SIMULATION STUDY OF TESTING AN INTERVENTION EFFECT IN ARIMA (1,0,0,L,δ) MODEL

1.0 Introduction

In order to test whether or not the intervention effect, δ which occurs between the pre-time series and the post-time series in an ntervention time series model is different from zero, under appropriate assumptions, the t- statistics test suggested by Glass, Wilson, and Gottman [2] may be used. In this study,the first order autoregressive ARIMA (1,0,0,L,δ) intervention model, which contains the same pre- and post- intervention first-order autoregressive parameter Ø, is chosen in order to study the validity of the t-statistics. The variable Z will represent the series measured across n equally spaced units of time. This series is labelled as $Z_1, Z_2, \ldots, Z_n$. We shall assume the intervention effect occurs between times n_1 and $n_1 + 1$, where $n_1 < n$, and where L is the level of the pre- intervention time series $Z_1, Z_2, \ldots, Z_{n_1}$, and (L + δ) is the level of post-intervention time series $Z_{n_1+1}, \ldots, Z_n$. Zinkgraft and Wilson [5] simulated the performance of this t- procedure under the null hypothesis assumption that Ho : $\delta = 0$ (no change) and reported that this procedure may not preserve an α - level of significance.

As the result of the simulation study in [5], the observed α- values for $n_1 + n_2 = 20$ and Ø = 0.6 are: 0.077, 0.178, and 0.259, compared to theoretical values of: 0.01, 0.05 and 0.10 respectively. Also from this simulation study, the observed (α- values) for $n_1 + n_2 = 50$ and Ø= 0.6 are: 0.034, 0.112 and 0.185. These observed values are also greater than the theoretical values: 0.01., 0.05 and 0.10 respectively.

Based on this preliminary and sketchy evidence, this study is being done to see whether this t-statistics can really control the type-I error (α) for a wider category of Ø values: Ø= 0.0, 0.3, 0.6 and 0.9, and a wider choice of sample sizes. The same procedure in [5] will be used.

2.0 ARIMA (1,0,0,L, δ) and statistical Analysis.

The ARIMA (1,0,0,L, δ) series is divided into two parts. The series before intervention is called the pre-intervention time series after the intervention effect is called the post-intervention time series.

Let $Z_1, Z_2, \ldots, Z_n$ be the realization of the time series for the times $t_1, t_2, \ldots, t_n$ respectively. Assume the intervention effect occurs between times t_{n_1} and $t_{n_1} + 1$

For the n_1 time points before the intervention, the series is modelled by:

$$Z_1 = L + U_1 \qquad \ldots\ldots\ldots\ (\text{i})$$

$$Z_t - L = \emptyset(Z_{t-1} - L) + U_t \qquad \ldots\ldots\ldots\ (\text{ii})$$

where Z_t = the realization of the time series at time t

L = the level of the series

U_t = the white noise series assumed to NID $(0, \sigma^2)$

and $\emptyset$= the first-order autoregressive coefficient

where $-1<\emptyset<1$

For the $n_2 = n - n_1$ time points after intervention, the series is

$$Z_{n_1} + 1 - (L + \delta) = \emptyset[Z_{n_1} - L] + U_{n_1} + 1 \quad ..(\text{iii})$$

$$Z_t - (L + \delta) = \emptyset[Z_{t-1} - (L + \delta)] + U_t \quad \ldots\ldots(\text{iv})$$

$$\text{for } t = n_1 + 2, \ldots, n_1 + n_2$$

where δ = the intervention effect at time $t = n_1 + 1$

In order to estimate the parameters L, δ and $\emptyset$, a linear transformation will be used, which will be denoted by Y_t.

Let $Y_1 = Z_1 = L + U_1$

and $Y_t = Z_t - \emptyset Z_{t-1}$ for $t \geqq 2$

Hence, we obtain:

$$Y_1 = Z_1 = L + U_1 \quad \text{..........(v)}$$
$$Y_t = Z_t - ØZ_{t-1} = (1 - Ø)L + U_t \quad \text{..........(vi)}$$

$$2 \leqq t \leqq n_1$$

After the intervention, the level of series is changed, the model is:

$$Y_{n_1} + 1 = (1 - Ø)L + \delta + U_{n_1} + 1 \quad \text{...........(vii)}$$

and $$Y_t = (1 - Ø)L + (1 - Ø)\ \delta + U_t \quad \text{.........(viii)}$$
for $n_1 + 1 < t \leqq n_1 + n_2$

This transformation can be represented in matrix form as follows:

$$\underline{Y} = \underline{X}\ \underline{ß} + \underline{\theta} \quad \text{..........(ix)}$$

where

$$\underline{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \\ . \\ Y_{n_1} \\ Y_{n_1} + 1 \\ Y_{n_1} + 2 \\ . \\ . \\ Yn \end{bmatrix}_{(n \times 1)} \qquad \underline{X} = \begin{bmatrix} 1 & 0 \\ 1-Ø & 0 \\ \\ . & . \\ 1-Ø & 0 \\ 1-Ø & 0 \\ 1-Ø & 1-Ø \\ . & . \\ . & . \\ 1-Ø & 1-Ø \end{bmatrix}_{(n \times 2)}$$

$$\underline{\beta} = \begin{bmatrix} L \\ \delta \end{bmatrix}_{(2 \times 1)} \qquad \underline{\theta} = \begin{bmatrix} U_1 \\ U_2 \\ . \\ . \\ U_{n_1} \\ U_{n_1+1} \\ . \\ . \\ U_n \end{bmatrix}_{(n \times 1)}$$

The general linear model in matrix form is:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_{n_1} \\ Y_{n_1+1} \\ Y_{n_1+2} \\ . \\ . \\ . \\ . \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1\text{-}\varnothing & 0 \\ . & . \\ . & . \\ 1\text{-}\varnothing & 0 \\ 1\text{-}\varnothing & 1 \\ 1\text{-}\varnothing & 1\text{-}\varnothing \\ . & . \\ . & . \\ 1\text{-}\varnothing & 1\text{-}\varnothing \end{bmatrix} \begin{bmatrix} L \\ \delta \end{bmatrix} + \begin{bmatrix} U_1 \\ U_2 \\ . \\ . \\ U_{n_1} \\ U_{n_1+1} \\ U_{n_1+2} \\ . \\ . \\ U_n \end{bmatrix}$$

$$\underline{Y} = \underline{X} \quad \underline{\beta} + \underline{\theta}$$

The estimated parameters, $\hat{L}$ and $\hat{\delta}$, are obtained by the method of least squares as described in [3]. The matrix equation for this estimation is:

$$\beta = (\underline{X}' \, \underline{X})^{-1} (\underline{X}' \, \underline{Y}) \qquad \text{..........(x)}$$

We will investigate the type-1 error control of the t-test statistics introduced by Glass, Wilson, and Gottmann in [2] for several values of sample sizes, n_1 and n_2, several values of autoregressive coefficient, and several values of α values are: $(n_1, n_2) = (10, 10), (10, 40), (40, 10), (50, 50)$; $\emptyset = 0.0, 0.3, 0.6$ and 0.9; and $\alpha = 0.01, 0.05, 0.10$ and 0.20. Altogether, 64 cases will be examined.

For each of these 64 cases, the parameters L and δ will be estimated from the equation (X). Because the coefficient ø in matrix X is unknown, the value of this coefficient will vary from -1 to 1 with the increment of 0.01. In each calculation, the estimated parameters L and δ will change according to the value of the autoregressive coefficient. To obtain the appropriate parameters, the sum of squares of deviations will be calculated from the equation:

$$SSE_{\emptyset} = \sum_{i=1}^{n} (Yi - \hat{Y}i)^2 \qquad \text{..........(xi)}$$

where $-1<\emptyset<1$ with steps of 0.01

so $\emptyset=-1, -.99, -.98, ..., .98, .99, 1.0$

$\hat{Y}i$ = estimated value of Y, obtained from replacing the estimated parameters L and δ in equation (v), (vi), (vii) and (viii).

The estimated parameters will be ones for which $SSE_{\emptyset}$ is minimized, say SSE:

$$SSE = \underset{\emptyset= -1, -.99, ..., .99, 1}{\text{minimum}}\ SSE_{\emptyset} \qquad \text{..........(xii)}$$

The statistical hypothesis for testing the intervention effect is:

$H_o ; \delta = 0$
$H_A ; \delta \neq 0$

The standard error and t-statistic for the least squares estimate $\hat{\delta}$ of δ are:

$$SE(\hat{\delta}) = SS(X_{22})^{1/2} \qquad \text{..........(xiii)}$$

and $$t_{\hat{\delta}} = \frac{\hat{\delta}}{SE(\hat{\delta})}$$ ....(xiv)

where $$SS = \sqrt{\frac{SSE}{n-2}}$$ ....(xv)

X_{22} is the (2,2) diagonal element in $(X'X)^{-1}$

The α- level t-test of the null hypothesis (Ho; δ =0) is:

$$|t_{\hat{\delta}}| > t_{\alpha/2,\, n-2}$$ ....(xvi)

Where $t_{\alpha/2}$, n - 2 is the upper α /2 point of central t-distribution with n - 2 degrees of freedom.

In order to calculate the type-1 error, the simulation will be repeated 5000 times. The type-1 error is the number of times $|t_{\hat{\delta}}| > t_{\alpha/2}, n_1 + n_2 - 2$ is observed, divided by the replications (i.e. 5000). Comparison between the actual type-1 error and theoretical value will show the effect of sample size and the autoregressive coefficient, on the validity or robustness of this model.

3.0 Method of Calculation and Computer Algorithm

The computer analysis will be obtained using the following 5-step procedure.

Step 1: Creating the time series model

The autoregressive model order 1 is generated by:

$$Z(1) = \emptyset Z(I-1) + S(I)$$

where S(I) is a white noise series with mean = 0, variance = 1 and we assume Z(0) = 0.

The FORTRAN function RAN is used to generate a series, X (I) which has a uniform distribution on (0,1). This is accomplished by:

X (I) = RAN[X(I)]

To obtain a normal series S(I) with mean = 0 and variance=1, the generation method due to Box and Muller [4] will be used. This method is:

$$S(I) = \sqrt{-2\ ALOG(\underline{X}(I))} * COS(2\pi * \underline{X}(I+1))$$

$$S(I) = VAR*S(I) + \underline{X}MEAN$$

$$S(I+1) = \sqrt{-2\ ALOG(\underline{X}(I))} * SIN(2\pi * \underline{X}(I+1))$$

$$S(I+1) = VAR*S(I+1) + \underline{X}MEAN$$

S(I) and S(I+1) will each have a normal distribution whose means equal XMEAN and whose variances equal VAR:

where $\underline{X}MEAN = 0$
$VAR = 1$

The series will be obtained from:

$$Z(I) = \emptyset * Z(I-1) + S(I)$$
for I = 1, 2, ..., N
$$Z(0) = 0$$

To transform the series Z(I) into the general linear model form.

We let $Y(I) = Z(I) - (1 - \emptyset)\, Z(I - 1)$

Step II: Calculating the estimated parameters

The values of the estimated parameters L and δ are obtained from equation:

$$\hat{\underline{\beta}} = \begin{bmatrix} \hat{L} \\ \hat{\delta} \end{bmatrix} = (\underline{X}'\ \underline{X})^{-1}\ (\underline{X}'\ \underline{Y})$$

For the first simulation, the value of Ø equal -1.0 and with fixed n_1 and n_2; $(\underline{X}'\ \underline{X})^{-1}$ and $\underline{X}'\ \underline{Y}$ are obtained from

$$(\underline{X}'\ \underline{X})^{-1} = \begin{bmatrix} C/FP & -B/FP \\ -B/FP & A/FP \end{bmatrix}$$

where $A = 1 + (n_1+n_2 - 1)\ (1 - Ø)^2,$

$B = (1-Ø) + (n_2 - 1)\ (1 - Ø)^2,$

$C = 1 + (n_2 - 1)\ (1-Ø)^2,$

$FP = 1 + (n_1 + 2n_2 - 3)\ (1-Ø)^2 - 2(n_2 - 1)(1-Ø)^3 + n_1(n_2 - 1)(1-Ø)^4$

and $(\underline{X}'\ \underline{Y}) = \begin{bmatrix} E \\ F \end{bmatrix},$

where $E = Y(1) + (1-Ø)\ (\sum_{i=2}^{n} Yi)$

$F = Y(n_1 + 1) + (1-Ø)\ (\sum_{i=n_1+2}^{n} Yi)$

The estimated parameters in computer program can be obtained from:

$$\hat{\beta} = \begin{bmatrix} \hat{L} \\ \hat{\delta} \end{bmatrix} = \frac{1}{FP} \begin{bmatrix} C & -B \\ -B & A \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix}$$

$$= \frac{1}{FP} \begin{bmatrix} CE - BF \\ AF - BE \end{bmatrix}$$

$$= \begin{bmatrix} \frac{M}{FP} \\ \frac{L}{FP} \end{bmatrix}$$

where $M = CE - BF$

and

$L = AF - BE$

Step III: Estimation of Y(I) and Calculation of SS

In order to obtain $\hat{Y}(I)$, the values of $\hat{L}$ and $\hat{\delta}$ are substituted into equations (v), (vi), (vii) and (viii) of 2.0.

The sum of squares deviation is obtained from

$$SS = \sum_{i=1}^{n} (Y(I) - \hat{Y}(I))^2$$

where SS = sum of squares deviation

Step IV: Changing the ø values and recomputing in Step I to Step III

The value of Ø will be changed in steps of 0.01 until it reaches the value of 1.0. Each value of Ø will be used in step II to compute estimated parameters, and step III to compute the $\hat{Y}(I)$ and the sum of squares deviation.

Step V: Testing the hypothesis

We use the estimated value of δ that has a minimum value of SS to test the hypothesis:

$$H_o : \delta = 0$$
$$H_A : \delta \neq 0$$

by calculating $$t = \frac{\hat{\delta} - 0}{SE(\hat{\delta})}$$

where $$SE(\hat{\delta}) = \sqrt{\frac{\min SS}{n-2}} \sqrt{X_{22}}$$

The value X_{22} = (2,2) diagonal element in $(\underline{X}'\underline{X})^{-1} = \frac{\underline{A}}{FP}$

We then compare this t value with $t_{\alpha/2, n-2}$. The null hypothesis (H_0) will be rejected if $|t| > t_{\alpha/2, n-2}$.

After finishing step v, the simulation will be repeated by generating a new series in step I, and repeating step II to step v. The number of times that H_0 is rejected will be counted during 5000 replications of this experiment. The observed type-1 error is computed from the number of times that the experiment rejects H_0 divided by total number of replications (5000 times).

4.0 Conclusion

In order to test whether the mean of post-intervention time and mean of pre-intervention time series of the ARIMA(1, 0, 0, L, δ) model have the same value, Glass and Willson suggest the use of the test-statistic $t = \frac{\hat{\delta} - 0}{\sigma_{\hat{\delta}}}$.

The observed probability of $|t| > t_{\alpha/2}$, when $\delta = 0$, is obtained from the simulation of 5000 replications of this experiment. The results are shown in table I.

To investigate whether the statistic that has been used to test the intervention effect has an approximate t-distribution, we use the following test.

Let P=Probability of $|t| > t_{\alpha/2}$ when $\delta=0$
(theoretical type-1 error P=0.20, 0.10, 0.05, 0.01)

$\hat{P}$=Probability of $|t| > t_{\alpha/2}$ when $\delta=0$
(actual type-1 error)

To test whether the actual type-1 error in column II of Table I is equal to theoretical type-1 error, we consider the hypothesis:

$H_o ; P \leq \alpha$
$H_A; P > \alpha$,

for = 0.20, 0.10, 0.05., 0.01

The test statistics is

$$Z = \frac{\hat{P} - \alpha}{\sqrt{\frac{\alpha(1-\alpha)}{5000}}}$$

We will reject Ho if $Z > 3$

Table II contains Z values for the simulation cases previously examined in Table I.

Table I : Result of the Experiment

Theoretical Type-1 error vs observed Type 1 error.

Case I $n_1 = 10$ $n_2 = 10$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	0.2650	0.1582	0.1024	0.0404
Ø = 0.3	0.3044	0.1964	0.1336	0.0588
Ø = 0.6	0.3756	0.2568	0.1906	0.0966
Ø= 0.9	0.4116	0.2978	0.2316	0.1398

Case II $n_1 = 10, n_2 = 40$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	0.2092	0.1128	0.0596	0.0166
Ø = 0.3	0.2218	0.1220	0.0708	0.0212
Ø = 0.6	0.2466	0.1416	0.0850	0.0290
Ø = 0.9	0.2844	0.1766	0.1132	0.0406

Case III $n_1 = 40, n_2 = 10$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	0.2166	0.1206	0.0666	o.0196
Ø = 0.3	0.2364	0.1352	0.0842	0.0256
Ø = 0.6	0.2806	0.1676	0.1070	0.0370
Ø = 0.9	0.2908	0.1778	0.1228	0.0468

Case iv $n_1 = 50, n_2 = 50$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	0.2122	0.1184	0.0632	0.0168
Ø = 0.3	0.2204	0.1264	0.0722	0.0214
Ø = 0.6	0.2460	0.1430	0.0894	0.0298
Ø = 0.9	0.2828	0.1722	0.1094	0.0448

Note : Each probability is based on 5000 samples.

Table II Z Values from result in Table I

Case I $n_1 = 10, n_2 = 10$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	11.50*	13.73*	17.01*	21.56*
Ø = 0.3	18.48*	22.74*	27.14*	34.61*
Ø = 0.6	31.08*	36.98*	45.65*	61.42*
Ø = 0.9	37.45*	46.65*	58.96	92.05*

Case II $n_1 = 10, n_2 = 40$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.50$	$\alpha = 0.01$
Ø = 0.0	1.63[a]	3.02*	3.12*	4.68*
Ø = 0.3	3.86*	5.19	6.75*	7.04*
Ø = 0.6	8.25*	9.81*	11.36*	13.48*
Ø = 0.9	14.94*	18.07*	20.52	21.70*

Case III $n_1 = 40, n_2 = 10$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	2.94[a]	4.86*	5.39*	6.80*
Ø = 0.3	6.44*	8.30*	11.10*	11.06*
Ø = 0.6	14.27*	15.94*	18.51*	19.14*
Ø = 0.9	16.07*	18.35*	23.64*	26.10*

Case iv $n_1 = 50, n_2 = 50$

	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Ø = 0.0	2.16[a]	4.34*	4.29*	4.82*
Ø = 0.3	3.61*	6.23*	7.21*	8.08*
Ø = 0.6	8.14*	10.14*	12.79*	14.04*
Ø = 0.9	14.65*	17.03*	19.28*	24.68*

Note: * indicates rejection of Ho ($P \leq \alpha$) and accepting of $H_A(P > \alpha)$

a indicates acceptance of Ho ($P \leq \alpha$)

From Table II, the test results show that actual type-1 error is always significantly greater than the theoretical type-1 error value, except when the autoregressive coefficient is equal to zero in three of four cases when $\alpha = 0.20$. Therefore, the results of simulation study to test the intervention effect in ARIMA (1,0,0,L, δ) model suggested by Glass, Wilsin and Gottman, do not preserve the α - level of prescribed significance. For all cases where Ø > 0, these test are liberal. The degree of liberality increases as Ø increases.

The computer program for the simulated series is shown in the appendix.

BIBLIOGRAPHY

1. Nelson, Charles R. Applied Time Series Analysis. San Francisco: Holden-Day, 1973.

2. Glass, G. V., Wilson, V. L. & Gottman, J. M. Design And Analysis Of Time Series Experiments. Boulder: Colorado Associated University Press, 1975.

3. Mendenhall, Williams, & Scheaffer, Richard L. Mathematical Statistics With Applications. North Seituate, MA: Duxbury Press, 1973.

4. Box, G.E.P., & Muller, M. E. A note on the generation of normal deviates. Annals Of Mathematical Statistics, 1958, 29, 610-611.

5. Zinkraft, Stephen A., & Willson, V. L. The Use Of The Box- Jenkins Approach In Casual Modeling: An Investigation Of The Cost Of The Misidentification Of Selected Stationary Models. Texas A & M University.

APPENDIX

```
C     THIS PROGRAM IS WRITTEN TO SIMULATE THE SERIES USED FOR
      TESTING
C     INTERVENTION EFFECT IN ARIMA(1,Ø,Ø,L,D)    MODEL

         INTEGER  NC
C     NC IS USED FOR COUNTING THE LOOPS

         INTEGER N1, N2
C     N1=NO. OF TIMES OF PRE-TIME SERIES
C     N2=NO. OF TIMES OF POST-TIME SERIES

         REAL PHI
C     PHI IS FIRST-ORDER REGRESSIVE COEFFICIENT

         REAL Y(3ØØ)
C     Y IS ERROR SERIES (NORMAL, MEAN=0, VAR=S**2)

         REAL Z(3ØØ)
C     Z IS AUTOREGRESSIVE MODEL SERIES

         REAL PHI1
C     PHI1 IS AUTOREGRESSIVE COEFFICIENT

         REAL A,B,C,E,F
C     ALL ARE ELEMENTS DURING CALCULATION

         REAL C11, C22
C     DIAGONAL ELEMENTS IN X-PRIME-X MATRIX

         REAL FP
C     FP IS DETERMINANT TO CALCULATE X-PRIME-X INVERSE

         REAL M,L
C     ELEMENTS IN MATRIX CALCULATION

         REAL YH(3ØØ)
C     YH IS ESTIMATED SERIES

         REAL SM(2Ø1)
C     SUM OF SQUARE ERROR

         REAL SEL, SET
C     SS OF LEVEL AND INTERVENTION EFFECT
```

```
      REAL SDM(2Ø1)
C     SDM(I) IS SQUARE ROOT OF SM(I)
      REAL TL, TD
C     TL AND TD SRE ESTIMATED T VALUES OF LEVEL AND INTERVENTION

      REAL ESTL(2Ø1),  ESTD( 2Ø1)
C     T-STATISTIC FOR LEVEL AND INTERVENTION EFFECT

      DO 33 I = 1, N
      Z(I) = PHI*Z(I-1)+S(1)
   33 CONTINUE

      TIME = 1ØØØØØ
C     TIME VALUE USED FOR SELECTIING THE MINIMUM SUM OF SQUARE
C     ERROR

      REAL  S(3ØØ)
C     WHITE NOISE SERIES (ERROR)
      VAR = 1.Ø
      XMEAN = Ø.Ø
C     INDICATE RANDOM  ERROR WITH MEAN = Ø, AND VAR = 1.Ø

      N1 = 1Ø
C     N1 IS NO. OF PRE-INTERVENTION TIME SERIES

      N2 = 4Ø
C     N2 IS NO. OF POST-INTERVENTION TIME SERIES

      N = N1 + N2
C     N IS THE TOTAL NO. OF SERIES

      V = N1 + N2 - 2
C     V IS DEGREE OF FREEDOMS

      NC = 1
C     INITIALIZED THE LOOP
      PHI = Ø.9
C     PHI IS THE VALUE OF AUTOAGRESSIVE COEFFICIENT

      T2Ø = 1.3ØØØ
      T1Ø = 1.68ØØ
      TØ5 = 2.ØØ5Ø
      TØ1 = 2.67ØØ

C     T2Ø, T15, TØ5, TØ1 ARE THE VALUES OF T AT ALPHA LEVEL OF .2Ø,  .15,  .05,
C        .Ø1
```

```
      TC2Ø = Ø
      TC1Ø = Ø
      TCØ5 = Ø
      TCØ1 = Ø
C     COUNTER OF REJECTION

  34Ø DO 77 I = 1,N
      Y(I) = RAN(Y(I))
   77 CONTINUE

C     TRANSFORM THE GENERATED RANDOM ERROR TO NORMAL DISTRIBUTION

      DO 12 I = 1,N,2
      S(I) = SQRT (-2*ALOG(Y(I))*COS(2*3.1415926*Y(I+1))
      S(I) = VAR*S(I)+XMEAN
      S(I +1) = SQRT(-2*ALOG(Y(I)))*SIN(2*3.1415926*Y(I+1))
      S(I +1) = VAR*S(I+1)+XMEAN
   12 CONTINUE

C     CREATE THE SERIES OF ALOGARITHM OF AUTOREGRESSIVE MODEL

      Z(Ø) = Ø
      DO 33 I = 1,N
      Z(I) =PHI*Z(I-1)+S(1)
   33 CONTINUE

      TIM = 1ØØØØØ
C     TIM  VALUE USED FOR SELECTING THE MINIMUM SUM OF SQUARE ERROR

      NCC = 5ØØØ
C     NO, OF TIMES TO CALCULATE

      PHI1 = -1.Ø1
      DO 53 I = 1,2Ø1
      PHI1 = PHI1 + Ø.Ø1
      S1=1-PHI1
      S2=(1-PHI1)**2
      S3=(1-PHI1)**3
      S4=1-PHI1)**4

      A=1.+(N1+N2-1)*S2
      B=S1+(N2-1)*S2
      C=1.+(N2-1)*S2
      FP=1. +(N1+2*N2-3)*S2-2.*(N2-1)*S3+N1*(N2-1)*S4
      C11=C/F
      C22=A/FP
```

```
C       TRANSFORMATIONS
          DO 136 J=2,N
          Y(J)=Z(J)-PHI1*Z(J-1)
      136 CONTINUE

C       CALCULATE X-PRIME -Z

          YSUM1 = Ø
          DO 137 = J = 2 , N
          YSUM1=YSUM1+Y(J)
      137 CONTINUE

          YBAR1 = YSUM1/(N-1)

          E = Y(1)+S1*(N1+N2-1)*YBAR1

          YSUM2=Ø
          DO 139 J=N1+2,N
          YSUM2=YSUM2+Y(J)
      139 CONTINUE

          YBAR2=YSUM2/(N2-1)

          F = Y(N1+1)+S1*(N2-1)*YBAR2

          M=C*E-B*F
          D=A*F-B*E

          ESTL(I)=M/FP
          ESTD(I)=D/FP

          YH(1)=ESTL(I)

          DO 643 KM=2,N1
          YH(KM)=S1*ESTD(I)
      643 CONTINUE

          YH(N1+1)=S1*ESTD(I)+ESTD(I)
          DO 644 KM=N1+2,N
          YH(KM)=S1*(ESTD(I)+ESTD(I))
      644 CONTINUE
```

```
        SS=Ø,Ø
       DO 639 KI=1,N
        SS=SS+(Y(KI)-YH(KI))**2
   639 CONTINUE

        SM(I)=SS/V
        SDM(I)=SQRT(SM(I))
        SEL=SDM(I)*SQRT(C11)
        SET=SDM(I)*SQRT(C22)
        TL=ESTD(I)/SEL
        TD=ESTD(I)/SET

        IF(SS .LT. TIM) GO TO 371
        GO TO 555

   321 TIM=SS
        TU=TD
   555 TIM=TIM

    53 CONTINUE
        IF(ABS(TU) .GT. T2Ø) TC2Ø=TC2Ø+1
        IF(ABS(TU) .GT. T1Ø) TC1Ø=TC1Ø+1
        IF(ABS(TU) .GT. TØ5) TCØ5=TCØ5+1
        IF(ABS(TU) .GT. TØ1) TCØ1=TCØ1+1
        NC=NC+1
        IF(NC .EQ. NCC) GO TO 14ØØ
        GO TO 34Ø

14ØØ     WRITE (5,3333)
3333     FORMAT(1H1,4X,'END',/)

          TP2Ø=TC2Ø/NCC
          TP1Ø=TC1Ø/NCC
          TPØ5=TCØ5/NCC
          TPØ1=TCØ1/NCC
          WRITE(5,3ØØØ)
3ØØØ     FORMAT(1HØ,7X,'TP2Ø',1ØX,'TP1Ø',1ØX,'TPØ5',1ØX,'TPØ1', /)
         WRITE(5,4434) TP2Ø, TP1Ø,TPØ5,TPØ1
4434     FORMAT(4F)
         WRITE(5,2222)  NCC
2222     FORMAT (1H1, 5X,'NCC',T,/)
         WRITE(5,5494)
5494     FORMAT(1H1,5X,'FROM THE VALUE OF N1,N2,RHO,T2Ø,T1Ø,TØ5,TØ1',/)
         WRITE(5,5449) N1,N2
5449     FORMAT(1H1, 'N1= ',I3,5X,'N2= ',I3,/)
         WRITE(5,6598) RHO
```

```
6598    FORMAT(1H1,'RHO IS = ',F2.1,/)
        WRITE(5,6599)
6599    FORMAT(1H1, 'VALUE OF T .2Ø,.1Ø,.Ø5,.Ø1',/)
        WRITE(5,8811) T2Ø,T1Ø,TØ5,TØ1
8811    FORMAT(4F)
        STOP

        END
```