Students' Evaluation on Lecturers using Fuzzy Numbers

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ABSTRACT

This paper presents an alternative method for students' evaluation on lecturers based on fuzzy approach. Owing to vague concepts frequently represented in decision data and decision making processes, the crisp data are inadequate to model real life situations. In this study, fuzzy number arithmetic operations and fuzzy ranking methods are applied to deal with the problem of uncertainty in the evaluation process. An empirical study has been conducted in Universiti Teknologi MARA (UiTM) Pahang where the process of traditional evaluation is basically based on the calculation of mean. Eleven items in the traditional process of evaluation are categorised into three basic criteria that are teaching, class management and professional and motivational attitude. The fuzzy weight of each criterion is generated from the students' opinion. This method can evaluate the performance of lecturers in class in a more holistic and effective approach and may give great satisfaction to all parties involved in the decision making process.

Keywords: *Fuzzy numbers, fuzzy ranking, lecturers, students' evaluation*

Introduction

Since the introduction of fuzzy set theory in 1965 by Lotfi Zadeh, its applications have steadily increased. The increasingly widespread use of fuzzy set theory can be attributed to its ability to solve problems in which descriptions of activities and observations are imprecise, vague and uncertain. Due to the vague concepts frequently represented in the real world, the crisp data are inadequate to model real life situations. One of the current issues is in the education system especially in the evaluation process where human decision making is uncertain, imprecise and fuzzy.

Echauz and Vachtsevanos (1995) developed a novel method using fuzzy set theory in improving the quality of the educational grading process. The method utilised students' and instructors' performance measures in order to produce a fair mark distribution. In another study, Biswas (1995) presented a fuzzy evaluation method (fem) and generalised fuzzy evaluation method (gfem) for students' answer scripts evaluation. Inspired by Biswas approach, Chen and Lee (1999) proposed two methods for students' answer scripts evaluation which overcame the complicated operations in Biswas (1995).

Law (1996) presented a structure model of a fuzzy educational grading system and developed an algorithm. He also proposed a method to build the membership functions of several linguistic values with different weights. Zhou, Ma, Tian and Kwok (1999) applied the fuzzy set approach to the assessment of students' projects. For that purpose, they developed a group decision support system (GDSS) which includes four stages, namely generation of basic assessment criteria, selection of assessment criteria, determination of assessment criteria weights and fuzzy grading of students' projects.

Weon and Kim (2001) presented a new learning achievement evaluation strategy in students' learning performance namely fuzzy evaluation. The inverse sigmoid function, fuzzy concentration function, fuzzy dilution function and fuzzy square method were used in the procedure. Vrettaros et al. (2004) developed a diagnostic system of taxonomies using fuzzy approach. The system is very useful for e-learning and distance diagnostic system. Cagman and Gokbulut (2005) presented a new method to build alternative membership functions for Law's fuzzy educational grading system. Recently, Wang and Chen (2006) presented a new method for appraising the performance of high school teachers based on fuzzy number arithmetic operations.

This paper presents an alternative method for students' evaluation on lecturers based on fuzzy approach by Wang and Chen (2006). The empirical study was conducted in Universiti Teknologi MARA (UiTM) Pahang where the process of traditional evaluation has been basically based on the calculation of mean. The fuzzy evaluation can discriminate the ranking order compared to the traditional evaluation. It can evaluate the performance of lecturers in class in a more holistic and effective approach and may give great satisfaction to all parties involved in the decision making process.

Preliminaries

Fuzzy Number

A fuzzy number is a fuzzy subset in the universe discourse that is both convex and normal.

Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}$, denoted by a triplet (a,b,c), is defined by a membership function $\mu_{\tilde{A}}(x)$ as

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a} & ,a\le x\le b\\ \dfrac{x-c}{b-c} & ,b\le x\le c\\ 0 & ,\text{otherwise}\end{cases}$$

Operations on Triangular Fuzzy Numbers

Let $\tilde{X}$ and $\tilde{Y}$ be two triangular fuzzy numbers parameterised by (x_1,x_2,x_3) and (y_1,y_2,y_3) respectively. The fuzzy number arithmetic operations between $\tilde{X}$ and $\tilde{Y}$ as presented in Chen and Hwang (1992) are as follows:

Addition operation: $$\tilde{X}\oplus\tilde{Y}=(x_1,x_2,x_3)\oplus(y_1,y_2,y_3)=(x_1+y_1,x_2+y_2,x_3+y_3)$$

Subtraction operation: $$\tilde{X}-\tilde{Y}=(x_1,x_2,x_3)-(y_1,y_2,y_3)=(x_1-y_1,x_2-y_2,x_3-y_3)$$

Multiplication operation: $$\tilde{X}\otimes\tilde{Y}=(x_1,x_2,x_3)\otimes(y_1,y_2,y_3)=(x_1\times y_1,x_2\times y_2,x_3\times y_3)$$

Division operation: $$\tilde{X}/\tilde{Y}=(x_1,x_2,x_3)/(y_1,y_2,y_3)=(x_1/y_3,x_2/y_2,x_3/y_1)$$

$$\tilde{X}/\alpha=(x_1,x_2,x_3)/(\alpha,\alpha,\alpha)=(x_1/\alpha,x_2/\alpha,x_3/\alpha)$$

Ranking of Fuzzy Numbers

In fuzzy multiple criteria decision making problem, the final scores of alternatives are represented in terms of fuzzy numbers. In order to choose the best alternatives, a method of ranking fuzzy number is needed.

Chen and Chen (2003) proposed a method for ranking generalised fuzzy number based on centre of gravity and standard deviations of generalised fuzzy numbers. The ranking value $Rank(\widetilde{A})$ of a generalised fuzzy number $\widetilde{A}=(a_1,a_2,a_3;w_{\widetilde{A}})$ is given as

$$Rank(\widetilde{A}) = x^*_{\widetilde{A}} + (w_{\widetilde{A}} - y^*_{\widetilde{A}})^{s_{\widetilde{A}}} \times (y^*_{\widetilde{A}} + 0.5)^{1-w_{\widetilde{A}}} \tag{1}$$

where

$$y^*_{\widetilde{A}} = \begin{cases} \dfrac{w_{\widetilde{A}} \times \left(\dfrac{a_3-a_2}{a_4-a_1}+2\right)}{6} & ; a_1 \neq a_4 \text{ and } 0 < w_{\widetilde{A}} \leq 1 \\ \dfrac{w_{\widetilde{A}}}{2} & ; a_1 = a_4 \text{ and } 0 < w_{\widetilde{A}} \leq 1 \end{cases} \tag{2}$$

and

$$x^*_{\widetilde{A}} = \frac{y^*_{\widetilde{A}}(a_3+a_2)+(a_4+a_1)(w_{\widetilde{A}}-y^*_{\widetilde{A}})}{2w_{\widetilde{A}}} \tag{3}$$

Based on the definition of standard deviations by Hines and Montgomery (1990, cited in Chen & Chen 2003), the standard deviation of the generalised fuzzy number $\widetilde{A}$ is defined as

$$\hat{s}_{\widetilde{A}} = \sqrt{\frac{\sum_{i=1}^{4}(a_i-\bar{a})^2}{4-1}} = \sqrt{\frac{\sum_{i=1}^{4}(a_i-\bar{a})^2}{3}} \tag{4}$$

where $\bar{a}=\dfrac{a_1+a_2+a_3+a_4}{4}$ is the mean values of a_1,a_2,a_3 and a_4 .

For a normal fuzzy number, equations (1) – (3) can be written as follows:

$$Rank(\widetilde{A}) = x^*_{\widetilde{A}} + (1 - y^*_{\widetilde{A}})^{s_{\widetilde{A}}} \tag{5}$$

$$y^*_{\widetilde{A}} = \begin{cases} \dfrac{1 \times \left(\dfrac{a_3-a_2}{a_4-a_1}+2\right)}{6} & ; a_1 \neq a_4 \\ \dfrac{1}{2} & ; a_1 = a_4 \end{cases} \tag{6}$$

$$x^*_{\widetilde{A}} = \frac{y^*_{\widetilde{A}}(a_3+a_2)+(a_4+a_1)(1-y^*_{\widetilde{A}})}{2} \tag{7}$$

Therefore, for fuzzy numbers $\widetilde{A}_i \in S$ and $\widetilde{A}_j \in S$ where S is a set of normal fuzzy numbers, the ranking order between $\widetilde{A}_i$ and $\widetilde{A}_j$ is as follows:

Case 1: If $Rank(\widetilde{A}_i) < Rank(\widetilde{A}_j)$, then $\widetilde{A}_i < \widetilde{A}_j$.

Case 2: If $Rank(\widetilde{A}_i) = Rank(\widetilde{A}_j)$, then $\widetilde{A}_i = \widetilde{A}_j$.

Case 3: If $Rank(\tilde{A}_i) > Rank(\tilde{A}_j)$, then $\tilde{A}_i > \tilde{A}_j$.

Methodology

Eleven items of the evaluation questionnaire on lecturers in UiTM Pahang are categorised into three basic criteria that are teaching, class management and professional and motivational attitude (Table 1). The evaluation involved six lecturers teaching the Diploma in Computer Science.

Table 1: The Criteria for Evaluating Lecturers' Performance in UiTM Pahang

Criteria	Sub-criteria	Contents
X_1 (teaching)	X_{11}	Be prepared to deliver teaching materials.
	X_{12}	Be knowledgeable and confident in using teaching materials.
	X_{13}	Presenting well-organised teaching materials.
	X_{14}	Having the ability to keep students' attention throughout the lesson.
X_2 (class management)	X_{21}	Giving opportunity for questions and discussions.
	X_{22}	Evaluating assignments, tests and quizzes fairly according to the standard of the course.
X_3 (professional and motivational attitude)	X_{31}	Always attending classes.
	X_{32}	Coming to class on time.
	X_{33}	Always showing interest and enthusiasm during teaching.
	X_{34}	Showing concern on students' attendance and motivating students to succeed.
	X_{35}	Treating students fairly.

Two sets of fuzzy linguistic questionnaires comprising the importance levels of each criterion and the satisfaction levels of each lecturer related to the sub-criterion were distributed among ten students (Table 2 and 3).

Table 2: Importance Levels of the Criterion X_i ($i = 1, 2, 3$)

Criterion	Very Low (LW)	Low (L)	Medium (M)	High (H)	Very High (VH)
X_i	0%	0%	10%	20%	70%

Table 3: Satisfaction Levels of Lecturers Related to the Sub-criterion X_{ij}

Criterion	LW	L	M	H	VH
X_{ij}	0%	0%	10%	20%	70%

This study implements the procedure by Wang and Chen (2006) which is presented as follows:

Step 1: The fuzzy weight $W(\tilde{x}_i)$ of each criterion is calculated using

$$W(\tilde{x}_i)=\frac{\sum_{k=1}^{5}\tilde{x}_{ik}f(\tilde{x}_{ik})}{\sum_{k=1}^{5}f(\tilde{x}_{ik})}$$

where i denotes the index of the criterion, k denotes the index of linguistic levels, $\tilde{x}_{ik}$ denotes the k-th importance level of the criterion X_i, $1\le k\le 5$, $\tilde{x}_{ik}\in\{\tilde{1},\tilde{2},\tilde{3},\tilde{4},\tilde{5}\}$, $f(\tilde{x}_{ik})$ denotes the degree of percentage the criterion X_i satisfies the k-th importance level and $\sum_{k=1}^{5}f(\tilde{x}_{ik})=1$. The fuzzy weight will be represented by a triangular fuzzy number (a, b, c) and must satisfy the rules: "If $a<1$, then let $a=1$; if $c>5$, then let $c=5$".

Step 2: For each criterion X_i $(1\le i\le 3)$ evaluated by the students, drop the fuzzy weight with the smallest ranking value and the largest ranking value. Then, calculate the average of the remaining fuzzy weight $\tilde{w}_i$ using addition and division operations of fuzzy numbers.

Step 3: The fuzzy grade $G(\tilde{x}_{ij})$ of each sub-criterion of each lecturer evaluated by each student is calculated using

$$G(\tilde{x}_{ij})=\frac{\sum_{k=1}^{5}\tilde{x}_{ijk}f(\tilde{x}_{ijk})}{\sum_{k=1}^{5}f(\tilde{x}_{ijk})}$$

where $\tilde{x}_{ijk}$ denotes the k-th linguistic satisfaction level of the sub-criterion X_{ij}, $\tilde{x}_{ijk}\in\{\tilde{1},\tilde{2},\tilde{3},\tilde{4},\tilde{5}\}$, $f(\tilde{x}_{ijk})$ denotes the degree the lecturer satisfies the k-th satisfaction level and $\sum_{k=1}^{5}f(\tilde{x}_{ijk})=1$. The fuzzy grade also satisfies the rules in Step 1.

Step 4: For each sub-criterion of each lecturer evaluated by each student, drop the fuzzy grades with the smallest ranking value and the largest ranking value. Then, calculate the average of the remaining fuzzy grades $\tilde{g}_{ij}$ using addition and division operations of fuzzy numbers.

Step 5: Build the fuzzy grade matrix $\tilde{G}$ defined as

$$\tilde{G}=\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array}\begin{bmatrix} X_1 & X_2 & \cdots & X_k \\ \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{bmatrix}$$

where $\tilde{g}_{ij}$ denotes the fuzzy grade of the i-th lecturer A_i with respect to the j-th criterion X_j, n denotes the number of lecturers and k denotes the number of criteria.

Step 6: Calculate the total fuzzy grade vector $\tilde{R}$ with

$$\tilde{R} = \tilde{G} \otimes \tilde{W} = \begin{bmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{bmatrix} \otimes \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_k \end{bmatrix} = \begin{bmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \vdots \\ \tilde{R}_k \end{bmatrix}$$

where $\tilde{R}_i$ denotes the total fuzzy grade of the i-th lecturer A_i and $1 \leq i \leq n$.

Step 7: Based on Equation 5, calculate the ranking value, $Rank(\tilde{R}_i)$.

Results and Discussion

The ranking value of each lecturer is shown in Table 4.

Table 4: Ranking Values of the Total Fuzzy Grades of Each Lecturer

Lecturers	R_i			$y^{\bullet}_{R_i}$	$x^{\bullet}_{R_i}$	s	Ranking value
A_1	31.51495	54.02995	71.79682	0.333333	104.8945	16.50203	52.44848
A_2	34.12554	57.36491	72.84558	0.333333	109.5573	15.96528	54.78022
A_3	37.57767	61.81267	74.34912	0.333333	115.8263	15.38705	57.91511
A_4	35.34759	58.95853	73.92682	0.333333	112.1553	15.94629	56.0792
A_5	37.83854	62.14073	74.56336	0.333333	116.3618	15.38004	58.18284
A_6	36.32195	60.1982	74.39556	0.333333	113.9438	15.79262	56.97356

This study also compares the ranking of each lecturer based on fuzzy evaluation and traditional evaluation (Table 5). The fuzzy evaluation can rank each lecturer distinctly according to the ranking value. The ranking order of the six lecturers is A_5, A_3, A_6, A_4, A_2 and A_1. However, by the traditional method, the value of mean for lecturers A_3 and A_6 is equal and, therefore, the ranking between these two lecturers cannot be discriminated.

Table 5: Ranking of Each Lecturer

Lecturers	Fuzzy evaluation		Traditional evaluation	
	Ranking value	Ranking	Mean	Ranking
A_1	52.44848	6	4.063636	5
A_2	54.78022	5	4.272727	4
A_3	57.91511	2	4.645455	**2**
A_4	56.0792	4	4.436364	3
A_5	58.18284	1	4.709091	1
A_6	56.97356	3	4.645455	**2**

Table 6: Ranking of Criteria for Each Lecturer

Lecturers	Criteria X_1		Criteria X_2		Criteria X_3	
	Ranking value	Ranking	Ranking value	Ranking	Ranking value	Ranking
A_1	4.51237	6	4.800103	6	4.755657	6
A_2	4.817646	5	4.823603	5	5.037312	4
A_3	5.204176	2	5.156077	1	5.155594	2
A_4	4.949211	4	5.09398	3	4.97291	5
A_5	5.236127	1	5.129335	2	5.211647	1
A_6	5.08505	3	5.071124	4	5.086427	3

Table 6 presents the ranking of criteria for each lecturer. Although overall, lecturer A_5 is on the top ranking compared to the other lecturers, he/she has the second ranking for the criteria of class management. However, for the other two criteria of teaching and professional and motivational attitude, lecturer A_5 has the first ranking.

Conclusion

This study presents an alternative method for students' evaluation on lecturers based on fuzzy approach by Wang and Chen (2006). This method considers the weight of each criterion and the satisfaction level of lecturers related to each sub-criterion. The ranking order of each lecturer is based on the ranking value compared to the mean value in the traditional process. In the traditional process, the mean value could sometimes be the same and, therefore, produce non-discriminating results. Although this approach leads to nearly the same ranking order as in the traditional evaluation, it considers the whole aspect of criteria such as the importance level and the fuzzy ratings. Therefore, this method can evaluate the performance of lecturers in class in a more holistic and effective approach and may give great satisfaction to all parties involved in the decision making process.

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