THEORETICAL EVALUATIONS OF SELF-ABSORPTION CORRECTION FACTORS

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ABSTRACT

The note describes how self-absorption of gamma-rays in gamma-ray spectrometry can be evaluated mathematically. It was shown that the correction factor depends on the mean absorption path length of gamma-rays in the particular sample. An experimental procedure to determine self-absorption correction factors of any sample-detector geometries is also briefly described.

INTRODUCTION

In quantitative measurements of radiation using gamma-ray spectrometry, accurate efficiency calibration is essential. This is normally carried out experimentally using multi-radionuclide standard samples emitting photons at a range of different energies. In principle, the simplest method for taking into account the influence of sample composition and geometry is to use standard sources having the same composition and geometry as the sample being analysed (Debertin and Ren, 1989). For many applications this approach is however impractical. The precise composition of the sample will be unknown, and limitations in sample size will mean that standard geometries cannot be adhered to exactly. A more flexible approach is to determine the nominal efficiency versus energy for a small number of standard geometries and to apply correction factors based on the sample density, composition and dimensions.

The present study evaluates the correction factors mathematically. The mathematical model is based on integration of the gamma radiation emitted from each small element of the sample. Exact calculations are very difficult in practice, though simplified models provide a useful algorithmic framework for interpreting numerical simulations and experimental results. Experimental measurements provide a means for evaluating both theoretical methods and for making further improvements to the algorithms. This note also proposes a procedure for an experimental study. The results can be used to validate theoretical algorithms and to reduce errors deriving from the mathematical simplifications on which they are based.

SELF-ABSORPTION CORRECTION FACTOR

For a voluminous sample, a significant fraction of the gamma radiation generated within the sample may be lost by the sample self-absorption before escaping from the sample. Therefore, besides depending on the geometry of the sample and detector and

their relative positions, the detection efficiency will also depend on the material composition of the sample.

Consider a homogeneous sample of mass m, taking up a geometry v. Suppose that during time t, the whole sample produces N_O gamma photons of energy E keV. The detection efficiency for the whole sample can be formulated in terms of the detection efficiency for emissions from each element of the sample. Consider an element dm of the sample placed at position p(x,y,z) producing dN_O gamma photons (towards 4π directions) in time t, as shown in Figure 1. Due to self-absorption effect the number of photons escaping the sample in a pencil of direction **n** subtending a solid angle dω at p will be $e^{-\rho\mu r} dN_O \frac{d\omega}{4\pi}$. Here ρ is the mass density of the sample, μ denotes the mass attenuation coefficient and r the distance traversed by the photons in the sample. The factor $e^{-\rho\mu r}$ represents the fraction of photons in the pencil that escape absorption in the sample. The total number of photons detected by the detector from the element dm during time t will be

$$dN(p,\mu) = \frac{dN_o}{4\pi} \int_{\Omega} \varepsilon(p,\mathbf{n})\, e^{-\rho\mu r} d\omega, \qquad (1)$$

where ε(p,**n**) the intrinsic detection efficiency for photons in the pencil (p,**n**), and Ω the solid angle subtended by the detector at p. The ratio of the number of photons detected to the number of photons created in the element is the *practical efficiency* for detection of photons from the element, thus

$$\varepsilon(p,\mu) \doteq \frac{dN(p,\mu)}{dN_o} = \frac{1}{4\pi} \int_{\Omega} \varepsilon(p,\mathbf{n})\, e^{-\rho\mu r} d\omega. \qquad (2)$$

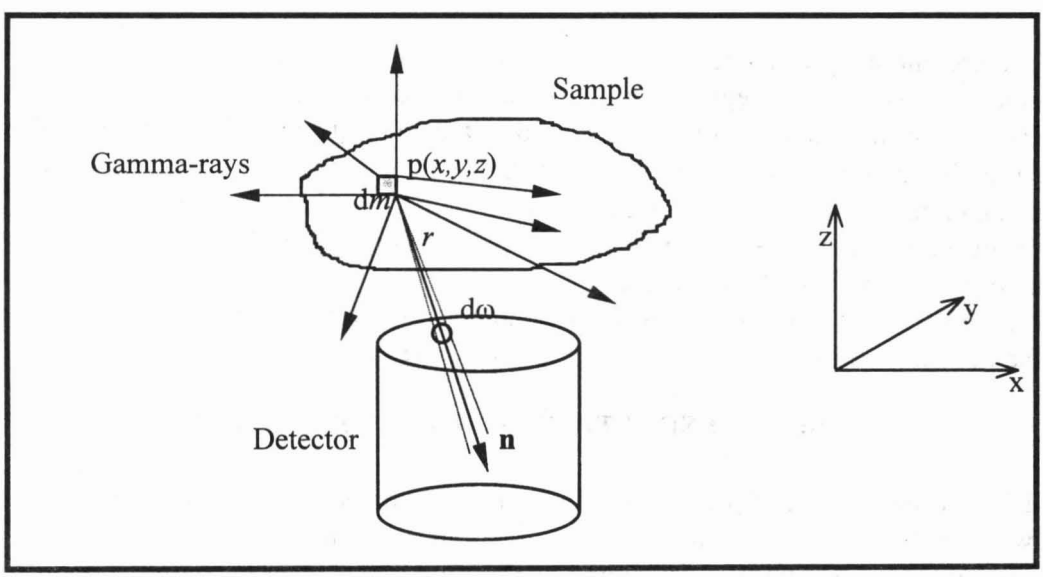

Figure 1. Gamma photons from a volume element of a voluminous sample.

If there had been no self-absorption effect in the sample, the number of photons from the element that would have been detected by the detector is

$$dN(p,0) = \frac{dN_o}{4\pi} \int_{\Omega} \varepsilon(p,\mathbf{n})\, d\omega. \tag{3}$$

The quantity

$$\varepsilon(p,0) = \frac{dN(x,0)}{dN_o} = \frac{1}{4\pi} \int_{\Omega} \varepsilon(p,\mathbf{n})\, d\omega. \tag{4}$$

is called the *nominal efficiency*. It depends only on the geometry of the sample and the properties of the detector.

The ratio of the actual number of photons detected by the detector to the nominal number detected in the absence of self-absorption is called the *self-absorption correction factor*, $f_a(p,\mu)$, for gamma photons emitted from point p. Thus from Equations (1) and (3)

$$f_a(p,\mu) = \frac{dN(p,\mu)}{dN(p,0)} = \frac{\varepsilon(p,\mu)}{\varepsilon(p,0)} = \frac{\int_{\Omega} \varepsilon(p,\mathbf{n}) e^{-\rho\mu r} d\omega}{\int_{\Omega} \varepsilon(p,\mathbf{n}) d\omega} \tag{5}$$

The total number of photons detected from the whole sample can be obtained by integrating the number of photons detected by the detector from every element in the sample. Therefore, during the time t the number detected is

$$N(v,\mu) = \int_{v} \varepsilon(p,\mu) dN_o = \frac{N_o}{m} \int_{v} \varepsilon(p,\mu) dm, \tag{6}$$

since $dN_O = N_O dm/m$. Rearranging Equation (5) we get $\varepsilon(p,\mu) = \varepsilon(p,0) f_a(p,\mu)$. Therefore

$$N(v,\mu) = \frac{N_o}{m} \int_{v} \varepsilon(p,0) f_a(p,\mu) dm. \tag{7}$$

If there has been no loss due to self-absorption in the sample, the number of gamma photons that would have been detected will be

$$N(v,0) = \frac{N_o}{m} \int_{v} \varepsilon(p,0) dm. \tag{8}$$

From Equations (7) and (8), the practical and nominal efficiencies for the whole sample will respectively be

$$\varepsilon(v,\mu) = \frac{N(v,\mu)}{N_o} = \frac{1}{m}\int_v \varepsilon(p,0) f_a(p,\mu)\mathrm{d}m, \tag{9}$$

and

$$\varepsilon(v,0) = \frac{N(v,0)}{N_o} = \frac{1}{m}\int_v \varepsilon(p,0)\mathrm{d}m. \tag{10}$$

And, finally the self-absorption correction factor for photon emissions from the whole sample is

$$F_a(v,\mu) = \frac{\varepsilon(v,\mu)}{\varepsilon(v,0)} = \frac{\int_v \varepsilon(p,0) f_a(p,\mu)\mathrm{d}m}{\int_v \varepsilon(p,0)\mathrm{d}m} \tag{11}$$

MEAN PATH LENGTH AND SELF-ABSORPTION

Photons emitted from point p(x,y,z) travel a range of different distances within the samples before escaping. The self-absorption correction factor for any individual photon travelling a distance r is $e^{-\rho\mu r}$. The self-absorption correction factor $F_a(v,\mu)$ may be represented in a similar way using the notion of a *mean absorption path length* for the whole sample. To establish this representation we first differentiate the self-absorption function $f_a(p,\mu)$ for emission from point p with respect to μ. Thus we obtain

$$\frac{\mathrm{d}f_a(p,\mu)}{\mathrm{d}\mu} = \frac{-\rho\int_\Omega r\varepsilon(p,\mathbf{n})e^{-\rho\mu r}\mathrm{d}\omega}{\int_\Omega \varepsilon(p,\mathbf{n})\mathrm{d}\omega}.$$

$$= -\rho\,\bar{r}\, f_a(p,\mu), \tag{12}$$

where

$$\bar{r} = \frac{\int_\Omega r\varepsilon(p,\mathbf{n})e^{-\rho\mu r}\mathrm{d}\omega}{\int_\Omega \varepsilon(p,\mathbf{n})e^{-\rho\mu r}\mathrm{d}\omega} \tag{13}$$

defines a mean absorption path length for gamma photons emitted at point p. The value of $\bar{r}$ depends on the nature of the sample and the sample-detector arrangement. If we assume that $\bar{r}$ is independent of the mass attenuation coefficient μ, we can get

the self-absorption correction factor $f_a(p,\mu)$ in term of $\bar{r}$ by solving the differential Equation (12). Since $f_a(p,0) = 1$, it follows that

$$f_a(p,\mu) = e^{-\rho\mu\bar{r}}. \tag{14}$$

A similar treatment on the self-absorption correction factor for the whole sample would produce the corresponding mean absorption path length for photon emission from the whole sample. Thus, differentiating $F_a(v,\mu)$ of Equation (11) with respect to μ and using the correction factor of Equation (14) gives

$$\frac{dF_a(v,\mu)}{d\mu} = \frac{-\rho\int_v \bar{r} f_a(p,\mu)\varepsilon(p,0)dm}{\int_v \varepsilon(p,0)dm}$$

$$= -\rho L F_a(v,\mu), \tag{15}$$

where

$$L = \frac{\int_v \bar{r} f_a(p,\mu)\varepsilon(p,0)dm}{\int_v f_a(p,\mu)\varepsilon(p,0)dm} \tag{16}$$

defines a mean absorption path length for emission from the whole sample.

Assuming the mean path length L is independent of the mass attenuation coefficient μ, then solving the differential Equation (15) will give the self-absorption correction factor in term of L

$$F_a(v,\mu) = e^{-\rho\mu L}, \tag{17}$$

since $F_a(v,0) = 1$. The correction factor in this form may not be valid for large values of $\rho\mu L$, however it will often be true for small values. For practical calculations it is more convenient to use the self-absorption correction factor in the form

$$F_a(v,\mu) = e^{-k\mu m}, \tag{18}$$

where m is the mass of the sample, $k = L/v$ is a *geometrical self-absorption attenuation parameter* and v is the sample volume.

Although in many sample-detector arrangements the mean absorption path length of Equation (16) would not easily be evaluated, it has been shown that approximate simple mathematical models L is possible. The self-absorption correction factors obtained using the simple mathematical models for various sample-detector arrangements based on the above discussions were found to agree reasonably well with experiments and Monte Carlo simulations (Ahmad (1999)).

EXPERIMENTAL TECHNIQUES

Based on the mathematical evaluations described, the self-absorption correction factor of any sample-detector geometry can be determined experimentally. The procedures are summarised here. The samples were counted on the appropriate detectors. The gamma photon spectrum of each sample was analysed to obtain the net full-energy peak (FEP) count, N during counting time t. The *practical specific activity* of UO_3 obtained using the count rate of the FEP of a gamma photon emitted from a sample of a certain geometry v is given by

$$A(v,\mu) = \frac{R(v,\mu)}{m_u}, \tag{19}$$

where m_u is mass (kg) of UO_3 in the sample and $R(v,\mu) = N/t$ is the count rate in the FEP. The corresponding *practical FEP efficiency* is

$$\varepsilon(v,\mu) = \frac{A(v,\mu)}{pA_o}, \tag{20}$$

where p and A_O are the photon yield and the corresponding radionuclide specific activity ($Bqkg^{-1}$) in UO_3 respectively.

For a series of samples of a certain geometry, v, at a particular gamma-ray of energy E keV, data of practical specific activities against the corresponding values of mass attenuation factor, μm, are obtained. Plotting the practical specific activities against μm and extrapolate to the intercept at $\mu m = 0$, will gives the *nominal specific activity*, $A(v,0)$. This corresponds to the specific activity that would be obtained if there is no self-absorption in the sample. Thus the corresponding *nominal FEP efficiency* will be

$$\varepsilon(v,0) = \frac{A(v,0)}{pA_o}. \tag{21}$$

Defining the experimental self-absorption correction factor, $F_a(v,\mu)$, as the ratio of the practical FEP efficiency to the nominal FEP efficiency, then

$$F_a(v,\mu) = \frac{\varepsilon(v,\mu)}{\varepsilon(v,0)}$$

$$= \frac{A(v,\mu)}{A(v,0)}. \tag{22}$$

Hence the self-absorption correction factors can be calculated from the ratio of practical specific activity to the corresponding nominal specific activity.

REFERENCES

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