EXTREMAL PROPERTIES OF A NEW GENERALIZED CLASS OF TILTED ANALYTIC UNIVALENT FUNCTIONS

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1. Introduction

Geometric function theory is a branch of complex analysis that aims to study the geometric properties of certain class of univalent functions. Other areas of mathematics, such as potential theory, hyperbolic geometry, and dynamical systems, have strong ties to the subject. Since a proper approach available in this project, determining the geometric properties is not an easy task since it involves many definitions and theorem. There are many classes of functions that can be explore. However, there are only a few methods that can be used to solve this problem. Method used to determine geometric properties of this function is using Herglotz Representative Theorem. We look at the function of Taylor Series, $f(z)=z+\sum_{k=2}a_n z^n$ which are analytic in the unit disc $E=\{z:|z|<1\}$. For $|\beta|<\pi, \cos\beta>\delta, 0\le\delta<1$, let $M(\beta,\delta)$ denote the class of function $g(z)=\frac{2z-z^2}{2}$ for which $\mathrm{Re}\left\{e^{i\alpha}\frac{2f'(z)}{g(z)}\right\}>\delta$. In this research, we obtained new representation theorem, new coefficient bound, new centre and radius and new distortion theorem of tilted analytic univalent functions. Finally, we have produced a new result for the class $M(\beta,\delta)$.

2. Methodology

The Herglotz representation theorem, published by Herglotz in 1911 to define the extremal properties of the class of function implemented. It has been chosen as our reference model and approach for this research. The Herglotz representation theorem is an accurate result that determine the extremal properties for a group's basic attributes. For the class of functions having a positive real value, this theorem is valid. Applying the Herglotz representation theorem to identify the extremal properties of coefficient bound by Akbarally et al. (2011), and Yahya et al. (2013). For this research, a new class of univalent functions will obtain using the Herglotz representation theorem. For the class of function that has been chosen, the Herglotz Representation Theorem is created with a positive real value.

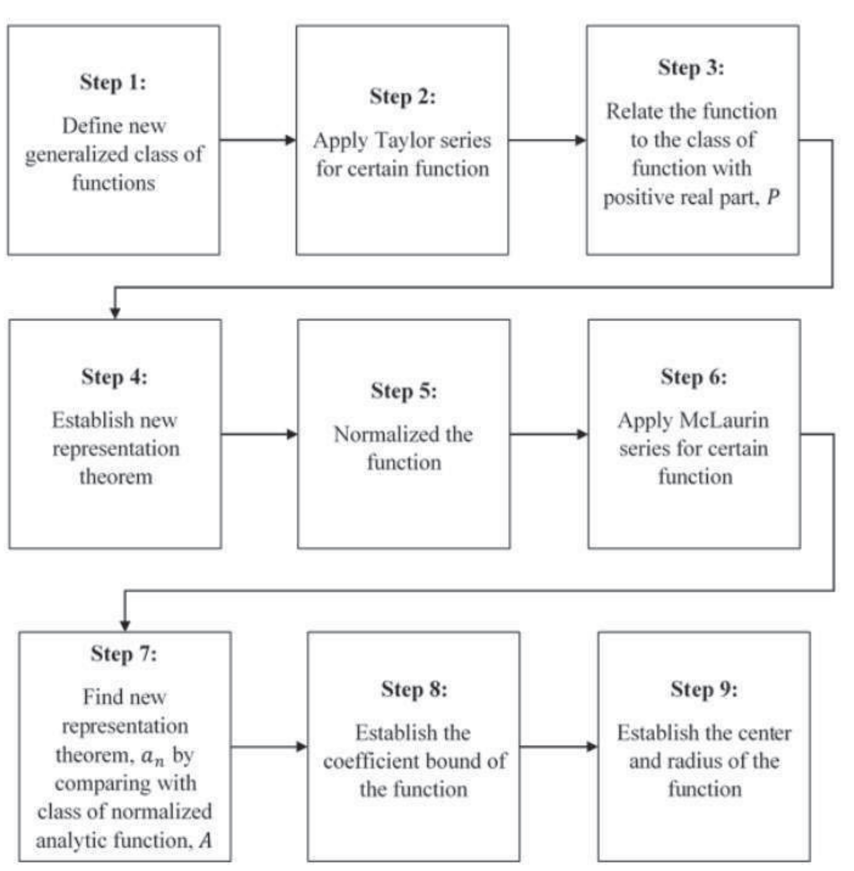

***Figure 1:** Methodology Steps*

3. Results and Discussion

3.1 Representation theorem

Theorem 1

Let $\dfrac{e^{i\alpha}\dfrac{zf'(z)}{g(z)}-\delta-i\sin\alpha}{\cos\alpha-\delta}=p(z)$ *given by (2.5). Then,* $f\in M(\beta,\delta)$ *if and only if*

$$\frac{e^{i\beta}\dfrac{zf'(z)}{g(z)}-\delta-i\sin\beta}{\cos\beta-\delta}=p(z).$$

3.2 Coefficient Bound

Theorem 2

Suppose that $f\in M$ *is given by* $f(z)=z\sum_{k=2}^{\infty}a_k z^k,(z\in U)$ *then the sharp result of coefficient bound is obtained as*

$$|a_n| \le \frac{1}{n}\left[A_{\beta\delta}e^{-i\beta}\right], \ n = 1,2,3,\ldots.$$

3.3 Center and Radius, Distortion theorem, $\operatorname{Re} f'$ and $\operatorname{Im} f'$.

Theorem 3

If $f \in M(\beta,\delta)$, then f' maps $|z| \le r$ into the disk D, with center

$$-e^{-i\beta}\left(e^{-i\beta} - 2\delta\right)\frac{\left(2-r^2\right)}{2} + \frac{A_{\beta\delta}e^{-i\beta}\left(2-r^2\right)}{\left(1-r^2\right)}$$

and radius

$$\frac{A_{\beta\delta}r\left(2-r^2\right)}{\left(1-r^2\right)}.$$

Theorem 4

If $f \in M(\beta,\delta)$, then

$$|f'(z)| \le C(r) + \frac{A_{\beta\delta}r\left(2-r^2\right)}{\left(1-r^2\right)}$$

where

$$C(r) = \left[1 + \frac{4r^2A_{\beta\delta}}{1-r^2}\left(\frac{A_{\beta\delta}}{1-r^2} + \delta\right)^{\frac{1}{2}}\right]\left[\frac{2-r^2}{2}\right]$$

and the bound is sharp for any extreme points of $f \in M(\beta,\delta)$.

Theorem 5

If $f \in M(\beta,\delta)$, then

$$\frac{\left(2-r^2\right)\left[4\delta\left(A_{\beta\delta}+\delta\right) + 1 + r^2\left(2A_{\beta\delta}\left(A_{\beta\delta}+\delta\right)-1\right) - 2A_{\beta\delta}r\right]}{2\left(1-r^2\right)} \le \operatorname{Re} f'(z)$$

$$\le \frac{\left(2-r^2\right)\left[4\delta\left(A_{\beta\delta}+\delta\right) + 1 + r^2\left(2A_{\beta\delta}\left(A_{\beta\delta}+\delta\right)-1\right) + 2A_{\beta\delta}r\right]}{2\left(1-r^2\right)}$$

and these bounds are sharp for any extreme points f of $M(\beta,\delta)$.

Theorem 6

If $f \in M(\beta,\delta)$, then

$$\frac{-A_{\beta\delta}r\left(2-r^2\right)\left(1+r\sqrt{1-\left(A_{\beta\delta}+\delta\right)^2}\right)}{\left(1-r^2\right)}\leq \operatorname{Im} f'(z)\leq \frac{A_{\beta\delta}r\left(2-r^2\right)\left(1-r\sqrt{1-\left(A_{\beta\delta}+\delta\right)^2}\right)}{\left(1-r^2\right)}$$

and these bounds are sharp for any extreme points f of $M(\beta,\delta)$.

3.4 Validation of Result

The function of $M(\beta,\delta)$ are normalized since $f(0)=0$ and $f'(0)=1$. Besides, the final result is unable to reduce to another results because this class of function is a new class.

4. Conclusion

The main goal of this study is to determine specific extremal properties such representation theorem, coefficient bound, center, and radius. The Herglotz Representative Theorem is then used to find representation theorem for a new generalized class. We also discover a new generalized class of tilted analytic functions for coefficient bound. There were three objectives that were successfully completed. We are able to develop a new generalized class of tilted analytic functions according to the first objective. Hence, we could also determine the new representation theorem for a new generalized class of $M(\beta,\delta)$ by using Herglotz Representative Theorem. Last but not least is to find the coefficient bound, center and radius of $M(\beta,\delta)$.

References

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Yahya, A., C. Soh, S., & Mohamad, D. (2013). Coefficient Bound of A Generalized Close-to-Convex function. *International Journal of Pure and Applied Mathematics*, *83*(2), 287–293. http://www.ijpam.eu