation:

$$\theta = \sin^{-1}\frac{\Phi\lambda}{2\pi dx} \tag{1}$$

where λ is the wavelength

The DOA can be estimated by using various techniques. The techniques are classified according to their performance, sensitivity and limitations. These techniques include Spectral Estimation Methods, Eigenstructure Methods, Multiple Signal Classification (MUSIC) Algorithm, Min-Norm Method, CLOSEST Method, Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and etc [5]. This paper only applies and discusses two methods; ESPRIT and MUSIC Algorithm.

B. ESPRIT

ESPRIT is a computationally efficient and robust method of DOA estimation. It uses two identical arrays in the sense that array elements need to form matched pairs with an identical displacement vector. The second element of each pair should be displaced by the same distance and in the same direction relative to the first element. However, it does not mean that it is a must to have two separate arrays. The array geometry should be such that the elements could be selected to have this property [5].

$$\text{Let } z_M = e^{jkd\cos\emptyset_m} \quad (1)$$

$$S = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-2} & z_2^{N-2} & \cdots & z_M^{N-2} \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M \end{bmatrix} \quad (2)$$

Based on this matrix, define two $(N-1) \times M$ matrices, S_0 and S_1,

$$S_0 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-2} & z_2^{N-2} & \cdots & z_M^{N-2} \end{bmatrix}$$

$$S_1 = \begin{bmatrix} z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-2} & z_2^{N-2} & \cdots & z_M^{N-2} \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix} \quad (3)$$

and note that $S_1 = S_0\Phi$ whereΦ is the $M \times M$matrix

$$\Phi = \begin{bmatrix} z_1 & 0 & \cdots & 0 \\ 0 & z_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & z_M \end{bmatrix}, \quad (4)$$

i.e. Φ is a diagonal matrix whose entries correspond to the phase shift from one element to the next due to each individual signal. If we can estimate Φ, we can estimate the DOA of all signals using Eqn. (1).

If $S_0$and S_1 were known, Φ can be solved easily. They are unknown matrices and we must use proxies to obtain the same result. The ESPRIT algorithm begins by recognizing that the steering vectors in matrix S span the same subspace the matrix Q_s, the $N \times M$matrix of 15 signal eigenvectors. Since both these matrices span the same subspace, there exists an invertible matrix Csuch that

$$Q_s = SC \quad (5)$$

Defining matrices $Q_0$and Q_1 derived from Q just as $S_0$and S_1 were derived from S, i.e., Q_0 comprises the first $(N-1)$ rows of Q and Q_1 the last $(N-1)$ rows of Q, and using Eqn. (5), we have

$$Q_0 = S_0C$$

$$Q_1 = S_1C = S_0\Phi C$$

Consider,

$$Q_1C^{-1}\Phi^{-1}C = S_0\Phi CC^{-1}\Phi^{-1}C = S_0C = Q_0 \quad (6)$$

Now, let

$$\Psi^{-1} = C^{-1}\Phi^{-1}C$$

$$\rightarrow Q_1\Psi^{-1} = Q_0$$

$$\rightarrow Q_1 = Q_0\Psi \quad (7)$$

Where

$$\Psi = C^{-1}\Phi C \quad (8)$$

Equation (7) implies that the matrix Φ is a diagonal matrix of the eigenvalues of Φ. Eqns. (7) and (8) are now the complete algorithm.

C. MUSIC Algorithm

Of all the techniques, MUSIC is probably the most popular technique. MUSIC, as are many adaptive techniques, is dependent on the correlation matrix of the data [1].

$$x = S_\alpha + n \quad (1)$$

$$S = [s(\emptyset_1), s(\emptyset_2), \ldots, s(\emptyset_M)] \quad (2)$$

$$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_M]^T \quad (3)$$

The matrix S is and $N \times M$ matrix of the Msteering vectors. Assuming that the different signals to be uncorrelated, the correlation matrix of xcan be written as:

$$R = E[xx^H] \quad (4)$$

$$= E[S\alpha\alpha^H S^H] + E[nn^H]$$

$$= SAS^H + \sigma^2 I \quad (5)$$

$$= R_s + \sigma^2 I \quad (6)$$

where

$$R_s = SAS^H \quad (7)$$

$$A = \begin{bmatrix} E[|\alpha_1|^2] & 0 & 0 \\ 0 & E[|\alpha_2|^2] & 0 \\ 0 & 0 & E[|\alpha_M|^2] \end{bmatrix} \quad (8)$$

The signal covariance matrix R_s is an $N \times N$ matrix with rank M. Therefore, it has $N - M$eigenvectors corresponding to the zero eigenvalue. Let q_m be such an eigenvector. Therefore,

$$R_s q_m = SAS^H q_m = 0 \quad (9)$$

$$\rightarrow q_m^H SAS^H q_m = 0 \quad (10)$$

$$\rightarrow S^H q_m = 0 \quad (11)$$

Where this final equation is valid since the matrix A is positive definite. Equation (12) implies that all $N - M$ eigenvectors (q_m) of R_s corresponding to the zero eigenvalues are orthogonal to all Msignal steering vectors. This is the basic for MUSIC. Call Q_n the $N \times (N - M)$ matrix of these eigenvectors. MUSIC plots the pseudo-spectrum

$$P_{MUSIC}(\emptyset) = \frac{1}{\sum_{m=1}^{N-M} |S^H(\emptyset) q_m|^2}$$

$$= \frac{1}{S^H(\emptyset) Q_n Q_n^H s(\emptyset)}$$

$$= \frac{1}{\|Q_n^H s(\emptyset)\|^2} \quad (12)$$

Since the eigenvectors making up Q_n are orthogonal to the signal steering vectors, the denominator becomes zero when $\emptyset$ is a signal direction. Therefore, the estimated signal directions are the M largest peaks in the pseudo-spectrum. However, in practically, the signal covariance matrix $R_s$would not be available. The most we can expect is to be able to estimate R the signal covariance matrix. The key is that the eigenvectors in Q_n can be estimated from the eigenvectors of R.

For any eigenvector $q_m \in Q$,

$$R_s q_m = \lambda q_m$$

$$\rightarrow R q_m = R_s q_m + \sigma^2 I q_m$$

$$= (\lambda_m + \sigma^2) q_m \quad (13)$$

i.e. any eigenvector of R_s is also an eigenvector of R with corresponding eigenvalue $\lambda + \sigma^2$. Let $R_s = Q \wedge Q^H$. Therefore,

$$R = Q[\wedge + \sigma^2 I]Q^H$$

$$= \begin{bmatrix} \lambda_1 + \sigma^2 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 + \sigma^2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda^2 m + \sigma^2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \lambda^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \lambda^2 \end{bmatrix} Q^H$$

(14)

Based on this eigendecomposition, we can partition the eigenvector matrix Q into a signal matrix Q_s with Mcolumns, corresponding to the M signal eigenvalues, and a matrixQ_n, with $(N - M)$columns, corresponding to the noise eigenvalues (σ^2). Note that Q_n, the $N \times (N - M)$ matrix of eigenvectors corresponding to the noise eigenvalue (σ^2), is exactly the same as the matrix of eigenvectors of Rs corresponding to the zero-eigenvalue. This is the matrix used in Eqn.(12).

Q_s defines the signal subspace, while Q_n defines the noise subspace. There are few important observations to be made:

- The $m -$ th signal eigenvalue is given by $\lambda_m + \sigma^2 = N|\alpha_m|^2 + \sigma^2$
- The smallest eigenvalues of R are the noise eigenvalues and are all equal to σ^2, i.e. one way of distinguishing between the signal and noise eigenvalues (equivalently the signal and noise subspaces) is to determine the number of small eigenvalues that are equal.
- By orthogonality of Q, $Q_s \perp Q_n$

Using the final two observations, we see that all noise eigenvectors are orthogonal to the signal steering vectors. This is the basis for MUSIC. Consider the following function of $\emptyset$:

$$P_{MUSIC}(\emptyset) = \frac{1}{\sum_{m=M+1}^{N} |q_m^H S(\emptyset)|^2} = \frac{1}{s^H(\emptyset) Q_n Q_n^H s(\emptyset)} \quad (15)$$

where q_m is one of the $(N - M)$ noise eigenvectors. If $\emptyset$ is equal to DOA one of the signals, $s(\emptyset) \perp q_m$ and the denominator is identically zero. Therefore, MUSIC identifies the directions of arrival with the peaks of the function is $P_{MUSIC}(\emptyset)$.

II. METHOGOLOGY

A. Project Flow

A study on the smart antenna system and DOA estimation includes the techniques and the algorithms have been done. The two algorithms being used are chosen due to the easy understanding of the mathematical equations. The software being used is MATLAB. The process and steps taken in running the program will be explained in detail in the next part. Various displacement vectors have been tested. Data were compiled and then being analyzed in obtaining the performance of the algorithms with the difference in the displacement vector. The above project flow is illustrated in Figure 1.

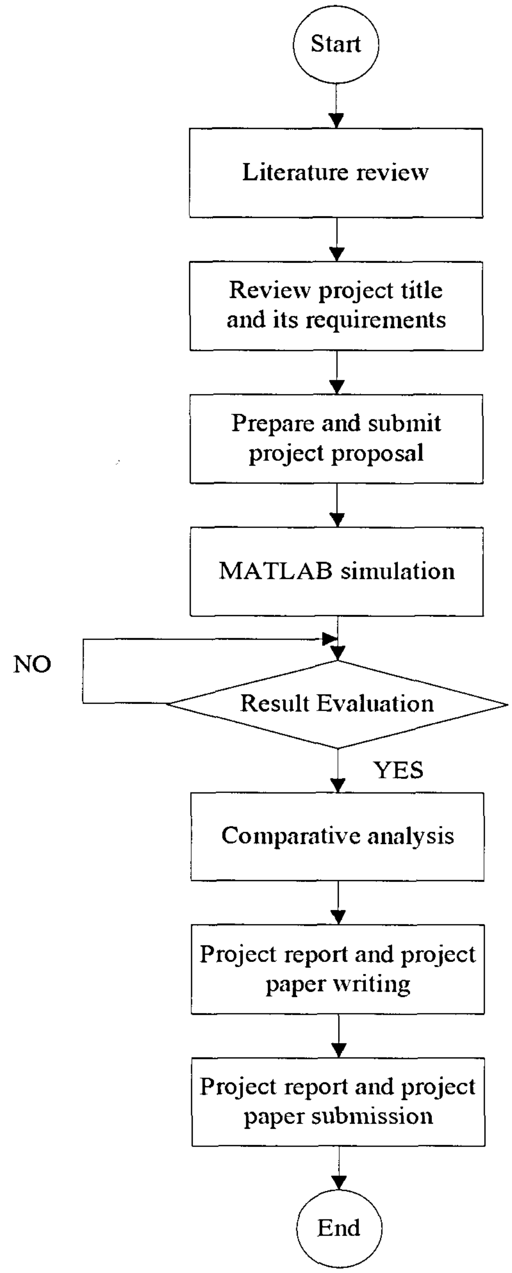

Figure 1. The project flow chart

B. Software Flow

The program is designed to have the mathematical model of ESPRIT and MUSIC algorithms that can be used to estimate the DOA of the incident signals. The parameters required in obtaining the DOA are defined at the very beginning of the program. They are number of sensors, sensors spacing and number of samples. The two parameters; number of sensors and number of samples are different for both ESPRIT and MUSIC Algorithms. They are set according to the values which both algorithms will give the best output of DOA. They are 15 sensors with 100 samples and 20 sensors with 500 samples are used for MUSIC and ESPRIT respectively, whereas the number of sources is set to be 2 [6]. The sensor spacing is varies time by time according to the data being tested.

The program continues by defining the parameters of the sources or incident signals. The sources being used is 20° and 70° for the two sources respectively. This applied for both ESPRIT and MUSIC algorithms. The effect of noise is included by inserting noise of 20dB. The noise variance added to the synthetic data has been set equal to that of the noise affecting the real data or environment [7]. Both parameters; number of sensors and number of samples will produce the covariance matrix and thus the array steering vector for the direction of the incident signals. The matrix is formed by applying the mathematical equations for both ESPRIT and MUSIC Algorithms. The output for ESPRIT gives two bearings in degrees whereas for MUSIC, the output is performed in the form of graph. The graph represents the power of the signal versus its DOA.

The program is tested for various elements spacing covering several ranges in multiple times. The readings are taken and analyzed until the best desired data is determined. The program mentioned above is illustrated in Figure 2.

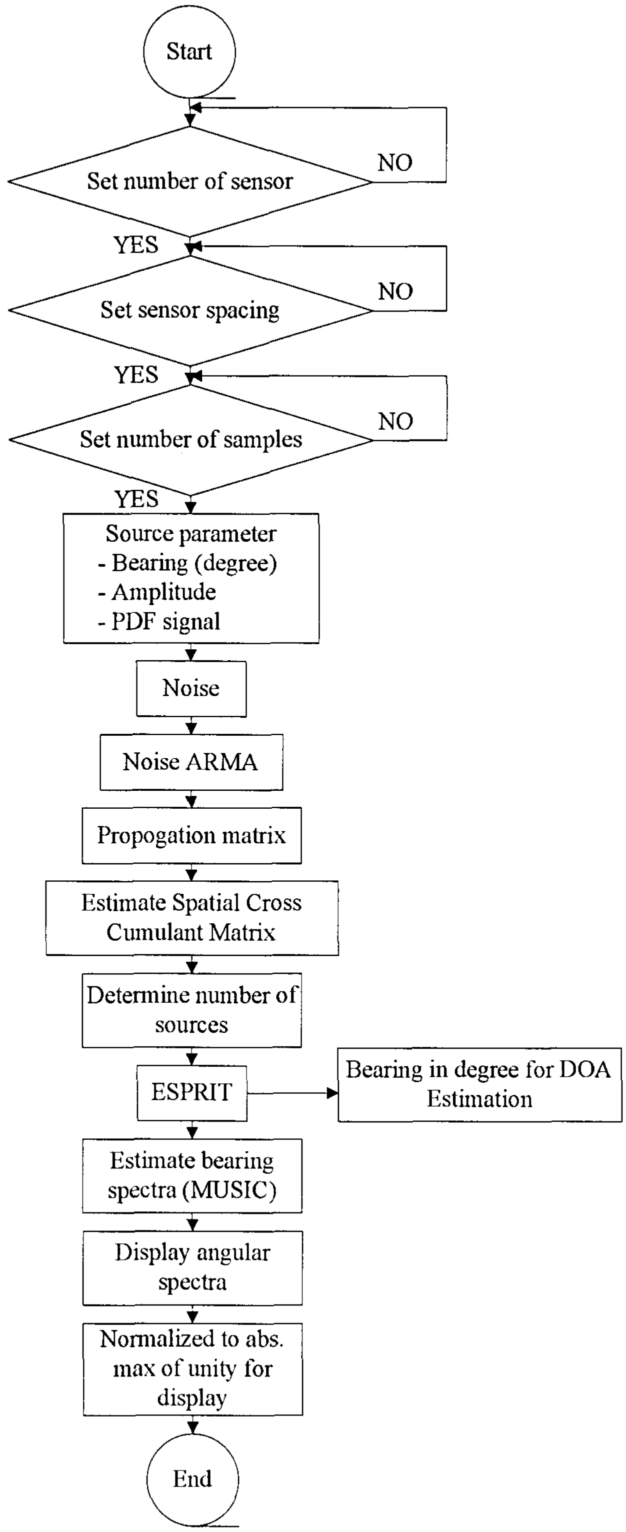

Figure 2. The software flowchart

III. RESULTS

It is said that the MUSIC and ESPRIT algorithms produce an ambiguity in the estimated DOA results, when the antenna elements spacing on a linear array is more than half a wavelength [8]. Many studies [8], [9] stated that the optimum smart antenna array spacing is half a wavelength. Therefore, the more accurate element spacing that should be used in estimating the DOA by using the stated algorithms is clarified in this paper.

This project consists of several parts where each stage acts as the input to the next stage. By using MATLAB simulation, the project has been run and the result is obtained. Repetitive readings have been taken for each array spacing range and the best reading for each spacing range is presented as the results of the project.

Taking half a wavelength as the benchmark of the element spacing, a wide range of element spacing values are tested to obtain the DOA. TABLE I shows the DOA of elements spacing from 0.1λ to 1.0λ by using ESPRIT algorithm. The range of 0.4λ to 0.6λ of elements spacing gives the nearest reading, hence has lower percentage of error of DOA with the true two impinged signals.

TABLE I
DOA BY USING ESPRIT FOR SENSOR SPACING OF 0.1λ - 1.0λ

Sensor spacing in unit of wave-length	ESPRIT			
	20°	% Error	70°	% Error
0.1	65.9877	69.7	47.5165	32.1
0.2	Unstable	Error	-20.029	71.4
0.3	11.5973	42	-12.5249	37.4
0.4	**15.1789**	**24.1**	**65.2861**	**6.7**
0.5	**20.8142**	**4.1**	**47.9481**	**31.5**
0.6	**18.2511**	**8.7**	**45.6016**	**34.9**
0.7	-20.2211	1.1	24.9533	64.4
0.8	17.469	12.7	-36.1147	48.4
0.9	19.2673	3.7	-15.3476	78.1
1.0	5.9368	70.3	29.1125	58.4

TABLE II shows the results of the second part of this project which is the DOA of elements spacing from 0.1λ to 1.0λ by using MUSIC algorithm. The similar range of elements spacing gives the nearest reading of DOA with the true impinged signals.

TABLE II
DOA BY USING MUSIC FOR SENSOR SPACING OF 0.1λ - 1.0λ

Sensor spacing in unit of wave-length	MUSIC			
	20°	% Error	70°	% Error
0.1	24	20	72	2.9
0.2	22	10	66	5.7
0.3	18	10	68	2.9
0.4	**22**	**10**	**70**	**0**
0.5	**20**	**0**	**70**	**0**
0.6	**20**	**0**	**68**	**2.9**
0.7	18	10	74	5.7
0.8	22	10	72	2.9
0.9	20	0	70	0
1.0	20	0	72	2.9

The next part is to simulate the program using the array spacing obtained from the previous parts applying smaller differences. This is done for both ESPRIT and MUSIC algorithms. TABLE III shows the results where the sensors spacing between 0.48λ and 0.52λ for both ESPRIT and MUSIC algorithms have DOA with lower percentage of error compared to the other.

TABLE III
DOA BY USING ESPRIT AND MUSIC FOR SENSOR SPACING OF 0.40λ - 0.60λ

Sensor spacing in unit of wave-length	ESPRIT				MUSIC			
	20°	% Error	70°	% Error	20°	% Error	70°	% Error
0.40	28.32	41.6	69.62	0.5	18	10	66	5.7
0.41	13.99	30.1	Unstable	Error	18	10	68	2.9
0.42	10.85	45.8	69.15	1.2	20	0	74	5.7
0.43	24.24	21.2	Unstable	Error	20	0	72	2.9
0.44	27.42	37.1	52.19	25.4	18	10	68	2.9
0.45	16.23	18.9	Unstable	Error	18	10	68	2.9
0.46	69.88	Error	-6.43	90.8	22	10	72	2.9
0.47	26.55	32.8	63.66	9.1	20	0	72	2.9
0.48	**21.41**	**7.1**	**75.08**	**25.4**	**18**	**10**	**72**	**2.9**
0.49	**22.75**	**13.8**	**76.30**	**9.0**	**20**	**0**	**68**	**2.9**
0.50	**25.76**	**28.8**	**66.44**	**5.1**	**22**	**10**	**76**	**8.6**
0.51	**31.01**	**35.5**	**58.73**	**16.1**	**20**	**0**	**64**	**8.6**
0.52	**21.93**	**8.8**	**50.26**	**28.2**	**20**	**0**	**78**	**11.4**
0.53	-2.61	87.0	-42.85	38.8	20	0	70	0
0.54	26.48	32.4	58.18	16.9	20	0	80	14.3
0.55	33.01	65.0	64.08	8.5	20	0	68	2.9
0.56	15.84	20.8	27.13	61.2	20	0	70	0
0.57	17.57	12.1	57.91	17.3	20	0	72	2.9
0.58	10.14	49.3	-53.93	23.0	22	10	70	0
0.59	33.85	69.2	-3.06	95.6	22	10	70	0
0.60	14.17	29.2	56.22	19.7	20	0	70	0

The simulation continues by narrowing down the array spacing range to gain a more accurate and precise result. The values of array spacing used are obtained from the result of the previous stage. TABLE IV shows that the elements spacing of 0.49λ gives the lowest percentage of error, thus the best DOA. The simulation ended as the value is obtained.

TABLE IV
DOA BY USING ESPRIT AND MUSIC FOR SENSOR SPACING OF 0.48λ - 0.52λ

Sensor spacing in unit of wave-length	ESPRIT				MUSIC			
	20°	% Error	70°	% Error	20°	% Error	70°	% Error
0.48	23.24	16.2	-9.43	86.5	10	50	70	0
0.49	**20.65**	**3.3**	**51.88**	**25.9**	**20**	**0**	**70**	**0**
0.50	19.34	3.3	47.99	31.4	20	0	70	0
0.51	19.22	3.3	48.21	31.1	22	10	72	2.9
0.52	7.08	64.6	49.85	28.8	20	0	70	0

IV. DISCUSSION

The number of sensors and samples are set according to the values which both algorithms will give the best output of DOA. They are 15 sensors with 100 samples and 20 sensors with 500 samples are used for MUSIC and ESPRIT respectively, whereas the impinging signals are set to be 20° and 70° for both algorithms. The effect of noise is included by inserting noise of 20dB. The project ran by gathering the data of DOA for various displacement vectors in wide range. The elements spacing which gives nearest DOA with the sources bearing are then being narrowed down with smaller range. This process continues until the best DOA is obtained.

From the observation and data analysis of this project, different value of element spacing will give different DOA. There is because only the suitable and correct spacing will produce a good DOA estimation. Besides, the DOA fluctuates as the sensors spacing is varied. This is due to the existence of noise as noise will interrupt and give disturbance to the DOA. This theory is applied in real signal processing of smart antenna system.

Towards the end of this project, the value of elements spacing which gives the best DOA reading is determined. It gives a value of 0.49λ, which is more accurate than the previous value being used; 0.5λ [6].

V. CONCLUSION

The target of this project is to find the exact or at least a more accurate value of element spacing for a set of linear array in the smart antenna system. It is quite difficult to accurately get the value because of the existence of noise, as noise will interrupt the output of DOA. But MATLAB simulation has made the objective achieved. The result shows that a more accurate value of sensor spacing in smart antenna system is obtained. Comparatively, the stated ideal spacing is half a wavelength, but through this project, it is found that the exact spacing is 0.49λ. Knowing the fundamental of array signal processing, more accurate and reliable spacing parameter can be found.

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