

**UNIVERSITI TEKNOLOGI MARA**

**A MODIFIED  $w$ -TH SECTION LINE  
SEARCH IN CONJUGATE  
GRADIENT METHODS FOR  
SOLVING UNCONSTRAINED  
OPTIMIZATION**

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## ABSTRACT

Conjugate Gradient (CG) method are well-known method for solving unconstrained optimization problems. A lot of efforts have been done in order to improve the efficiency of this CG methods. For unconstrained optimization, line search act as a pillar for solving optimization problems. In this research, a new modification of inexact line search in CG methods is proposed. It is based on classical bisection line search method. Generally, bisection method is the easiest method to solve root of a function. Thus, it is an ideal method to employ as a line search in CG methods. This new modification that was proposed by Hayati (2015) is named as n-th section method. However, n-th section method inherits the same behaviour as bisection method which is slow convergence. Thus, a re-modification on n-th section algorithm is conducted to reduce its CPU time per execution. Overall, six line search methods that are employed in CG methods which are classical bisection, 4<sup>th</sup> section, 6<sup>th</sup> section, modified bisection, modified 4<sup>th</sup> section and modified' 6<sup>th</sup> section are compared through performance profile analysis based on number of iterations and CPU times. These line search methods are tested based on six standard optimization test problems. The CG methods used in this research are classical formulas known as Fletcher-Reeves (FR), Polak-Ribiere-Polyak (PRP) and Rivaie-Mustafa-Ismail-Leong (RMIL). All algorithms are written and executed using Maple 16 software. Numerical results show that modified 6<sup>th</sup> section is the best method in term of number of iterations while modified 4<sup>th</sup> section is the best based on CPU times. Other than that, the modified version of n-th section line search method has less amount of CPU time allocated to execute CG's algorithm when compared to the original version of n-th section line search method. In a nutshell, modified n-th section line search method is more promises and efficient compared to the classical bisection line search.

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## TABLE OF CONTENTS

|  | <b>Page</b> |
|--|-------------|
| <b>CONFIRMATION BY PANEL OF EXAMINERS</b>                              | <b>ii</b>   |
| <b>AUTHOR'S DECLARATION</b>  | <b>iii</b>  |
| <b>ABSTRACT</b>  | <b>iv</b>   |
| <b>ACKNOWLEDGEMENT</b>   | <b>v</b>    |
| <b>TABLE OF CONTENTS</b>   | <b>vi</b>   |
| <b>LIST OF TABLES</b>  | <b>ix</b>   |
| <b>LIST OF FIGURES</b>   | <b>x</b>    |
| <b>LIST OF ALGORITHMS</b>  | <b>xii</b>  |
| <b>LIST OF SYMBOLS</b>   | <b>xiii</b> |
| <b>LIST OF ABBREVIATIONS</b>   | <b>xiv</b>  |
| <b>LIST OF NOMENCLATURES</b>   | <b>xv</b>   |
| <br>   |             |
| <b>CHAPTER ONE: INTRODUCTION TO RESEARCH</b>                           | <b>1</b>    |
| 1.1 Introduction   | 1           |
| 1.2 Research Background  | 1           |
| 1.3 Problem Statement  | 4           |
| 1.4 Overview of the Research   | 5           |
| 1.5 Objective of Research  | 9           |
| 1.6 Conceptual Framework   | 9           |
| 1.7 Scope and Limitation   | 11          |
| 1.8 Benefits of Research   | 13          |
| 1.9 Significance of Research   | 14          |
| 1.10 Organization of the Thesis  | 15          |
| <br>   |             |
| <b>CHAPTER TWO: FUNDAMENTAL CONCEPTS OF UNCONSTRAINED OPTIMIZATION</b> | <b>17</b>   |
| 2.1 Introduction   | 17          |
| 2.2 The Basic Concepts   | 18          |

|   |   |           |
|---|---|-----------|
| 2.3   | The Local and Global Minimizer                    | 20        |
| 2.4   | The Convexity                                     | 23        |
| 2.5   | The Convexity and Derivatives                     | 26        |
| 2.6   | Line Search                                       | 30        |
|   | 2.6.1 Exact Line Search                           | 31        |
|   | 2.6.2 Inexact Line Search                         | 31        |
| 2.7   | Search Direction                                  | 34        |
|   | 2.7.1 Steepest Descent Method (SD)                | 34        |
|   | 2.7.2 Newton's Method                             | 35        |
|   | 2.7.3 Quasi-Newton's Method (QN)                  | 36        |
|   | 2.7.4 Conjugate Gradient Method (CG)              | 37        |
| 2.8   | Stopping Criteria                                 | 38        |
| 2.9   | Rate of Convergence                               | 39        |
| 2.10  | Conclusion  | 40        |
| <b>CHAPTER THREE: CONJUGATE GRADIENT METHODS</b>        |   | <b>42</b> |
| 3.1   | Introduction                                      | 42        |
| 3.2   | The History of CG Development                     | 42        |
| 3.3   | The Overview of CG Methods for Nonlinear Problems | 45        |
| 3.4   | Classes of CG Methods                             | 46        |
|   | 3.4.1 Classical CG Methods                        | 46        |
|   | 3.4.2 Hybrid CG Methods                           | 50        |
|   | 3.4.3 Parameterized CG Methods                    | 50        |
|   | 3.4.4 Scaled CG Methods                           | 51        |
|   | 3.4.5 Modified CG Methods                         | 52        |
| 3.5   | Recent Modification of CG Method                  | 53        |
| 3.6   | Convergence Analysis for Descent Condition        | 55        |
| 3.7   | Conclusion  | 56        |
| <b>CHAPTER FOUR: MODIFICATION OF LINE SEARCH METHOD</b> |   | <b>57</b> |
| 4.1   | Introduction                                      | 57        |
| 4.2   | Bisection Method                                  | 57        |
| 4.3   | The n-th Section Method                           | 63        |