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Enhancement of Gielis' Supershapes in Generating Nature Motifs

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ABSTRACT

Nature motifs have played an important role in designing and generating most products such as jewellery, fashion, furniture, textile, or visual arts. The designers may translate their ideas by using the mathematical equations to design the products inspired by the nature motifs. One of the mathematical equations that can be used in creating or designing nature motifs is the Gielis' Supershape (GS). This formula has been introduced by Johan Gielis, who is a botanist and mathematician. In this paper, we discuss the nature motif that can be created using the GS. We also proposed the Enhanced Gielis' Supershape (EGS) and present some comparisons. The result shows that the nature shape created by using the EGS was more impressive compared to the shape created using the original GS.

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1. Introduction

Many great designers derive their inspiration from nature. In fact, the most famous inventions and breakthroughs in design and technology have been inspired by nature [1],[2]. Those designs and images made up of thousands of curves, angles, and straight lines. The curves and shapes created are very common in the field of Computer Aided Geometric Design (CAGD). Fractals and spline curves techniques are widely used in creating curves and shapes[3]–[6]. It is difficult to produce a mathemathical formula, which fitted with the nature motifs[7]. However, Johan Gielis had built a superformula which fitted in most of the nature motifs[8]–[10]. We proposed the new formula in creating nature motifs namely EGS. This paper discusses the comparison on using two different mathematical functions namely GS and EGS. The EGS modifies the GS's formula by including a

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new mathematical function. The proposed EGS is to find the best mathematical function in creating nature motifs.

2. Literature Review

GS was the modification of curve formula developed by the French mathematician, Gabriel Lamẻ in the year 1818, and the inventive mathematician Piet Hein in the year 1959[10]. Gabriel Lame defined the curves in the xy-plane by the following formula:

$$\left|\frac{x}{a}\right|^{n} + \left|\frac{y}{b}\right|^{n} = 1 \tag{1}$$

where a, b and n are any positive real numbers.

Formula (1) is known as Lame curves. Then, Piet Hein has revived the Lame curves by creating a superellipse with n>2. This formula has been used in designing a variety of shapes of home interior products and turn out to be a symbol of Scandinavian designs.

Johan Gielis, the Dutch botanist then innovated the formula created by Lame and Hein to produce a superformula [11], [12] defined as:

$$r(\theta) = \frac{1}{\sqrt[n_1]{\left|\frac{1}{a}\cos\left(\frac{1}{4}m\theta\right)\right|^{n_2} + \left|\frac{1}{b}\sin\left(\frac{1}{4}m\theta\right)\right|^{n_3}}} \tag{2}$$

Formula (2) consists of six free parameters that play their roles in generating natural motifs. Gielis then introduced a new function $f(\theta) = \cos\left(\frac{m}{2}\theta\right)$ [13]. This function is important to make the curve more precisely. The improvement formula[14] is defined as follows:

$$r(\theta) = \frac{1}{\sqrt[n_1]{\left|\frac{1}{a}\cos\left(\frac{1}{4}m\theta\right)\right|^{n_2} + \left|\frac{1}{b}\sin\left(\frac{1}{4}m\theta\right)\right|^{n_3}}}.f(\theta) \tag{3}$$

Gielis then called Formula (3) as Gielis Supershapes (GS) as it can generate a variety of natural shapes and motifs. Figure 1 shows the process of creating natural patterns bu using GS. Six different values of the free parameters were used to create various natural motifs.

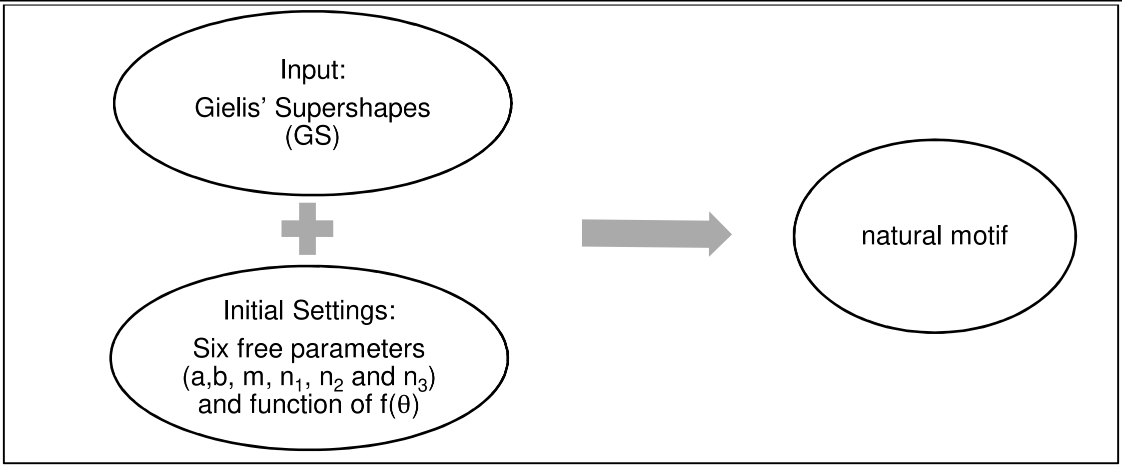

Figure 1. The process of generating the natural motif using Gielis' Supershapes

3. Methodology

This study involves two different mathematical functions, namely GS and EGS, in creating nature motifs in order to achieve the objectives. Equation (3) has been modified to improve certain shape that was generated using the GS with the introduction of $f(\theta)$ with the product of $\cos\frac{m}{2}\theta$ and $\sin\frac{m}{2}\theta$ as defined below:

$$f(\theta)=\cos\frac{m}{2}\theta\sin\frac{m}{2}\theta \tag{4}$$

We name this new modification as Enhanced Gielis' Supershapes (EGS). The EGS has been defined as:

$$r(\theta)=\frac{\left(\cos\frac{m}{2}\theta\sin\frac{m}{2}\theta\right)}{\sqrt[n_1]{\left|\frac{1}{a}\cos\left(\frac{1}{4}m\theta\right)\right|^{n_2}+\left|\frac{1}{b}\sin\left(\frac{1}{4}m\theta\right)\right|^{n_3}}} \tag{5}$$

The selection of $f(\theta)=\cos\frac{m}{2}\theta\sin\frac{m}{2}\theta$ is because both functions are under the polar coordinate system. The original ES employs an even function, $\cos\left(\frac{m}{2}\theta\right)$. In this extended function, we use products of $\cos\left(\frac{m}{2}\theta\right)$ and $\sin\left(\frac{m}{2}\theta\right)$ which produce the odd function. When compared to the original GS, the possible shapes generated by the EGS will be different.

In order to compare the output of both equations, we generate a natural shape using GS and EGS. Figure 1 depicts the overall procedures. We use the same values of six free parameters for both GS and EGS to observe the output.

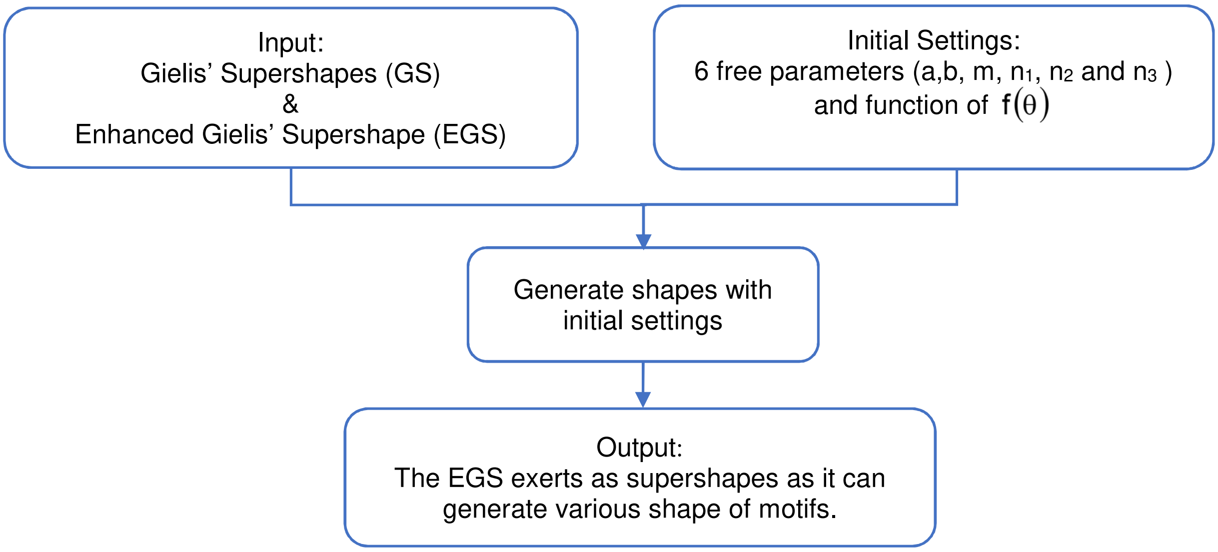

Figure 2. The general processes in generating the shape of motifs

4. Results and Discussion

4.1 Polygon Generated by Gielis' Supershapes

If we set the parameter a = b = 1 and n_2 = n_3, we can generate the polygon [14] by using Equation (2) as shown in Figure 3. What can be clearly seen in this figure is the shapes of polygons depend on parameter m. As an example, if m = 3, the shape of polygon is a triangle.The parameter m controls the rotational symmetry. The values of a and b control the size, and the exponents n_1, n_2 and n_3 control the curvature of the sides[15].

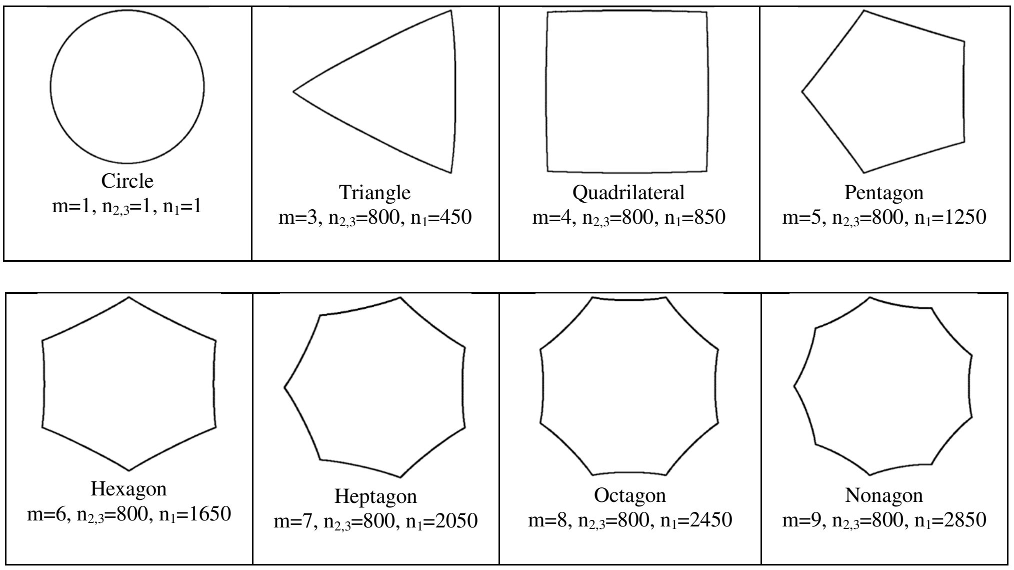

Figure 3. Circle and regular polygons generated by using Gielis' Supershape

4.2 The Nature Motifs Generated by Gielis' Supershapes

Figure 4 illustrates various shapes of flower by using Formula (3). What is interesting in this figure is the dominance of parameter m in determining the number of petals of the flowers. For instance, if we set the parameter a = b = 1 and m = 10, the petals of the flower are 5 and if m = 30, the petals of the flower are 15. The numbers of petals are half of the value of m. While, it is clearly seen in this figure, the shape of petal is likely influenced by the parameter n.

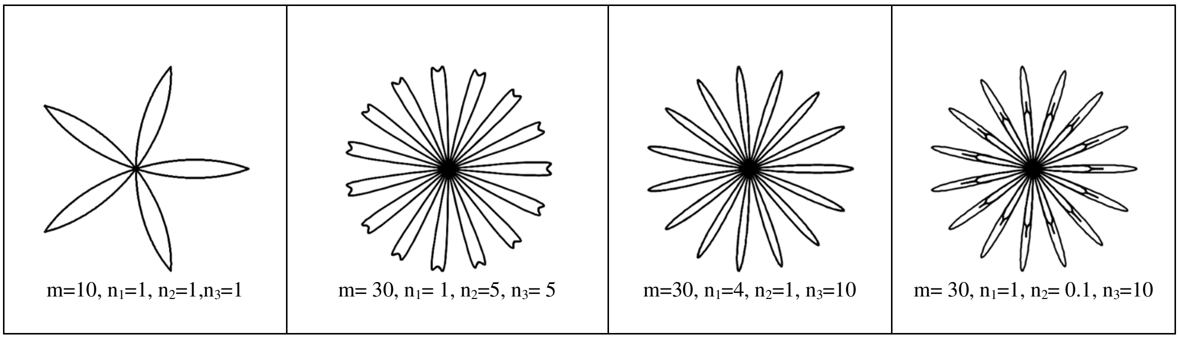

Figure 4. Flower shapes generated by using Gielis' Supershape where $f(\theta)=\cos\frac{m}{2}\theta$

Equation 3 also can be used to generate the shape of leaves as shown in Figure 5. For example, if we set the parameter a = b = 1 and m = 1, the leaflet is in the shape of heart, if m =1/2 the shape is circular and m = 3 will give the three-leaflet shape. Figure 5 reveals that the numbers of leaves are the same as the value of m. While, the shape of leaves is likely influenced by the parameter n.

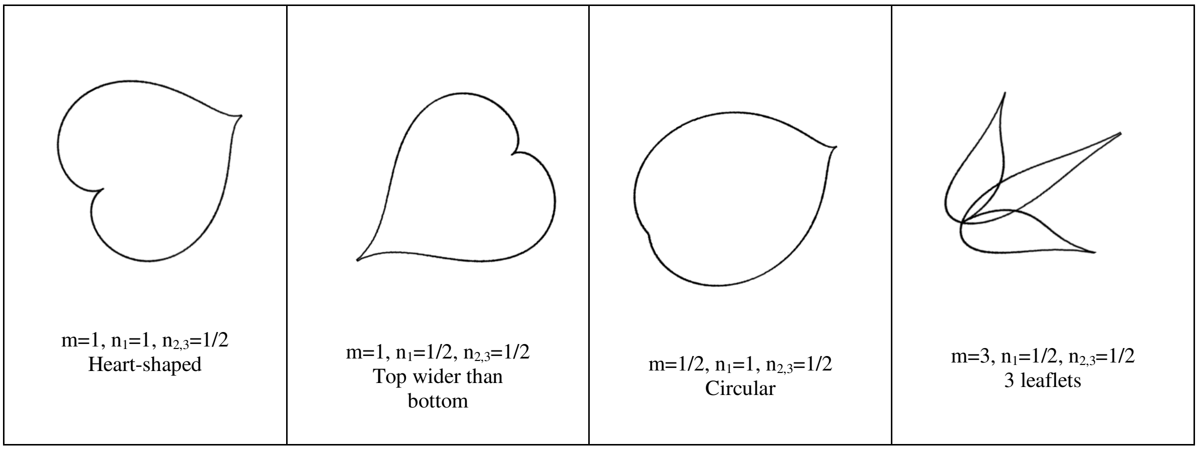

Figure 5. Leaflet shapes generated by using Gielis' Supershape where $f(\theta)=\cos\frac{m}{2}\theta$

4.3 The Nature Motifs Generated by Difference Function of $f(\theta)$

We commence by studying the effect of using difference function of $f(\theta)$. Thus, the function $f(\theta)=\sin\frac{m}{2}\theta$ has been introduced and the effect to the shape is observed. The selection of $\sin\frac{m}{2}\theta$ because it is also under the polar coordinate system. The function $\sin\frac{m}{2}\theta$ is an odd, while $\cos\frac{m}{2}\theta$ is an even function. As shown in Figure 6, if all the parameters are integers, then the shape generated is a flower. The numbers of petals are half of the value of m, the same result as shown in Figure 6. However, the shape of petals is different compared to the result shown in Figure 4. The shape of petal is seen not likely influenced by the parameter n. The parameter n controls the size of the petal.

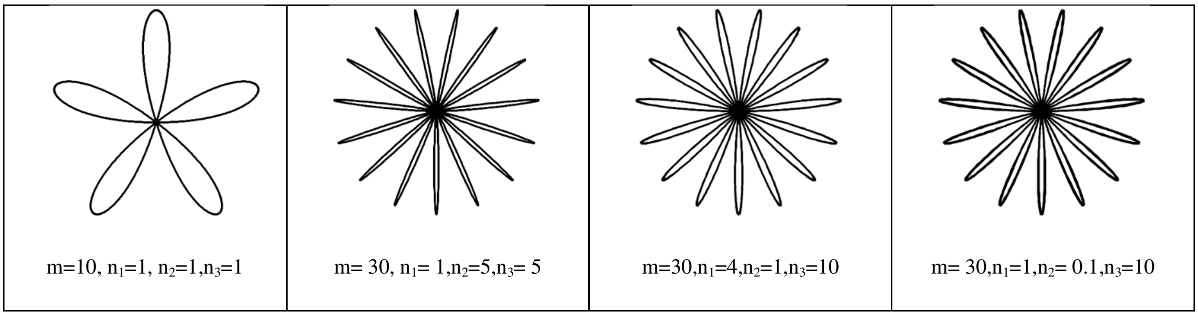

Figure 6. Flower shape generated by using Gielis' Supershape where $f(\theta)=\sin\frac{m}{2}\theta$

If the parameter is set to the rational number, the shape generated is in the shape of cycloidal curves and rose as shown in the Figure 7. The cycloidal arch was used by architect Louis Kahn in his design for the Kimbell Art Museum in Fort Worth, Texas. It was also used in the design of the Hopkins Center in Hanover [15].

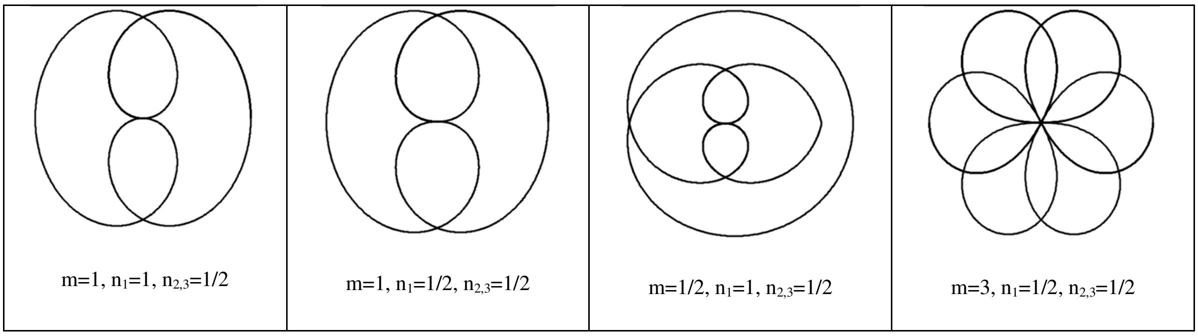

Figure 7. Cycloidal curves and rose generated by using GS where $f(\theta)=\sin\frac{m}{2}\theta$

4.4 The Nature Motifs Generated by Enhancement of Gielis' Supershapes

From $f(\theta)$, it is obvious that the number of petal or leaflet that depend on parameter m will increase. Then, this enhanced equation is being explored to study the shape of the curves. Our most exciting finding is that by using Equation 5, it reveals that the nature shapes generated is more impressive compared to the shape generated using Equation 3. The parameter n plays a role in determining the length of the petal as appeared in Figure 8. Equation 5 also produces the flower shape with double length while Equation 3 only produces one length of petal.

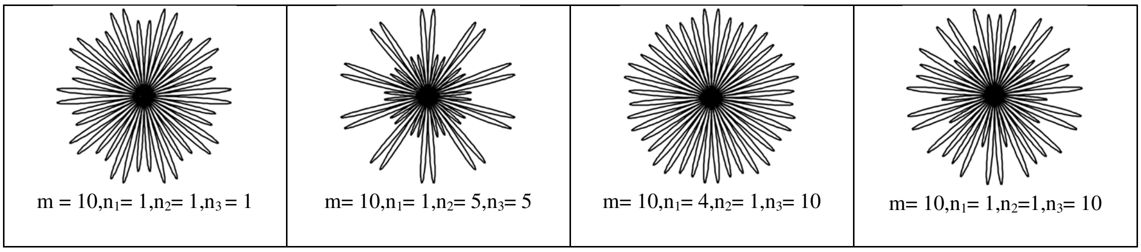

Figure 8. Flower shape generated by using EGS

Equation 5 may generate the shape of butterfly's wing as shows in Figure 9. Figure 8 and Figure 9 also illustrates that Equation 5 can generate shapes with two different lengths. We cannot use Equation 3 to produce nice shape of a petal and a butterfly's wing because it can only produce the same length of petals.

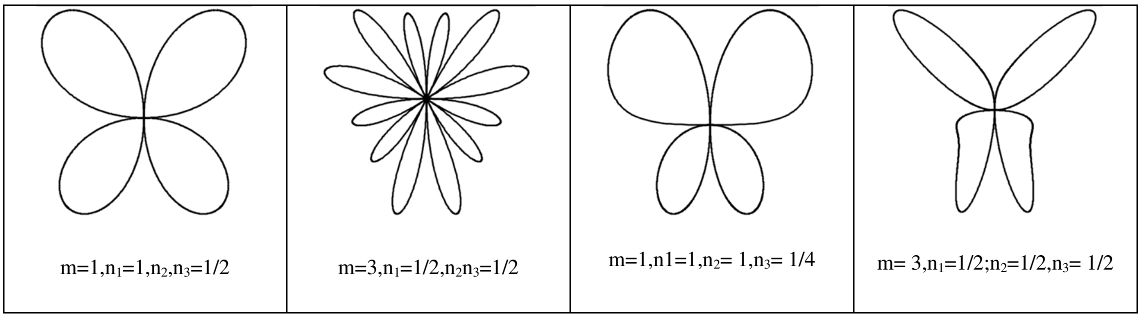

Figure 9. Butterfly's wing shape generated by using Enhanced Gielis' Supershape

The difference in length of each shape is influenced by the product of $\cos\frac{m}{2}\theta$ and $\sin\frac{m}{2}\theta$ .The simplest form of the of product of $\cos\frac{m}{2}\theta$ and $\sin\frac{m}{2}\theta$ is only in terms of $\sin(m\theta)$ known as an odd function and is shown in following derivation.

$$f(\theta) = \cos\frac{m}{2}\theta \sin\frac{m}{2}\theta$$

$$= \frac{1}{2}\left[\sin\left(\frac{m}{2}\theta + \frac{m}{2}\theta\right) - \sin\left(\frac{m}{2}\theta - \frac{m}{2}\theta\right)\right]$$

$$= \frac{1}{2}\sin(m\theta) \qquad (6)$$

As a result, the argument in Equation (6) will be in terms of $m\theta$. It produces more rotational symmetrical shapes compared to the original GS because in GS the argument of the function is $\frac{1}{2}m\theta$. .

5. Conclusion

GS is found to be one of the great findings in mathematical world. In addition to GS, the EGS exerts a powerful effect upon the shape of motifs. It is a powerful tool in generating shapes since we can control the number of petals for the flowers and size of the wings for the butterfly. This finding would be helpful for designers in venturing more into nature motifs for their designs.

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References

[1] S. Pathak, "Biomimicry:(Innovation Inspired by Nature)," Int. J. New Technol. Res., vol. 5, no. 6, pp. 34–38, 2019.

[2] G, Poteraş, György Deák, M. V. Olteanu, I-F. Burlacu, C. Sîrbu,N. Muhammad, and N.A.Mohd Mortar, "Profiles of blades and paddles for turbines with geometric design inspired by nature," in AIP Conference Proceedings, 2020, vol. 2291, no. 1, p. 20016.

[3] K. I. Tsianos and R. Goldman, "Bézier and B-spline curves with knots in the complex plane," Fractals, vol. 19, no. 01, pp. 67–86, 2011.

[4] M. Hoffmann and I. Juhász, "On interpolation by spline curves with shape parameters," in International Conference on Geometric Modeling and Processing, 2008, pp. 205–214.

[5] H. Hang, X. Yao, Q. Li, and M. Artiles, “Cubic B-Spline curves with shape parameter and their applications,” Math. Probl. Eng., vol. 2017, 2017.
[6] G. Irving and H. Segerman, “Developing fractal curves,” J. Math. Arts, vol. 7, no. 3–4, pp. 103–121, 2013.
[7] P. do E. Silva, Caroline, J. A. De Souza, and A. Vacavant, “Automated Mathematical Equation Structure Discovery for Visual Analysis,”,” J. Mach. Learn. Res., vol. 22, pp. 1–25, 2021.
[8] S. K. Mishra, “On estimation of the parameters of gielis superformula from empirical data,” SSRN Electron. J., 2006.
[9] P. Shi, D. A. Ratkowsky, and J. Gielis, “The generalized Gielis geometric equation and its application,” Symmetry (Basel)., vol. 12, no. 4, p. 645, 2020.
[10] J. Gielis, D. Caratelli, P. Shi, and P. E. Ricci, “A note on spirals and curvature,” Growth Form, vol. 1, pp. 1–8, 2020.
[11] M. Matsuura, “Gielis’ superformula and regular polygons,” J. Geom., vol. 106, no. 2, pp. 383–403, 2015.
[12] J. Gielis, “Natural Polygons and Gielis Transformations,” 2019, p. 53.
[13] J. Gielis, The geometrical beauty of plants. Springer, 2017.
[14] S. Primo, Brandi; Anna, “A MAGIC FORMULA OF NATURE BRANDI Primo (IT), SALVADORI Anna (IT),” in 17th COnference on Applied Mathematics, 2018, pp. 110–126.
[15] J. A. Norato, “Topology optimization with supershapes,” Struct. Multidiscip. Optim., vol. 58, no. 2, pp. 415–434, 2018.