CHROMATIC UNIQUENESS OF CERTAIN COMPLETE TRIPARTITE GRAPHS WITH SOME EDGES DELETED

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ABSTRACT

Since the introduction of the concepts of chromatically unique graphs and chromatically equivalent graphs, many families of such graphs have been obtained. In this thesis, we continue with the search of new families of chromatically unique graphs

In Chapter 1, we define the concept of graph colouring, the associated chromatic polynomial and some properties of chromatic polynomial. We also give some necessary conditions for graphs that are chromatically unique or chromatically equivalent.

Chapter 2 begins with the definition of complete tripartite graphs and some preliminary results related to complete tripartite graphs. In particular, certain chromatically unique complete tripartite graphs are given. Some new chromatically unique complete tripartite graphs are then presented at the end of this chapter.

In Chapter 3, we first give some necessary conditions for complete tripartite graphs with some edges deleted to be chromatically equivalent. We then proved that certain complete tripartite graphs with one edge deleted are also chromatically unique.

The determination of the chromaticity of other complete tripartite graphs with certain edges deleted is left as an open problem for future research. We end the thesis by making a conjecture that all complete tripartite graphs with each partite set having at least two vertices are chromatically unique.

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CHAPTER ONE

INTRODUCTION

1.1 Introduction

A graph G is a collection of the edge set and the vertex set, denoted $E(G)$ and $V(G)$, respectively. An edge e of G is a line joining two vertices in $V(G)$, say v_i and v_j, denoted v_iv_j or (v_i, v_j). The theory of graph was invented by Euler [10] more than two centuries ago in trying to solve the famous Königsberg bridge problem. For a century, one of the most famous problems in mathematics and graph theory was to prove the four colour conjecture which stated that "For any positive integer λ and any map M, it is possible to colour the map M with at most λ colours such that no two adjacent regions are assigned the same colour". This has spawned the development of many useful tools for solving graph colouring problems.

In a paper in 1912, Birkhoff [2] proposed a way of tackling the four-colour problem by introducing a function $P(M, \lambda)$, defined for all positive integers λ, to be the number of proper λ-colourings of a map M. It turns out that $P(M, \lambda)$ is a polynomial in λ, called the chromatic polynomial of M. If one could prove that $P(M, 4) > 0$ for all maps M, then this would give a positive answer to the four-colour conjecture. It was hoped that many useful tools from algebra and analysis could be used to find or estimate the roots of the polynomial and hence lead to the resolution of the problem.

The notion of a chromatic polynomial was later generalized to the vertex colouring of an arbitrary graph by Whitney (1932) [21], who established many fundamental