Enhancement of Filter Design and EEG Power Ratio Features in IQ Pattern Analysis

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***Abstract*—Power ratio is an established electroencephalogram (EEG) feature that has been used to study cognitive performance. Essentially, the technique computes the normalized power for each of the brainwave components prior to pattern analysis. The method however, is subject to further improvement as previous pre-processing approach rely on low-order filter designs. As a result, the obtained features are less accurate due to the presence of spectral leakages within the pre-processing element. Hence, this paper propose an improved extraction algorithm based on the use of high-order equiripple filters. Pre-existing intelligence quotient data are acquired from 50 samples and their EEG is recorded from the left pre-frontal cortex. The power ratio features are obtained from the energy spectral density of theta, alpha and beta bands. While results maintain conformity with the Neural Efficiency Hypothesis of human intelligence, comparative study shows that with equiripple filters, the revised power ratio is more suitable for IQ pattern analysis.**

***Index Terms*—EEG, equiripple filter, intelligence, power ratio**

I. Introduction

Electroencephalogram (EEG) is a non-invasive method electrical recording of brain activities [1]. The aggregated potential from a population of neurons are measured from the surface of the scalp using specialized biopotential electrodes [2]. EEG is essentially an oscillating signal which consist of different frequency harmonics and amplitudes ranging from ±10 μV to ±100 μV [3]. Brainwaves can be segregated into delta, theta, alpha and beta waves. Delta band range from 0.5 Hz to 4 Hz, theta band range from 4 Hz to 8 Hz, alpha band range from 8 Hz to 13 Hz, and beta band ranges from 13 Hz to 30 Hz [4]. Each of the frequency bands is uniquely associated with different cognitive states. Delta waves are generally related to deep sleep and comatose condition. Meanwhile, theta waves are associated with light sleep [5]. Studies have also shown that the frequency band is involved in working memory demands [6]. Conversely, alpha waves is most dominant when the brain is relaxed, but in conscious state. When the brain is mentally engaged however, the faster beta waves would become more prominent [7].

Cognitive ability is comprised of different aspects of mental constituents. Its performance can be assessed using parameters such as intelligence [8], attentional state [9], working memory [10] and learning styles [11]. For each of these constituents, the varying brain states is being reflected by specific patterns between the EEG bands. The Neural Efficiency hypothesis of human intelligence states that brighter individuals exhibit efficient use of brain areas compared to the less gifted ones [12]. They have demonstrated lower glucose consumption by the cerebrum [13]. These are attributed by higher probability of error-less transmissions due to the well-functioning neurotransmitters which leads to lower cerebral arousability [14]. Subsequently, the higher alpha power indicates reduced stress levels [15]. Meanwhile, the alpha suppression theory also states that there is a reciprocal relationship between theta and alpha waves. As such, the desynchronization of alpha waves will result in the synchronization of theta waves. Hence, the lower alpha power will also be reflected in higher theta power, and vice versa [16].

Previous study has established an intelligence quotient (IQ) classification model based on the brainwave pattern. The recording protocol and pre-processing procedure adopts a standardized approach; focusing on EEG from the left prefrontal cortex. The model was successfully developed using power ratio features and artificial neural network [17, 18]. The feature extraction algorithm however, still requires further improvement as its pre-processing relies on low-order filters that were designed using Hamming window [19]. The frequency response characteristics which were not optimized resulted in spectral leakages; causing loss of information at the band edges. These alters the power spectral density (PSD) for each of the sub-bands and consequently affects the accuracy of power ratio features. The preceding results have also indicated the presence of outliers in theta, alpha and beta power ratio [17].

To overcome the deficiencies, this paper proposes enhanced filter designs for EEG power ratio feature extraction. Instead of relying on conventional window method, the filters will be designed using Parks-McClellan algorithm. This is to ensure

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optimum frequency response characteristics with uniformed approximation error and narrow transition bands [20]. The new filter designs will be implemented on similar set of data that was previously used to analyze brainwave patterns for varying IQ levels.

II. Methods

The study generally comprise of data collection, EEG pre-processing, extraction of power ratio, and feature evaluation based on varying IQ levels. Fig.1 summarizes the methods implemented throughout the study.

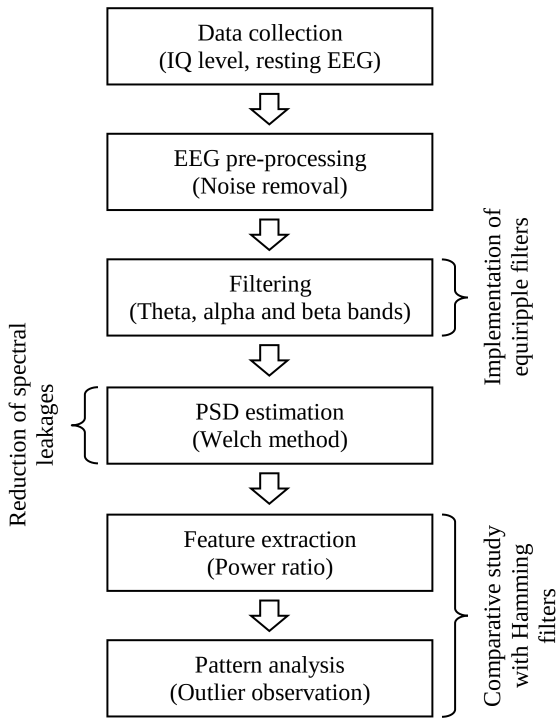

Fig. 1.Framework of research methods.

A. Data Collection

50 healthy university students from various disciplines have participated in the study. The age range is between 18 to 40 years (mean age / standard deviation = 23.9 / 3.5 years). All subjects are right-handed and not under any prescribed medications. The overall procedure has been briefed to the subjects prior to data collection and subjects have completed the consent form.

Subjects are required to relax in seated position with both eyes closed. EEG is recorded from Fp1 scalp location using g.MOBIlab$^+$. The positioning comply with the International 10-20 System of electrode placement. EEG is sampled at 256 Hz. Subsequently, the 3 minutes signal is analyzed offline in MATLAB.

After completing the EEG recording, subjects are required to answer the online Raven's Progressive Matrices IQ test. The IQ scores range from 0 to 150. Subjects are then grouped into high, medium and low IQ level based on the established mean and standard deviation [19].

B. EEG Pre-Processing

The study focuses on the left prefrontal cortex as region is involved in sequential and logic processes [21]. Initially, the signals are pre-processed to obtain artefact-free EEG. Epochs that exceed amplitudes of ±100 µV are regarded as noise from electrooculagram and thus, rejected [22]. After pre-processing, the 2 minutes 30 seconds EEG is then segregated into theta, alpha and beta waves using band-pass filters [23].

In the preceding studies, the filters were established using window technique with order N = 74. The desired impulse response $h_d(n)$ is mathematically expressed by (1), where $h(n)$ is the impulse response for band-pass filter and $w(n)$ is the window function in time-domain.

$$h_d(n) = h(n).w(n). \tag{1}$$

$h(n)$ is generally shown by (2), where ω_1 is the lower cut-off frequency, ω_2 is the upper cut-off frequency, N is the filter order and $\alpha = N / 2$.

$$h(n) = \frac{\sin \omega_2 (n-\alpha)}{\pi(n-\alpha)} - \frac{\sin \omega_1 (n-\alpha)}{\pi(n-\alpha)}. \tag{2}$$

Meanwhile, Hamming window function $w(n)$ is represented by (3), where N is the filter order and $\alpha = N / 2$.

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right). \tag{3}$$

The frequency range for each band is specified as follow; theta waves from 4 Hz to 8 Hz, alpha waves from 8 Hz to 13 Hz, and beta waves from 13 Hz to 30 Hz. Fig. 2, Fig. 3 and Fig. 4 each shows the frequency response for the respective filters that are designed using window method.

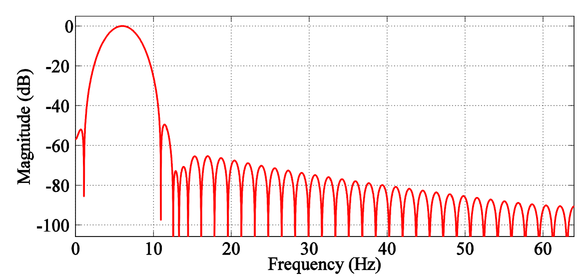

Fig. 2.Frequency response for theta filter (Hamming window).

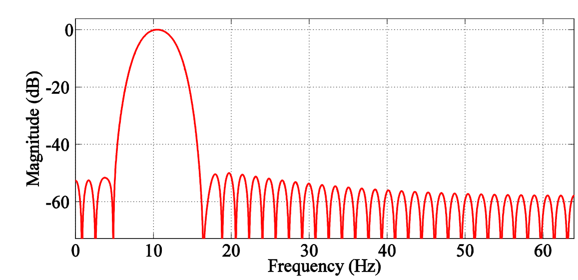

Fig. 3.Frequency response for alpha filter (Hamming method).

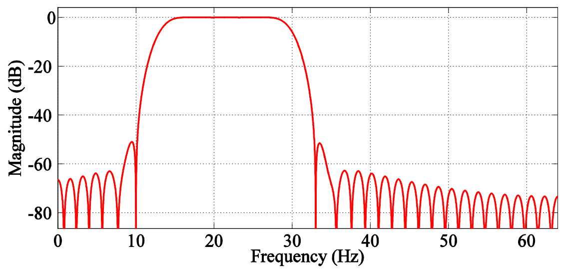

Fig. 4.Frequency response for beta filter (Hamming method).

In this study however, equiripple filters are proposed and designed using optimum method. The technique adopts the Parks-McClellan algorithm to minimize the maximum ripple in the pass-band and stop-band regions [20]. To attain narrow transition region of 0.2 Hz with attenuation of –60 dB in the stop-band, the desired characteristics is achieved at order N = 1264. Fig. 5, Fig. 6 and Fig. 7 frequency response for the respective filters that are designed using optimum method.

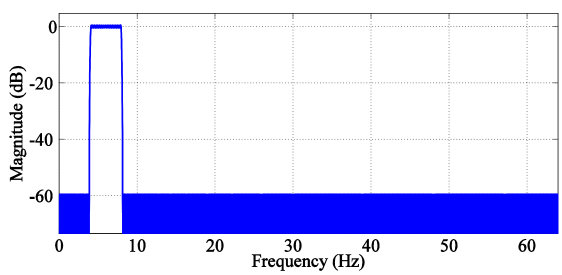

Fig. 5.Frequency response for theta filter (optimum method).

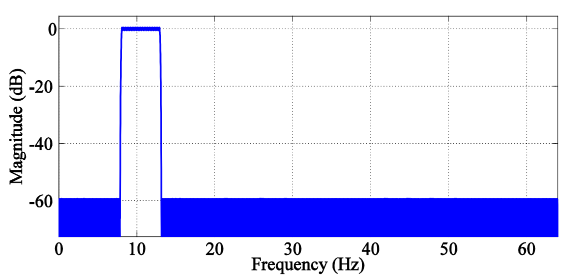

Fig. 6.Frequency response for alpha filter (optimum method).

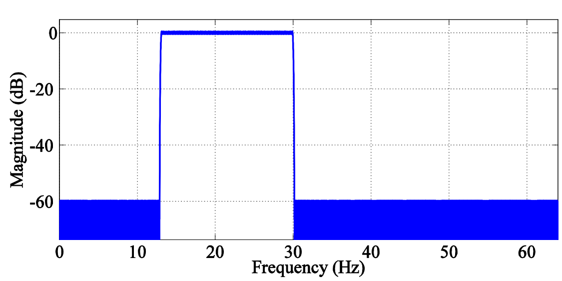

Fig. 7.Frequency response for beta filter (optimum method).

Significant improvement has been observed for filters design using optimum method. The frequency response shows narrow transition band and uniformed ripple in both the pass-band and stop-band of the equiripple filters. These reduce the spectral leakages at the band edges and avoid information loss.

C. Power Spectral Density and Power Ratio Features

To verify the effectiveness of designed filters, power spectral density (PSD) for each of the EEG bands are estimated using Welch method. The computation involves the use of Hamming window with 50% overlapping epoch. Energy spectral density (ESD) is then calculated as the area under the PSD curve. Power ratio is subsequently established to normalize the energy content between the respective bands. The method has proven to be effective in eliminating extreme outliers compared to the ESD features. Theta, alpha and beta ratio is shown by (4), (5) and (6), respectively. θ refers to the ESD in theta band, α is the ESD of alpha band, and β represents the ESD for beta band [17].

$$\text{Theta Ratio} = \frac{\theta}{\theta + \alpha}. \tag{4}$$

$$\text{Alpha Ratio} = \frac{\alpha}{\alpha + \beta}. \tag{5}$$

$$\text{Beta Ratio} = \frac{\beta}{\alpha + \beta}. \tag{6}$$

Subsequently, the power ratio features are clustered into the respective IQ groups and analyzed for conformity with the Neural Efficiency Hypothesis of human intelligence. Outliers are also identified for each of the IQ groups.

III. Results and Discussion

The findings initially discuss on the effects of equiripple filters on theta, alpha and beta waves. As differences are not clearly evident in time-domain, results between Hamming and equiripple filters are compared using PSD as the presence of spectral leakages is much visible in the frequency-domain.

A. Power Spectral Density

The following results are obtained from the EEG of Sample 1. Fig. 8 and Fig. 9 each shows the PSD for theta waves that are filtered using Hamming and equiripple filters. The lower order design that was implemented in the preceding studies exhibit inferior filtering capabilities compared to the proposed alternatives. These can be observed from the wide transition bands and truncation of critical information at the band edges. As a result, noise with frequencies lower than 4 Hz and higher than 8 Hz were not sufficiently filtered from the theta waves.

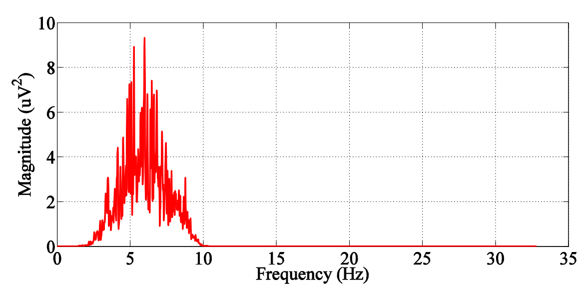

Fig. 8.PSD for theta waves (Hamming filter).

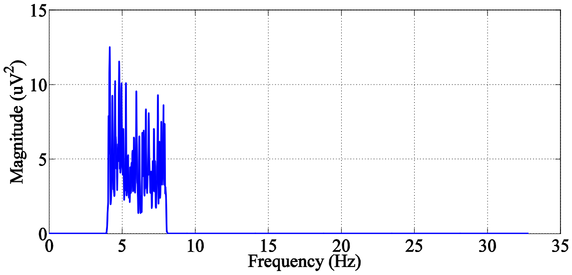

Fig. 9.PSD for theta waves (equiripple filter).

For theta waves that are filtered using equiripple filters however, noise with frequencies lower than 4 Hz and higher than 8 Hz have been sufficiently attenuated. Hence, the PSD at both cut-off frequencies form a steep slope with transition band of 0.2 Hz between the pass-band and stop-band regions. Spectral information within the edges of pass-band remains preserved. This is due to the absence of window function that is not present in optimum design method.

Meanwhile, Fig. 10 and Fig. 11 each demonstrates the PSD for alpha waves that are attained using Hamming and equiripple filters. Notably, noise is not effectively attenuated by Hamming filters near the cut-off frequencies. Hence, the resultant alpha waves are also interfered by unwanted signal with frequency content lower than 8 Hz and higher than 13 Hz. Meanwhile, attenuation of information at the edges of pass-band is not clearly evident as spectral power is most dense at the middle of alpha frequency range.

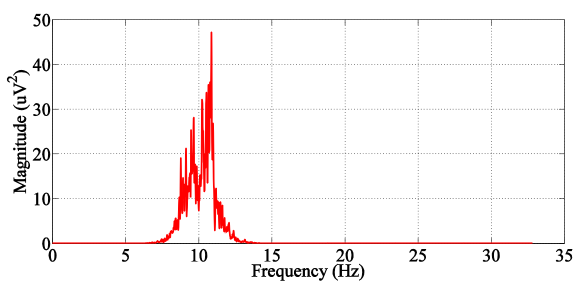

Fig. 10.PSD for alpha waves (Hamming filter).

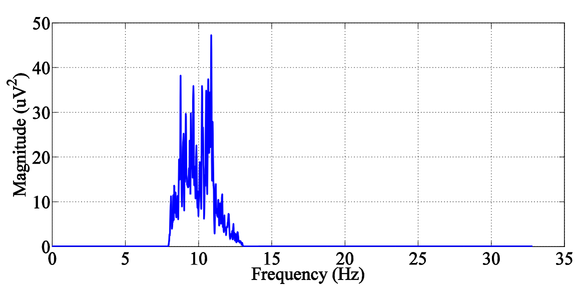

Fig. 11.PSD for alpha waves (equiripple filter).

Comparatively, alpha waves that are obtained via equiripple filters have demonstrated sufficiently attenuated noise at the stop-band region. Steep slopes with transition band of 0.2 Hz is also visible from the PSD curve. Also, incidental truncation of information at the edges of pass-band does not occur.

The PSD of beta waves obtained using Hamming and equiripple filters are each shown by Fig. 12 and Fig. 13. Similar differences have been observed at the band edges. The Hamming filter is unable to efficiently attenuated unwanted signals less than 13 Hz and higher than 30 Hz. Additionally, truncation of information at the edges of pass-band is glaringly visible.

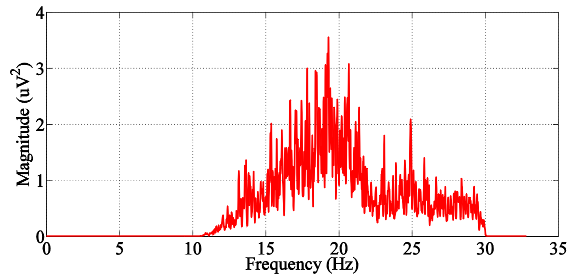

Fig. 12.PSD for beta waves (Hamming filter).

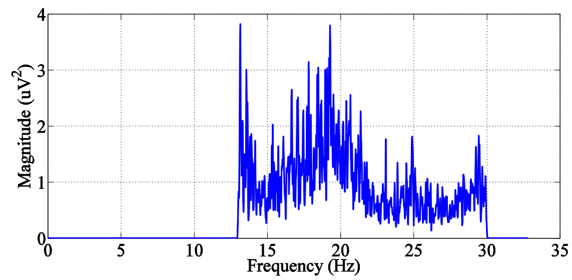

Fig. 13.PSD for beta waves (equiripple filter).

PSD for theta, alpha and beta bands successfully show the efficiency of equiripple filters in retaining precise information for the specific EEG bands. Hence, its implementation in the power ratio extraction algorithm is expected to yield features that are more accurate than the previous studies. It should be noted however, that the proposed filter design impose a higher computational load due to its very high filter order.

B. Power Ratio and Observation of Outliers

Based on the preceding research, six samples have been identified as high IQ, 39 samples are medium IQ, and the remaining five samples as low IQ [17]. Fig. 14 , Fig. 15 and Fig. 16 each shows the box plot for theta, alpha and beta power ratio for the respective IQ groups.

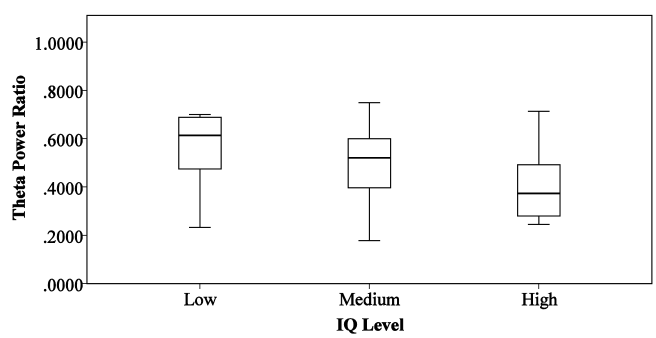

Fig. 14.Box plot for theta power ratio (N = 50).

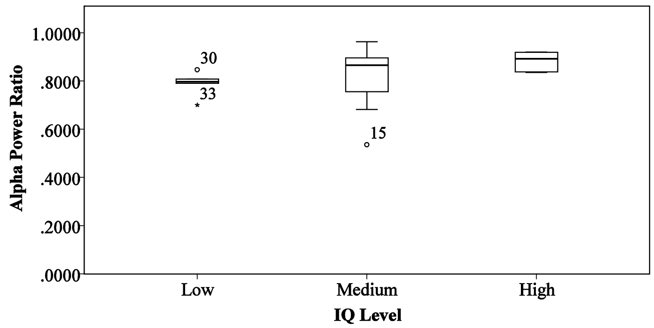

Fig. 15.Box plot for alpha power ratio (N = 50).

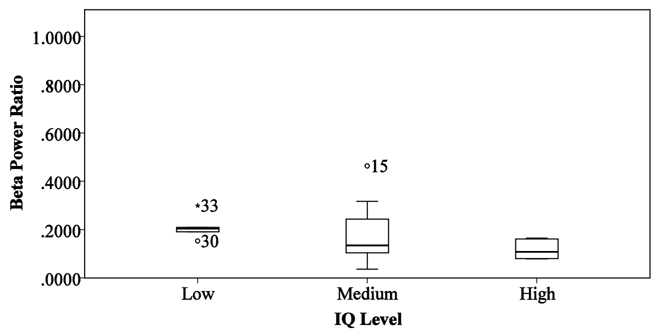

Fig. 16.Box plot for beta power ratio (N = 50).

The median of theta power ratio shows a descending pattern from low to high IQ group. Meanwhile, the pattern is inversed for alpha power ratio where the median increases from low to high IQ group. Findings are valid as the reciprocal relationship between both theta and alpha bands are in compliance with the alpha suppression theory. Results in the alpha band shows that the high IQ group exhibit the highest median for alpha power ratio. Hence, the brain is in more relaxed state and exhibit lower cortical activation. These demonstrate much efficient use of the brain compared to the less intelligent groups [12]. Meanwhile, the trend of median power ratio in the alpha band increases from low to high IQ level. Findings are therefore, in agreement with Neural Efficiency Hypothesis of human intelligence. The well-functioning inhibitory and excitatory neurotransmitters in the brain of brighter individuals result in low constant probability of transmission errors. These lead to smaller variation of rhythmic EEG and lower cortical noise. Such phenomena is thus, reflected in higher alpha power [24].

Outliers have been observed for alpha and beta power ratio features. Two samples are identified for low IQ group; one of which is considered as an extreme outlier. Meanwhile, one sample from the medium IQ group is identified as an outlier. Comparatively, features that are extracted using Hamming filters only exhibit two outliers. Hence, current results are unfavourable as features extracted via equiripple filters have resulted in the presence of an extreme outlier compared to those using Hamming filters. To compensate for this, the energy content of individual EEG band will be normalized against the ESD of theta, alpha and beta bands. Therefore, the revised equations for theta, alpha and beta power ratio is each presented by (7), (8) and (9).

$$\text{Theta Ratio} = \frac{\theta}{\theta + \alpha + \beta}. \tag{7}$$

$$\text{Alpha Ratio} = \frac{\alpha}{\theta + \alpha + \beta}. \tag{8}$$

$$\text{Beta Ratio} = \frac{\beta}{\theta + \alpha + \beta}. \tag{9}$$

Following the revision, the box plots for theta, alpha and beta power ratio is regenerated and shown by Fig. 17, Fig. 18 and Fig. 19, respectively. Overall, the pattern fo median for each of the EEG bands maintains agreement with the Neural Efficiency Hypothesis of intelligence and alpha suppression theory.

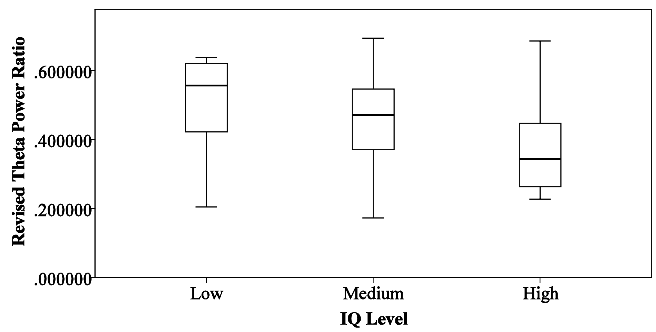

Fig. 17.Box plot for revised theta power ratio (N = 50).

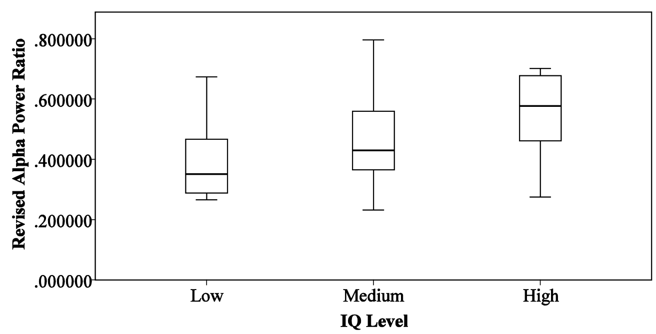

Fig. 18.Box plot for revised alpha power ratio (N = 50).

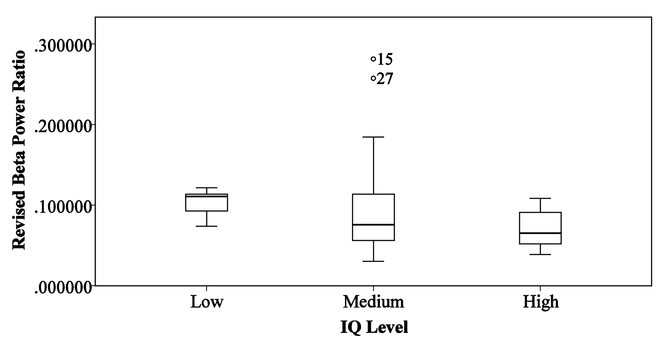

Fig. 19.Box plot for revised beta power ratio (N = 50).

The presence of outliers has been minimized. None is identified as extreme outlier. Only two samples have been identified for the revised beta power ratio features; both of which are for the medium IQ group. Hence, formulation of revised power ratios have demonsrated desirable results and is recommended with implementation of equiripple filters.

IV. CONCLUSION

The study has successfully proposed equiripple filter designs in EEG power ratio extraction algorithm. Results have shown optimized filter response characteristics compared to those of the Hamming filters; leading to accurate computation of power ratios. Implementation for IQ pattern analysis however, have resulted in the presence of extreme outliers. To overcome the problem, the power ratio features have been revised with formulation. The method has successfully minimized the presence of outliers and eliminated the extreme ones. Conclusively, the proposed equiripple design provides better alternative than the Hamming filters. Thus far, the drawback of its implementation is in the increased computational load that are caused by very high filter order.

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