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THE HYBRIDIZATION OF ANALYTIC HIERARCHY PROCESS AND GRAPH THEORY MATRIX APPROACH (AHP-GTMA) TO SOLVE SOLID WASTE TRANSSHIPMENT SITE SELECTION PROBLEM

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ABSTRACT

Proper selection of solid waste facilities is a complex issue and requires a broad assessment measure. Hence, the selection of these sites is a multiple criteria decision-making issue. This study proposed a hybridization of the Analytic Hierarchy Process and Graph Theory Matrix Approach (AHP-GTMA) method in selecting the best solid waste transshipment site. The objectives of this study are to apply the AHP-GTMA method to solve the solid waste transshipment site selection problem, and to determine the best solid waste transshipment site. Real-life empirical data about the solid waste transshipment site selection in Istanbul were used to demonstrate the application of the AHP-GTMA method. The data consisted of five alternatives of the solid waste transshipment site candidates in Istanbul labeled as A1, A2, A3, A4, and A5, and five criteria which were proximity to industrial solid waste (C1), proximity to household solid waste (C2), transportation simplicity (C3), necessity (C4) and proximity to residential area (C5). The findings revealed the ranking order of the criteria was C4 > C1 > C2 > C3 > C5 and the ranking order for the alternatives was given by A1 > A5 > A3 > A4 > A2. In conclusion, AHP-GTMA is successfully applied to solve the solid waste transshipment site selection problem.

Keywords: *AHP, GTMA, MCDM, Solid Waste Transshipment Site Selection.*

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1. Introduction

The election of an adequate solid waste site is a major task, requiring a thorough assessment process within the field, which complies with the environmental and scientific criteria as well as local and governmental regulations (Siddiqui *et al.*, 1996). The judgment of waste management sites and recycling stations is challenging work as the population of solid waste grows. A regional solid waste management strategy considers the selection of collection and recycling stations, the solid wastes appropriation and waste residues from the generator to the collection and recycling stations, and the option of conveying paths (Nema & Gupta, 1999).

In addition, the selection for solid waste facility placement should include several possible factors, such as climate, land slope, distances from neighborhoods and main streets, and capital expenditures (Önüt & Soner, 2008). Thus, the selection of a site may be perceived as an issue involving Multi-criteria Decision Making (MCDM).

MCDM became one of the operations research's most relevant and quickest increasing areas of study. This method can be used to tackle a variety of decision-making problems. Decision makers can use MCDM methods to select, evaluate, or rank alternatives depending on the weight of each criterion (Nădăban *et al.*, 2016). There are three crucial categories of MCDM methods that use distinct philosophies and concepts (Abdelli *et al.*, 2020). The first class of MCDM integrates the outcomes rating associated with the parameters using utility functions. The second class is based on the idea of creating a relationship of choice between alternatives. The other is strategies with different paradigms.

This study employed Analytic Hierarchy Process (AHP) and Graph Theory Matrix Approach (GTMA) to solve the problem of selecting solid waste transshipment sites. AHP is one of the common MCDM methods that enable decision makers to face multiple conflicting and subjective criteria. This method is used to decompose a hierarchical decision problem structure into easily understandable criteria (Ramanathan, 2001). One of the most significant aspects is that independence should be assured by each criterion. Each criterion may be a tangible or intangible element of the decision problem (Saaty, 2008). After the development of the structured hierarchy, for any aspect of criteria, a pairwise comparison is determined by the decision makers. The pairwise comparison is the mechanism that assesses each element's relative impact or significance. The decision makers may use concrete data about the elements in the pairwise comparison process or use their intuitive and competent judgment about the elements to achieve a priority scale (Rao & Pawar, 2018).

Furthermore, GTMA is suitable for representing and evaluating types of structures in many areas of science and engineering in combination with operational analysis, transport networks, and stochastic process behavior (Muduli *et al.*, 2013). There are three components of this method which are digraph representation, matrix representation, and permanent function representation. The elements and interrelationships are performed by a digraph in terms of edges and nodes. The digraph then is converted into a matrix form. To derive the index value, a permanent function is used to obtain optimal comparison, ranking, and selection (Grover *et al.*, 2006).

In this study, real-life data about the selection of solid waste transshipment sites in Istanbul (Önüt & Soner, 2008) is implemented to demonstrate the application of the hybridization of AHP and GTMA methods. AHP is implemented for multi-criteria issues includes a sizable number of criteria and alternatives and enables evaluation of the accuracy of the relative importance of attribute judgments. While GTMA will assist in determining attributes and their interrelations and provides a much preferable visual evaluation of the attributes (Lanjewar *et al.*, 2015). In selecting the solid waste transshipment site, the AHP method is used to check the consistency of the decision makers' assessment of their preferences for the criteria, and the AHP-GTMA evaluates and ranks alternative solutions.

Accordingly, deciding the area of solid waste transshipment sites is a complex decision-making issue for districts and relies upon social, ecological, and guidelines (Unal *et al.*, 2020). The picked site ought to stay away from impacts as much as could be expected on the environment, public well-being, and safety, and forestall any cooperation with other natural and human-made systems (Chabok *et al.*, 2020). Other than that, if any wasteful municipal solid waste administration system occurs, ecological effects like irresistible infections, land and water contamination, obstruction of drains, and loss of biodiversity might happen (Önüt & Soner, 2008).

Therefore, in this study, the AHP-GTMA method is applied to solve the solid waste transshipment site selection problem and to determine the best solid waste transshipment site.

This paper is divided into 5 sections which are (1) Introduction, (2) Literature Review, (3) Methodology, (4) Results and Discussion, and (5) Conclusion.

2. Literature Review

MCDM methods are designed to determine the better alternative, distinguish alternatives in a limited number of categories, and rank alternatives in extremely subjective order of choice (Mardani *et al.*, 2015). MCDM methods are often categorized into two which are discrete MCDM or discrete Multi-attribute Decision-Making (MADM) and continuous Multi-objective Decision-Making (MODM) techniques (Zavadskas *et al.*, 2019). The major benefit of the MCDM methods is the opportunity to tackle the problem characterized by various conflict interests. These strategies require little maintenance and can be improved (Daengdej *et al.*, 1999).

AHP is the most remarkable and ordinarily utilized MCDM method (Wind & Saaty, 1980). It is a compelling apparatus for solving complex and unstructured issues that may have connections and likenesses between different goals and objectives (Talib *et al.*, 2011). It is created to break down a complex multi-criteria issue into a few progression levels with the high level as the goal or objective, while the criteria are the middle levels, and the most bottom level gives alternatives (Agarwal *et al.*, 2014). Decision makers are consulted, and pairwise correlation assessments are applied to sets of homogeneous criteria, in this manner making general inclinations for alternative rankings (Wind & Saaty, 1980). AHP is preferably prepared to help model vulnerability and danger circumstances as it can remove scales where there is typically no estimation (Millet & Wedley, 2002). There is some worry about specific issues in the AHP strategy regardless of the accomplishment of the AHP. AHP utilizes excess decisions to check the accuracy, and this will essentially extend the number of decisions to be gotten from decision makers (Ramanathan, 2001).

Furthermore, GTMA method may be a systematic approach that begins with combinatory science. It examines and understands the structure as a complete by differentiating structure and substructure up to the phase level (Darvish *et al.*, 2009). It is employed in modeling and determining the decision makers' downside with multiple and reticular attributes (Geetha & Sekar, 2018). GTMA is beneficial in straightforwardly analyzing digraph models by explaining the system and problems in various science and technology (Rao & Padmanabhan, 2007). However, where there are pros, they will be accompanied by cons. The statement could be proven since the model provided by this method does not cover all the potential factors and criteria related to the study selection and the criteria and the interrelations between the criteria shown in the framework could be specific to a particular case (Darvish *et al.*, 2009).

The two methods are being hybridized to interpret a dynamic actualize that combines the scientific assets and the clear solidness check of similar hugeness decisions determined by the AHP strategy alongside the visual assessment of the attributes and the associations used by the GTMA strategy. In a study by Tuljak-Suban and Bajec (2020), many benefits can be gained from the hybrid method. The researchers have expressed that the AHP-GTMA method can deliver equal outcomes and is a simple method used to take care of the troublesome dynamic issues when there is a gigantic measure of standards and options inside the features. Moreover, AHP-GTMA enables decision makers to create a proper solution to rank comparable alternatives since the results obtained could be highly distinguished to direct the choice makers to identify the distinction between the options without any problem.

3. Methodology

This section consists of the framework and the implementation of the hybridization of the Analytic Hierarchy Process and Graph Theory Matrix Approach (AHP-GTMA).

3.1 Diagram of Hybridization of Analytic Hierarchy Process and Graph Theory Matrix Approach (AHP-GTMA)

Figure 1 depicts the proposed framework based on this hybrid methodology.

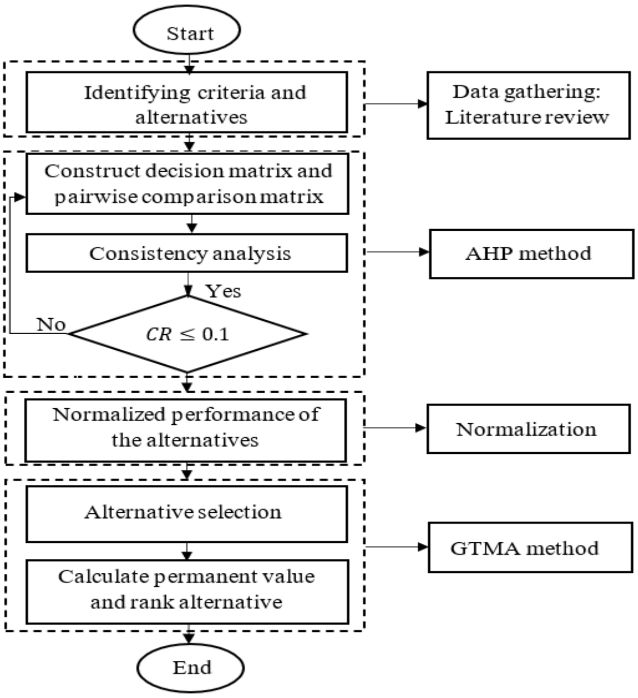

Figure 1. Proposed Framework of AHP-GTMA

3.2 Framework of Proposed Approach

In this paper, a pairwise comparison matrix for criteria is constructed, and AHP method is applied to perform the consistency analysis. Then, GTMA is utilized to find the permanent values of the alternatives. Thus, the best option could be decided from the obtained results. Steps 1-6 below explain the framework of the AHP-GTMA method (Saeed *et al.*, 2018).

Step 1: Set Up the Decision Matrix, D.

Table 1. Decision Matrix D

		C1	C2	$\cdots$	CN
	A1	a_{11}	a_{12}	$\cdots$	a_{1n}
$D =$	A2	a_{21}	a_{22}	$\cdots$	a_{2n}
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	AM	a_{m1}	a_{m2}	$\cdots$	a_{mn}

Step 2: Construct Pairwise Comparison Matrix, B.

This matrix is formed using the measurement scale for pairwise comparison (Önüt & Soner, 2008) shown in Table 2. The size of pairwise matrix is equal to the number of criteria used in decision making process. The diagonal elements of the matrix are always 1 and the upper triangular matrix is filled with actual judgment and reciprocal values. The reciprocal values of the upper diagonal are used to fill the lower triangular matrix. If b_{ij} is the element of row i column j of the matrix, then the lower diagonal is filled using this formula $b_{ji} = \frac{1}{b_{ij}}$ (Saaty, 1990). Below shows the construction of the matrix.

$$[B] = \begin{bmatrix} 1 & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & 1 & b_{23} & \cdots & b_{2n} \\ b_{31} & b_{32} & 1 & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & b_{n3} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & b_{12} & b_{13} & \cdots & b_{1n} \\ \frac{1}{b_{12}} & 1 & b_{23} & \cdots & b_{2n} \\ \frac{1}{b_{13}} & \frac{1}{b_{23}} & 1 & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{b_{1n}} & \frac{1}{b_{2n}} & \frac{1}{b_{3n}} & \cdots & 1 \end{bmatrix} \tag{1}$$

Table 2. Pairwise Comparison Scale

Intensity of importance	Definition
1	Equally preferred
3	Moderately preferred
5	Strongly preferred
7	Very strongly preferred
9	Extremely preferred
2, 4, 6, 8	Compromise between above values

Step 3: Consistency Analysis.

A comparison matrix $[B]$ is said to be consistent if $x_{ij}x_{jk} = x_{jk}$ for all i, j and k. In this study, a Formal Consistency Analysis of AHP is used. The sum of each column in pairwise comparison matrix $[B]$ is calculated. Then, each element of the matrix is divided by the sum of its column to find a normalized pairwise matrix by using:

$$c_{ij} = \frac{b_{ij}}{\sum_{i=1}^{n} b_{ij}} \tag{2}$$

The sum of each column in the normalized pairwise matrix must be 1. The weight coefficient of each criterion can be obtained by averaging across the rows by using:

$$w_i = \frac{\sum_{j=1}^{n} c_{ij}}{n} \tag{3}$$

The weight coefficient is used to find the maximum eigenvalue as follows:

$$\lambda_{\max} = \frac{1}{n}\sum_{i=1}^{n} \frac{(Aw)_i}{w_i} \tag{4}$$

Then, the Consistency Index (CI) is computed using:

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{5}$$

Table 3 shows the values of the Random Consistency Index (*RI*) (Saaty & Vargas, 2000).

Table 3. Random Consistency Index

n	1	2	3	4	5	6	7	8
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40

Finally, the Consistency Ratio (*CR*) is calculated to find the consistency, which is a sort of correlation between *CI* and *RI* (Saaty, 2003) using:

$$CR = \frac{CI}{RI} \tag{6}$$

Step 4: Normalized Performance of the Alternatives.

Values in matrix $[D]$ are normalized by using the maximum value in the column as the pivot for beneficial criteria and using the minimum value as the pivot for non-beneficial criteria respectively using:

$$\overline{A_{ij}} = \frac{a_{ij}}{a_j \max} \tag{7}$$

$$\overline{A_{ij}} = \frac{a_j \min}{a_{ij}} \tag{8}$$

The normalized pairwise comparison matrix, $[D_N]$ is then formed using Equation (7) and (8).

Step 5: Alternatives Selection.

A Decision matrix is formed for each alternative considered in the decision problem. In this step, pairwise comparison matrix $[B]$ is used for the construction of a decision matrix for each alternative. The normalized value matrix $[D_N]$ is used to replace the main diagonal elements of pairwise matrix, $[B]$ for each alternative as follows:

$$[E_i] = \begin{bmatrix} y_1 & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & y_2 & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & y_3 & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & y_n \end{bmatrix} \tag{9}$$

where $i = 1,2,3,...,m$ alternatives.

Step 6: Permanent Values for Alternatives.

The permanent function of the attribute matrix E_i, i.e. $Per(E_i)$ is defined as the characteristic permanent function. A characteristic permanent function is a complete representation of the attributes of a system and retains all possible information about the attributes and their interrelations. The permanent is defined as similar to the matrix determinant with all the determinant terms as positive terms. In addition, no negative sign appears in the permanent function of a matrix and therefore no information will be lost (Lanjewar *et al.*, 2015). The permanent function for a matrix E_i is written as:

$$\begin{aligned} Per(E_i) &= \prod_{i=1}^{n} S_i + \sum_i \sum_j \sum_k \cdots \sum_n (x_{ij}x_{ji}) y_k y_i \cdots y_n \\ &+ \sum_i \sum_j \sum_k \cdots \sum_n (x_{ij}x_{ji}x_{ki} + x_{ik}x_{kj}x_{ji}) y_1 y_n \cdots y_n \\ Per(E_i) &= \left[\sum\sum\sum \cdots \sum (x_{ij}x_{ji}) y_n y_n \cdots y_n \right] \\ &+ \left[\sum_i \sum_j \sum_k \cdots \sum_n (x_{ij}x_{jk}x_{kl}x_{li} + x_{il}x_{lk}x_{kj}x_{ji}) y_n y_n \cdots y_n \right] \\ &+ \left[\sum_i \sum_j \sum_k \cdots \sum_n (x_{ij}x_{ji})(x_{kl}x_{ln}x_{nk} + x_{kn}x_{nl}x_{lk}) y_n y_o \cdots y_n \right] \\ &+ \left[\sum_i \sum_j \sum_k \cdots \sum_n (x_{ij}x_{jk}x_{kl}x_{ln}x_{ni} + x_{in}x_{nl}x_{lk}x_{kj}x_{ji}) y_n y_o \cdots y_n + \cdots \right] \end{aligned} \tag{10}$$

where $i = 1,2,3,...,m$ alternatives.

The calculated permanent value is called the permanent index. The alternatives are ranked based on the index score. The higher the index, the best the alternatives among others.

3.3 Implementation of Proposed Approach

Real-life data on selecting the best solid waste transshipment sites in Istanbul (Önüt & Soner, 2008) is used to demonstrate the implementation of the AHP-GTMA method. The solid waste that has been collected will be transported to the closest solid waste station. The waste should be transferred to a place that is 32-meter-high silos. These shipment processes are conducted by the silos to transport the solid waste to solid waste transfer stations.

The decision makers oversaw selecting the candidate sites. There were five criteria, which were proximity to industrial solid waste (C1), proximity to household solid waste (C2), transportation simplicity (C3), necessity (C4), and proximity to residential area (C5). C5 is the non-beneficial criterion while the others are beneficial criteria. Besides, there were five candidate transshipment sites as depicted in Figure 2 labeled as A1, A2, A3, A4, and A5.

Figure 2. Alternative Transshipment Sites in Istanbul

Note. Reprinted from "Transshipment site selection using the AHP and TOPSIS approaches under fuzzy environment", by Önüt, S. and Soner, S., 2008, *Waste Management*, *28*(9), p. 1557. Copyright 2008 by Elsevier Inc.

GTMA facilitates the identification of attributes and provides a better visual appraisal of the attributes and their interrelations (Rao, 2007). AHP can be used to solve multi-criteria problems with many alternatives and criteria, and it enables to check the consistency of relative importance judgments of the attributes (Lanjewar *et al.*, 2015). AHP-GTMA method is then applied to solve the problem under study.

Step 1: Set up the decision matrix, D.

Table 4. Fuzzy Conversion Scale

Linguistic Terms	Score	Fuzzy Preference
Very Low	1	(0, 0, 0.2)
Low	2	(0, 0.2, 0.4)
Fairly Low	3	(0.2, 0.4, 0.6)
Fairly High	4	(0.4, 0.6, 0.8)
High	5	(0.6, 0.8, 1)
Very High	6	(0.8, 1, 1)

Each related factor is represented by a linguistic variable. The decision matrix of the problem obtained from Önüt and Soner (2008) contains fuzzy data expressed in linguistic terms that had been transformed into fuzzy numbers shown in Table 5. Then, the data is transformed into crisp numbers using a conversion scale (Ölçer & Odabaşi, 2005) shown in Table 4 above. Table 6 shows the decision matrix $[D]$ for the five candidate sites.

Table 5. Fuzzy Preferences for the Five Candidate Sites

	C1	C2	C3	C4	C5
A1	(0.8, 1, 1)	(0.6, 0.8, 1)	(0.4, 0.6, 0.8)	(0.6, 0.8, 1)	(0.2, 0.4, 0.6)
A2	(0.4, 0.6, 0.8)	(0.4, 0.6, 0.8)	(0.6, 0.8, 1)	(0, 0.2, 0.4)	(0.6, 0.8, 1)
A3	(0.2, 0.4, 0.6)	(0.8, 1, 1)	(0.2, 0.4, 0.6)	(0.6, 0.8, 1)	(0.4, 0.6, 0.8)
A4	(0.6, 0.8, 1)	(0.4, 0.6, 0.8)	(0.2, 0.4, 0.6)	(0.4, 0.6, 0.8)	(0.8, 1, 1)
A5	(0.4, 0.6, 0.8)	(0.6, 0.8, 1)	(0, 0.2, 0.4)	(0.8, 1, 1)	(0, 0.2, 0.4)

Table 6. Decision Matrix, D

		C1	C2	C3	C4	C5
	A1	6	5	4	5	3
	A2	4	4	5	2	5
$D =$	A3	3	6	3	5	4
	A4	5	4	3	4	6
	A5	4	5	2	6	2

Step 2: Construct Pairwise Comparison Matrix, B.

Table 7 shows the pairwise comparisons matrix $[B]$ constructed using the scale presented in Table 2.

Table 7. Pairwise Comparison Matrix, B

		C1	C2	C3	C4	C5
	C1	1	3	5	$\frac{1}{5}$	3
	C2	$\frac{1}{3}$	1	3	$\frac{1}{7}$	3
$B =$	C3	$\frac{1}{5}$	$\frac{1}{3}$	1	$\frac{1}{9}$	3
	C4	5	7	9	1	9
	C5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	1
	$\sum$	$\frac{103}{15}$	$\frac{35}{3}$	$\frac{55}{3}$	$\frac{493}{315}$	19

Step 3: Consistency Analysis

The sum of its column of matrix $\left[B\right]$ is calculated and shown in Table 7. Then, the normalized pairwise comparison matrix, $\left[B_N\right]$ is set up using Equation (2) and depicted in Table 8 below.

Table 8. Normalized Pairwise Comparison Matrix, B_N

		C1	C2	C3	C4	C5
$B_N =$	C1	$\frac{15}{103}$	$\frac{9}{35}$	$\frac{3}{11}$	$\frac{63}{493}$	$\frac{3}{19}$
	C2	$\frac{5}{103}$	$\frac{3}{35}$	$\frac{9}{55}$	$\frac{45}{493}$	$\frac{3}{19}$
	C3	$\frac{3}{103}$	$\frac{1}{35}$	$\frac{3}{35}$	$\frac{35}{493}$	$\frac{3}{19}$
	C4	$\frac{75}{103}$	$\frac{3}{5}$	$\frac{27}{55}$	$\frac{315}{493}$	$\frac{9}{19}$
	C5	$\frac{5}{103}$	$\frac{1}{35}$	$\frac{1}{55}$	$\frac{35}{493}$	$\frac{1}{19}$

Next, the weight coefficients are calculated by using Equation (3).

$$w = \frac{1}{5}\begin{bmatrix} \frac{15}{103} + \frac{9}{35} + \frac{3}{11} + \frac{63}{493} + \frac{3}{19} \\ \frac{5}{103} + \frac{3}{35} + \frac{9}{55} + \frac{45}{493} + \frac{3}{19} \\ \frac{3}{103} + \frac{1}{35} + \frac{3}{55} + \frac{35}{493} + \frac{3}{19} \\ \frac{75}{103} + \frac{3}{5} + \frac{27}{55} + \frac{315}{493} + \frac{9}{19} \\ \frac{5}{103} + \frac{1}{35} + \frac{1}{55} + \frac{35}{493} + \frac{1}{19} \end{bmatrix} = \begin{bmatrix} 0.1922369963 \\ 0.1094133932 \\ 0.06822634968 \\ 0.5863387749 \\ 0.04378448597 \end{bmatrix}$$

The weights of the criteria were calculated by using the AHP method. Based on result above, the weights were determined as C1 (19.22%), C2 (10.94%), C3 (6.82%), C4 (58.63%) and C5 (4.38%). C4 has a higher value of weight than the other criteria, indicating that it is a very important criterion. In contrast, C5 has the lowest value among the criteria, indicating that it is not as important as the other criteria. The maximum eigenvalue is calculated using Equation (4) and the consistency index value (CI) is calculated using Equation (5).

$$\text{weighted sum matrix} = 0.1922369963\begin{bmatrix} 1 \\ \frac{1}{3} \\ \frac{1}{5} \\ 5 \\ \frac{1}{3} \end{bmatrix} + 0.1094133932\begin{bmatrix} 3 \\ 1 \\ \frac{1}{3} \\ 7 \\ \frac{1}{3} \end{bmatrix} + 0.06822634968\begin{bmatrix} 5 \\ 3 \\ 1 \\ \frac{1}{9} \\ \frac{1}{3} \end{bmatrix} + 0.5863387749\begin{bmatrix} \frac{1}{5} \\ \frac{1}{7} \\ \frac{1}{9} \\ 1 \\ \frac{1}{9} \end{bmatrix}$$

$$+0.04378448597\begin{bmatrix} 3 \\ 3 \\ 3 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.10230137 \\ 0.593287581 \\ 0.3396470907 \\ 3.32151503 \\ 0.2322254851 \end{bmatrix}$$

$$\lambda_{\max} = \frac{1}{5}\left[\frac{1.110230137}{0.1922369963} + \frac{0.593287581}{0.1094133932} + \frac{0.3396470907}{0.06822634968} + \frac{3.32151503}{0.5863387749} + \frac{0.2322254851}{0.04378448597}\right] = 5.428933812$$

$$CI = \frac{5.428933812}{5-1} = 0.107233453$$

The random index value given for the 5 parameters in Table 8 was taken as $RI = 1.11$. Then, CR is calculated using Equation (6).

$$CR = \frac{0.107233453}{1.11} = 0.09660671441 < 0.1$$

Since the $CR < 0.1$, it was decided that the judgments of the decision makers were consistent.

Step 4: Normalized Performance of the Alternatives

The elements in Table 6 are normalized by using Equation (7) for beneficial criteria which are C1, C2, C3, and C4, and using Equation (8) for non-beneficial criteria which is C5. Table 9 shows the normalized decision matrix $\left[D_N\right]$.

Table 9. Normalized Decision Matrix, D_N

$D_N =$

	C1	C2	C3	C4	C5
A1	1	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{2}{3}$
A2	$\frac{2}{3}$	$\frac{2}{3}$	1	$\frac{1}{3}$	$\frac{2}{5}$
A3	$\frac{1}{2}$	1	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{1}{2}$
A4	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{1}{3}$
A5	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{2}{5}$	1	1

Step 5: Alternatives Selection

For this step, a decision matrix is formed for each alternative. The diagonal elements of matrix $[B]$ are replaced with the normalized value for row in matrix $[D_N]$, starting from the elements in the first row of matrix $[D_N]$ to the last row of matrix $[D_N]$ respectively.

Table 10. Alternative Selection for A1, E_1

$E_1 =$

	C1	C2	C3	C4	C5
C1	1	3	5	$\frac{1}{5}$	3
C2	$\frac{1}{3}$	$\frac{5}{6}$	3	$\frac{1}{7}$	3
C3	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{4}{5}$	$\frac{1}{9}$	3
C4	5	7	9	$\frac{5}{6}$	9
C5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{3}$

Table 11. Alternative Selection for A2, E_2

$E_2 =$

	C1	C2	C3	C4	C5
C1	$\frac{2}{3}$	3	5	$\frac{1}{5}$	3
C2	$\frac{1}{3}$	$\frac{2}{3}$	3	$\frac{1}{7}$	3
C3	$\frac{1}{5}$	$\frac{1}{3}$	1	$\frac{1}{9}$	3
C4	5	7	9	$\frac{1}{3}$	9
C5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{5}$

Table 12. Alternative Selection for A3, E_3

$E_3 =$

	C1	C2	C3	C4	C5
C1	$\frac{1}{2}$	3	5	$\frac{1}{5}$	3
C2	$\frac{1}{3}$	1	3	$\frac{1}{7}$	3
C3	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{9}$	3
C4	5	7	9	$\frac{5}{6}$	9
C5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{2}$

Table 13. Alternative Selection for A4, E_4

$E_4 =$

	C1	C2	C3	C4	C5
C1	$\frac{5}{6}$	3	5	$\frac{1}{5}$	3
C2	$\frac{1}{3}$	$\frac{2}{3}$	3	$\frac{1}{7}$	3
C3	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{9}$	3
C4	5	7	9	$\frac{2}{3}$	9
C5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{3}$

Table 14. Alternative Selection for A5, E_5

$E_5 =$

	C1	C2	C3	C4	C5
C1	$\frac{2}{3}$	3	5	$\frac{1}{5}$	3
C2	$\frac{1}{3}$	$\frac{5}{6}$	3	$\frac{1}{7}$	3
C3	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{9}$	3
C4	5	7	9	1	9
C5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	1

Step 6: Permanent Values for Alternatives

The permanent function is calculated for each alternative rather than the determinant function because the negative sign is replaced with a positive sign. Maple software is used to find the permanent values. The alternative with the highest index score is the best choice in decision making problem. Table 15 shows the index score for each alternative.

Table 15. Index Score

A_i	Index Score	Rank
A1	169.2507	1
A2	138.2563	5
A3	151.3916	3
A4	141.9855	4
A5	162.8370	2

From the Table 15 above, A1 has the highest score of 169.2507, thus it is the most favorable alternative chosen by decision makers, meanwhile, A2 has the lowest score and is ranked last. The ranking order of the alternatives is A1 > A5 > A3 > A4 > A2. This concludes that A1 is the best site to choose among others.

4. Results and Discussion

This section discusses the result of criteria and alternatives using the hybridization of the Analytic Hierarchy Process and Graph Theory Matrix Approach (AHP-GTMA).

4.1 Results of the Criteria

Real-life data on the selection of a solid waste transshipment sites in Istanbul (Önüt & Soner, 2008) is used to demonstrate the application of the AHP-GTMA method. Table 16 shows the result of the criteria weights. Subsequently, based on result of the criteria weights in Table 16, it is shown that the ranking order is C4 > C1 > C2 > C3 > C5. Figure 3 illustrates the weightage of each criterion in percentage.

Table 16. Result of Criteria Weight

Criteria	Weight	Percentage	Rank
Proximity to industrial solid waste (C1)	0.1922369963	19.22%	2
Proximity to household solid waste (C2)	0.1094133932	10.94%	3
Transportation simplicity (C3)	0.06822634968	6.82%	4
Necessity (C4)	0.5863387749	58.63%	1
Proximity to residential area (C5)	0.04378448597	4.38%	5

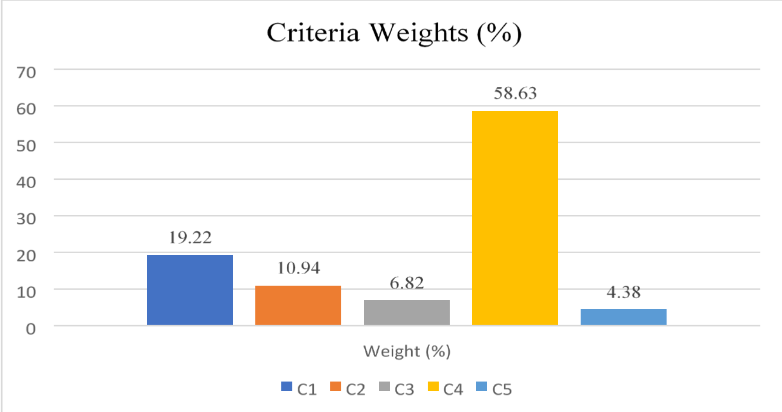

Figure 3. Weight of Each Criteria in Percentage

The results in Figure 3 indicate that necessity (C4) has the highest weight which is 58.63%. It must mean that it is necessary to have a new transshipment site for solid wastes. However, proximity to residential area (C5) is the least preferred criterion because has the lowest weight which is 4.38%. If the distance is too close, the transshipment site may infect the residents with diseases or odors, whereas placing a dumpsite too far away from the source of the waste is both uneconomical and time-consuming (Abdulhasan *et al.*, 2019). The AHP-GTMA method is used to determine a suitable site that is far from the residential area.

4.2 Results of the Alternatives

In the hybridization of the AHP-GTMA method, the value of the index determined the ranking of the alternative transshipment sites. Table 17 shows the index score and closeness coefficient (CC_j) values for each alternative of transshipment sites by using AHP-GTMA and Fuzzy AHP and Technique for Order of Preference by Similarity to Ideal Solution (Fuzzy AHP-TOPSIS) from Önüt and Soner (2008). Figure 4 illustrates the index score for each alternative.

Table 17. The Final Ranking of the Alternative Transshipment Sites

Alternatives	AHP-GTMA		Fuzzy AHP-TOPSIS (Önüt & Soner, 2008)	
	Index Score	Ranking	CC_j	Ranking
A1	169.2507	1	0.353	1
A2	138.2563	5	0.269	5
A3	151.3916	3	0.331	3
A4	141.9855	4	0.314	4
A5	162.8370	2	0.352	2

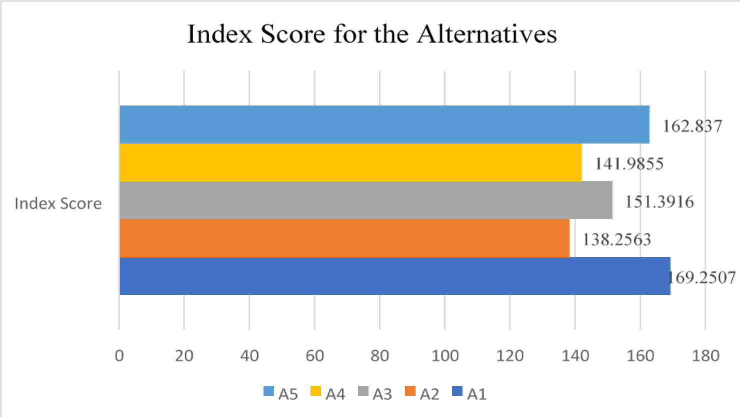

Figure 4. Ranking Order of the Alternatives

It can be seen from the data in Table 17 that the ranking order of the alternatives for both methods are the same which is A1 > A5 > A3 > A4 > A2. As can be seen from Figure 4, A1 has the highest index score and it is determined as the most preferable alternative whereas A2 is the least preferable alternative as it has the lowest index score. The scores used to construct the decision matrix, D are then classified into three classes for analysis purposes and shown in Table 18.

Table 18. Classes of Scores

Scores	Class
1	Low
2	
3	Medium
4	
5	High
6	

According to the decision matrix $[D]$ provided in Table 6, the criteria that contribute to the selection of A1 as a solid waste transshipment site include proximity to industrial solid waste (C1), proximity to household solid waste (C2), and necessity (C4) since it gets a high score of 5 and 6 on this site. Hence, A1 is required as the new solid waste transshipment site. This area offers the greatest benefits to both district municipalities and the residents. Based on Figure 4, the least preferable site is A2. Compared to the other sites, this site gets the poorest evaluation on necessity (C4), the criterion that contributes the highest weight. Therefore, A2 is not in demand as a solid waste transshipment site. Moreover, it gets a high score for proximity to residential area (C5) which is close to the residential area.

The results agree that the three most preferable sites are A1, A5, and A3. A4 is ranked second to last, so it is not a recommended option since A4 is the nearest to the residential areas. The single most striking observation to emerge from the data comparison is the final ranking for both methods is the same. Therefore, the results are consistent with Önüt and Soner (2008).

5. Conclusion

The aim of this study is to utilize the hybridization of the Analytic Hierarchy Process and Graph Theory Matrix Approach (AHP-GTMA) to solve the solid waste transshipment site selection problem and assist decision makers to rank relevant alternatives. AHP is used to check the consistency of the judgment of the decision makers regarding their preferences on the criteria, and AHP-GTMA is used to evaluate and rank alternative solutions in selecting the solid waste transshipment sites.

The results of the AHP method demonstrate that the criteria are ranked in the following order, C4 > C1 > C2 > C3 > C5, with necessity (C4) being the most important criterion and proximity to residential areas (C5) being the least important criterion. The ranking order for the alternatives is A1 > A5 > A3 > A4 > A2 according to the result of AHP-GTMA method. A1 becomes the most potential location as it provides the most benefits to both district municipalities and the residents. As the results achieved for the problem are reasonable, the AHP-GTMA method can be applied in various industries. It can improve the efficiency of complicated decision-making problems and minimizes the possibility of selecting an unsuitable alternative (Tuljak-Suban & Bajec, 2020).

However, since the AHP-GTMA method also has a few downsides, thus, it can be overcome by proposing the integration of Fuzzy AHP with the GTMA (Fuzzy AHP-GTMA) method. In a study established by Kahraman *et al.* (2003), the Fuzzy AHP method can assist decision makers to state their choices for each performance attribute using natural language terms. The pairwise comparisons in the judgment matrix in the Fuzzy AHP method are fuzzy numbers that are adjusted by the designer's emphasis. Chang (1996) has proposed to employ triangular fuzzy numbers for the pairwise comparison scale of Fuzzy AHP and the extent analysis method for the synthetic extent values of the pairwise comparisons.

To recap, selecting the ideal solid waste transshipment site should be done wisely to avoid any negative impact on the solid waste industry.

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