RISK MINIMIZATION FOR A PORTFOLIO USING MEAN ABSOLUTE DEVIATION AND CONDITIONAL VALUE-AT-RISK

Iylia Lyiana Rahim, Mohd Azdi Maasar, Siti Ayuni Mohd Jamil, and Nur Anis Mohd Aziz
Universiti Teknologi MARA Negeri Sembilan
Corresponding author: azdimaassar@tmsk.uitm.edu.my

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1. Introduction

Portfolio selection is one of the major problems in finance since the expected return of the assets is unknown at the time of investment is made. Usually, investors will seek to invest in assets that will yield higher returns and possess lower risks. In order to make better decision-making, investors should conduct a thorough decision analysis in selecting the stocks for their portfolio investment, as well as to determine the ways of distributing amount of money to several assets. Below is the simplest form of determining the weight of the portfolio:

$$X = \{(x_1, ...x_n) | \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j = 1...n\} \quad (1)$$

Referring to Equation 1, a portfolio is denoted by $x = (x_1, ...x_n)$ where x_j is categorized as the proportion of the capital invested in asset j, and $j = 1...n$. $(x_1, ...x_n)$ which known as portfolio weights, is necessary for investment decision as it plays a role in forming a feasible set X, by which it must not be a negative value and the sum of the portfolio weights must be equals to 1. The return of the portfolio $x = (x_1, ...x_n)$ is a random variable, denoted by R_x and its dependency on individual asset returns $R_1, ...R_n$ is expressed as in Equation 2 below:

$$R_x = x_1 R_1 + ... + x_n R_n \quad (2)$$

Based on Roman and Mitra (2009) , a portfolio selection is made by solving an optimization problem, involving minimizing the risk and setting a desired level of target returns as a constraint. Hence, it is crucial to comply with these conditions in order to obtain the feasible set X as in Equation 1. In the field of portfolio theory, Modern Portfolio Theory (MPT) pioneered by Harry Markowitz in 1952, had used variance as a risk measure. The theory was the major breakthrough in financial decision-making and has led to the introduction of several alternative risk measures. Consequently, various applications of mean-risk models motivated from MPT are studied and presented. (see Abdul Razak et al. (2019), Maasar et al. (2016), Maasar et al. (2020), Maasar et al. (2021) for examples).

In this study, the mean absolute deviation (MAD) and the conditional value-at-risk (CVaR) are chosen to be the risk measures for the risk-minimizing portfolios. MAD and CVaR which comes from different types of risk measure, have different computational method since both risk measure holds onto different purposes. While MAD is the attempts to linearize the procedure of portfolio optimization that can hardly be solved using variance since it is a quadratic program, CVaR, on the other hand, can easily be handled in optimization and statistics. According to Krokhmal et al. (2002) , the mean-CVaR model is efficient for computing a numerous assets to be combined in a portfolio. From the outcome, the relationship between portfolio returns with each of the mean-risk model therefore definitely be different. Hence, the evidence on the efficiencies of the mean-MAD and mean-CVaR model are evaluated based on the models' performance in Malaysia's stock market.

2. Methodology

This section presents the methodology applied in this study involving data collections and formulation of the models.

The monthly historical closing price of 23 companies in FBMKLCI were collected within 10 years, starting from January 2008 until January 2018 from Yahoo! Finance. From these closing price, the monthly returns for each assets are carefully simulated to represent the random returns r_{ij} for asset i in scenario j.

3. Construction of In-sample Portfolio

From the simulated scenario returns, as many as 20 in-sample portfolios were obtained from the available scenarios. The first 100 scenarios were used to construct the first in-sample portfolio, and the second 100 scenarios for the second in-sample portfolio, until a total of 20 in-sample portfolios were accumulated. The expected return, μ_j of 23 assets were taken into consideration to determine the desired level of target return, d since it must be in the range of μ_j to optimize the in-sample portfolios. The level of d were specified into three levels; 1.6%, 2.0% and 2.7% as low risk, medium return and high return respectively. The notation used in this research computational analysis are shown in the next page:

S = the number of scenarios
n = number of assets
r_{ij} = return of asset j under scenario i, $j \in \{1 \dots n\}$, i $\in \{1 \dots S\}$
μ_j = the expected return of asset j
x_j = the fraction of the capital invested in asset j
y_i = deviations of portfolio returns
d = the desired level of expected return for the portfolio
v = the negative of α - quantile of the portfolio return distribution

3.1. *Mean-Mean Absolute Deviation (MAD)*

The first model used in this research was mean-MAD. This model was proposed by Konno and Yamazaki as an alternative to the Markowitz model. Mean-MAD model can be demonstrated to generate an optimal portfolio much faster than Markowitz model because it can be reduced to a linear programming problem instead of quadratic programming problem. Besides, it is easier to compute than Markowitz model because it eliminates the need for covariance matrix. The Markowitz model is difficult to solve for large data sets, hence MAD model was mainly introduced. Additionally, mean-MAD model minimize the risk where the measure in this case is the mean absolute deviation. Large mean absolute deviation means the portfolio is high in risk. Mean-MAD model corresponds to $MAD(R_x) = E[|(R_x - E(R(x))|]$ can also be formulated as follows:

$$\min \frac{1}{S} \sum_{i=1}^{S} y_i \tag{3}$$

Subject to:

$$\sum_{j=1}^{n} (r_{ij} - \mu_j)\, x_j \leq y_i, \forall_i \in \{1...S\}$$

$$\sum_{j=1}^{n} (r_{ij} - \mu_j)\, x_j \geq -y_i, \forall_i \in \{1...S\}$$

$$y_i \geq 0, \forall_i \in \{1...S\}$$

$$\sum_{j=1}^{n} \mu_j x_j \geqslant d; x \in X$$

where $(x_1, ..., x_n)$ represents the portfolio weights, S decision variable $y_i = 1, \ldots, S$ represents the absolute deviations of the portfolio return R_x from its expected value, for every scenario $i \in \{1, \ldots, S\}$:

$$y_i = \left| \sum_{j=1}^{n} (r_{ij} - \mu_j)\, x_j \right|, \forall_i \in \{1...S\}$$

3.2. *Mean-Conditional Value-at-Risk (CVaR)*

Conditional Value at Risk (CVaR) attempts to address the shortcomings of the VaR method, a mathematical methodology used to measure the level of financial risk within a company or portfolio of securities over a specific time frame. Whereas VaR represents a worst-case loss associated with a probability and time horizon, if that worst-case threshold is ever crossed, CVaR is the predicted loss. In other terms, CVaR quantifies the expected losses over and above the VaR breakpoints.

$$y_i = \begin{cases} -v - \sum_{j=1}^{n} r_{ij} x_j, & \text{if } \sum_{j=1}^{n} r_{ij} x_j \leq -v \\ 0, & \text{otherwise} \end{cases}$$

The algebraic formulation of the mean-CVaR model is given below:

$$\min v + \frac{1}{\alpha S} \sum_{i=1}^{S} y_i \tag{4}$$

Subject to:

$$\sum_{j=1}^{n} -r_{ij} x_j - v \leq y_i,\ \forall i \in \{1 \ldots S\}$$

$$y_i \geq 0,\ \forall i \in \{1 \ldots S\}$$

$$\sum_{j=1}^{n} \mu_j x_j \geq d;\ x \in X$$

In order to determine the CVaR, there are few values that need to be identified such as number of scenarios (Data Count) used, rank, VaR, expected returns, $r_{ij} x_j$ and deviations of portfolio return, y_i.

4. Results and Discussion

In this study, 10-years historical closing price of 23 risky assets in the FBMKLCI, amounting a total of 120 monthly price has been collected. A total of 119 scenario returns are then simulated from the monthly prices of the risky assets. These simulated (also called scenarios) returns are implemented in the selected mean-risk models to obtain the in-sample portfolios . The three appropriate target return chosen for this study are, 1.6% as a low risk-low return, 2.0% as medium risk-medium return and 2.7% as a high risk-high return investing.

All 119 scenario returns has been used in the construction of 20 in-sample portfolios, in which each in-sample used 100 different scenarios in sequential order. Then, the 119 scenario returns are then reused to test and validate the 20 in-sample portfolios using out-of-sample analysis. The construction of the in-sample portfolio and validation of the portfolio using out-of-sample analysis is necessary for both mean-risk models in the process of minimizing the risk. What we can see from the in-sample portfolio results for mean-MAD and mean-CVaR is, the higher the target return, the higher the risk and the lower the target return, the lower the risk. The mean-CVaR also shows the same pattern of results with the mean-MAD. The first objective that we set has been achieved, where we have successfully minimized the risk for a portfolio using MAD and CVaR as risk measures.

Other than that, we found that both mean-risk models are not dominated by each other. This is because each mean-risk model minimized different risks. Mean-MAD will only minimize the absolute deviation, while mean-CVaR only minimize the CVaR. The results show that CVaR for MAD-minimizing portfolio is not the minimum CVaR since the CVaR is higher than the optimal-CVaR. Similarly to the MAD for CVaR-minimizing portfolio, which is not the minimum MAD compared to the optimal-MAD. This result matches the second objective that have been set earlier, which is to verify whether mean-MAD dominates the mean-CVaR and vice versa. For that, the second objective have been also successfully achieved.

5. Conclusion

After the validation of the portfolio using out-of-sample analysis is carried out, we can conclude that the lower the expected realized returns, the lower the realized risks. The mean-risk model is said to be favourable if it possesses lower risks and high returns. As a whole, we can infer that MAD-minimizing portfolio is more favourable since MAD gives higher realized returns than CVaR, and the CVaR for mean-MAD only give small marginal difference over the optimal-CVaR. Hereby, we have successfully achieved our last objective, which is to compare which mean-risk model is more favourable in terms of in-sample performance and out-sample realized performance.

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