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A Guideline on Constructing Multivariate Control Chart using Excel Spreadsheet

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ABSTRACT

Multivariate process control chart (MSPC) is the most popular method in statistical process control which involves more than one quality characteristic. It is much simpler if the process has only one quality characteristic. If there are several quality characteristics involved, a specific and costly statistical programming package is needed to assist the construction of the multivariate control chart. This poster presentation will put researchers and the users of MSPC at ease by providing a step by step guideline using EXCEL spreadsheet for that purpose. This guideline will enable any interested researchers to write the language programming on constructing multivariate process control chart.

Key words: *EXCEL spreadsheet, multivariate control chart, statistical process control*

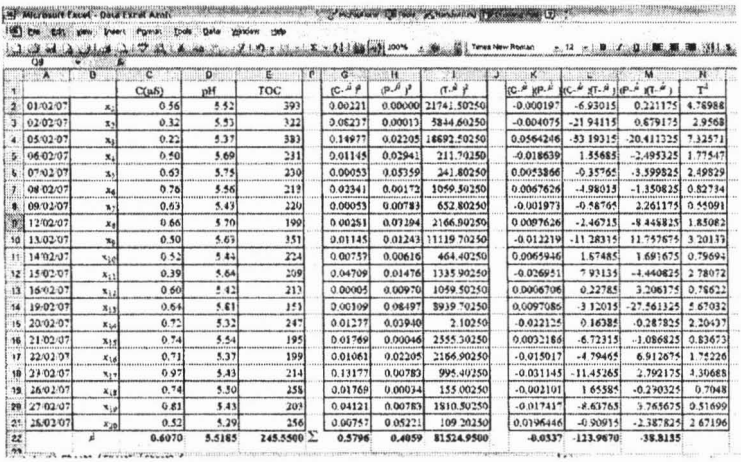
Introduction

Many problems in modern industrial quality control will involve a vector of measurements of several quality characteristics, rather than only a single characteristic (Abdollahian, Abachi & Nahavandi, 2000). Multivariate statistical process control (MSPC) chart is the most popular tool for that purpose (Woodhall & Montgomery, 1999). It enables the identification of the sample that causes the out-of-control situation with taking into account the correlations among the quality variables (Marroquin & Solis, 2006).

This presentation provides researchers and MSPC chart user a step by step guideline using EXCEL spreadsheet in constructing MSPC chart. Three quality characteristics of water supply which are used in producing pharmaceutical products in the form of tablet or syrup were taken. The quality characteristics are conductivity (C), pH and the amount of total organic carbon (TOC). The Hotelling T^2 multivariate control chart will be constructed on these three quality characteristics.

Methodology

All the necessary formulas for the EXCEL procedures are given and it should be emphasized that along the explanation, one must always refer to the specific column and row in the spreadsheet (Figure 1).



		C(µS)	pH	TOC								
1												
2	01/02/07	0.56	5.52	393	0.00221	0.00000	21741.50250	-0.000197	-6.93015	0.221175	4.78988
3	02/02/07	0.32	5.53	322	0.05207	0.00013	5844.60250	-0.004075	-21.94115	0.679175	2.9568
4	05/02/07	0.22	5.37	383	0.14977	0.02205	18692.50250	0.0564246	-83.19315	-20.411325	7.32571
5	06/02/07	0.50	5.69	231	0.01145	0.02941	211.70250	-0.018639	1.55685	-2.495325	1.77547
6	07/02/07	0.63	5.75	230	0.00053	0.05359	241.80250	0.0053366	-0.35765	-3.599825	2.49829
7	08/02/07	0.76	5.56	219	0.02341	0.00172	1059.50250	0.0067626	-4.98015	-1.350825	0.82734
8	09/02/07	0.63	5.43	220	0.00053	0.00783	652.80250	-0.001973	-0.58765	2.261175	0.55091
9	12/02/07	0.66	5.70	199	0.00251	0.07204	2166.90250	0.0097626	-2.46715	-8.448825	1.85082
10	13/02/07	0.50	5.69	351	0.01145	0.01243	11119.70250	-0.012219	-11.28315	11.757675	3.20133
11	14/02/07	0.52	5.44	224	0.00757	0.00616	464.40250	0.0065946	1.67485	1.691675	0.79694
12	15/02/07	0.39	5.64	209	0.04709	0.01476	1335.90250	-0.026951	7.93135	-4.440825	2.78072
13	16/02/07	0.60	5.42	213	0.00005	0.00970	1059.50250	0.0006706	0.22785	3.206175	0.78622
14	19/02/07	0.64	5.61	353	0.00309	0.08497	8939.70250	0.0097086	-3.12015	-27.561325	5.67032
15	20/02/07	0.72	5.32	247	0.01277	0.03940	2.10250	-0.022125	0.16385	-0.287825	2.20437
16	21/02/07	0.74	5.54	195	0.01769	0.00046	2555.30250	0.0032186	-6.72315	-1.086825	0.83673
17	22/02/07	0.73	5.37	199	0.01061	0.02205	2166.90250	-0.015017	-4.79465	6.912675	1.75226
18	23/02/07	0.97	5.43	214	0.13177	0.00783	995.40250	-0.031145	-11.45265	2.792175	3.30688
19	26/02/07	0.74	5.50	258	0.01769	0.00034	155.00250	-0.002101	1.65585	-0.290325	0.7048
20	27/02/07	0.81	5.43	203	0.04121	0.00783	1810.50250	-0.01741	-8.63765	3.765675	0.51699
21	28/03/07	0.52	5.29	256	0.00757	0.05221	109.20250	0.0196446	-0.90915	-2.387825	2.67196
22		0.6070	5.5185	245.5500	0.5796	0.4059	81524.9500	-0.0337	-123.9670	-38.8135	

Figure 1: The Spreadsheet Containing All the Values from the 8 Step EXCEL Application.

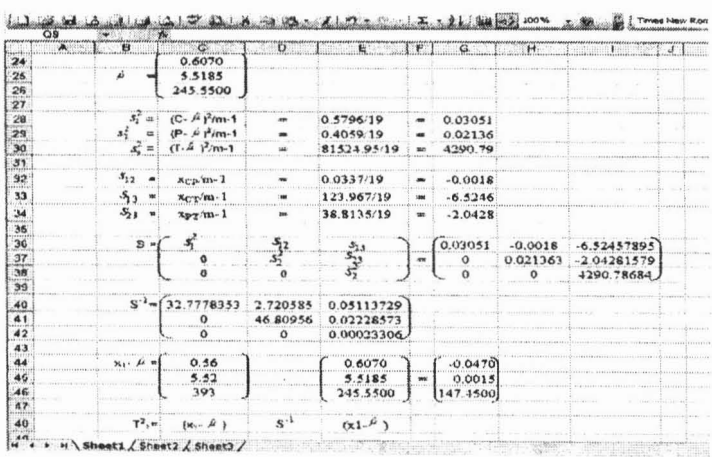
There are eight steps involved in constructing T^2 multivariate control chart. The first step is to find the mean of every variable, in this case is the quality characteristic. The mean was obtained from the equation below.

$$\bar{x}_j = \frac{\sum_{k=1}^{m} x_{jk}}{m}$$

Where, j = jth quality characteristic
k = kth observation; k = 1,2,…,n ;
n = the number of observation

$\bar{x}_j$ = the mean of jth quality characteristic

x_{jk} = k-th observation of j-th quality characteristic
m = the number observation for quality characteristic
(for this case, m = 20)



24		0.6070									
25	$\bar{x}$ =	5.5185									
26		245.5500									
27											
28	s_1^2 =	(C- $\bar{x}$)²/m-1	=	0.5796/19	=	0.03051					
29	s_2^2 =	(P- $\bar{x}$)²/m-1	=	0.4059/19	=	0.02136					
30	s_3^2 =	(T- $\bar{x}$)²/m-1	=	81524.95/19	=	4290.79					
31											
32	s_{12} =	x_{CP}/m-1	=	0.0337/19	=	-0.0018					
33	s_{13} =	x_{CT}/m-1	=	123.967/19	=	-6.5246					
34	s_{23} =	x_{PT}/m-1	=	38.8135/19	=	-2.0428					
35											
36	D =	s_1^2	s_{12}	s_{13}	=	0.03051	-0.0018	-6.52457895			
37		0	s_2^2	s_{23}		0	0.021363	-2.04261579			
38		0	0	s_3^2		0	0	4290.78684			
39											
40	S^{-1} =	32.7778353	2.720585	0.05113729							
41		0	46.80956	0.02228573							
42		0	0	0.00023306							
43											
44	$x_{i1} - \bar{x}$ =	0.56		0.6070	=	-0.0470					
45		5.52		5.5185		0.0015					
46		393		245.5500		147.4500					
47											
48	T^2 =	$(x_{i1} - \bar{x})$	S^{-1}	$(x1-\bar{x})$							

Figure 2: The Spreadsheet Showing the Results of EXCEL Application

In EXCEL application, the mean can be obtained by putting the cursor at the end of the first quality characteristic column. Click the *Insert* menu, followed by *Function* and *Average*, and then click *OK*. The same process has to be repeated to the other quality characteristic columns in order to get the other mean values. The second step is to obtain the mean vector.

$$\text{Mean vector} = \hat{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}$$

An empty cell needs to be activated before beginning the procedures. Click *Insert* menu followed by *Function*, *Transpose* and *OK*. Highlight the cells of the three means and then click *OK*. Once again, highlight three empty cells but this time in horizontal position. Press the F2 button and three other buttons which are *CTRL, SHIFT* and *ENTER* simultaneously. The transposed mean matrix will appear in the highlighted cells.

$$\text{Variance} = s_j^2 = \frac{\sum_{j=1}^{3}(x_{jk} - \bar{x}_j)}{m-1}$$

The third step is to find the variances using the equation above. You need to type "=(C2- $\bar{X}_1$) ^2" in an empty cell in a new column and then click *ENTER*. Repeat the same step to the other column of quality characteristics and to the other cells in the same column. Allocate the cursor at the end of the column. Find the summation of the values obtained by clicking the sigma, $\sum$ toolbar. Repeat the same procedure at the end of the other two columns. Divide the three values obtained at the end of the columns with m-1 = 19 and it gives you the variance of the variables which are s_1^2 , s_2^2 and s_3^2 .

$$\text{Covariance} = s_{jh} = \frac{\sum_{k=1}^{m}(x_{jk} - \bar{x}j)(x_{hk} - \bar{x}h)}{m-1}$$

The fourth step is to find the covariance. By referring to the equation above, type "=(C2- $\bar{x}_j$) *(D2- $\bar{x}_h$)" in an empty cell of a new column and then click *ENTER*. Repeat the same procedure to the other cells in the same column and to the other two columns of variables but this time you need to type in "=(C2- $\bar{x}_j$)*(D2- $\bar{x}_h$)" and "=(C2- $\bar{x}_j$)*(D2- $\bar{x}_h$)" according to the column sequence of the other quality characteristics. Allocate the cursor at the end of the columns, (it need to be done one by one), sum up the values in the column by clicking the sigma, $\sum$ toolbar. The three values obtained from the previous summation are divided by 19 to get the covariance values, s_{12}^2 , s_{13}^2 and s_{23}^2 . All the variances and covariance values need to be arranged in a matrix form like below in order to get a covariance matrix.

$$\text{Covariance matrix} = s = \begin{bmatrix} s_1^2 & s_{12} & s_{13} \\ & s_2^2 & s_{23} \\ & & s_3^2 \end{bmatrix}$$

The next step is to find the inverse of the covariance matrix. Once again, put the cursor at a new empty cell. Click *Insert* menu followed by *Function*, *MINVERSE* and *OK*. The function arguments dialog box will appear. Click the button *OK*. Highlight a 3 x 3 cell, press the F2 button followed by pressing three buttons, *CTRL*, *SHIFT* and *ENTER* simultaneously.

$$S^{-1} = \begin{bmatrix} s_1^2 & s_{12} & s_{13} \\ & s_2^2 & s_{23} \\ & & s_3^2 \end{bmatrix}^{-1}$$

Hotelling T^2 statistic is obtained by using the equation below. Once again, click the *Insert* menu followed by *Transpose* and then highlight the first observation of the three quality characteristics. Subtract the transposed matrix with the mean matrix. Again, click *Insert* menu and this time followed by *MMULT*. The function arguments dialog box for *MMULT* will appear. Assign the first matrix in Array 1 and the second matrix in Array 2. Click *OK* and repeat the same process to the other rows.

$$T_k^2 = (x_k - \hat{\mu})' S^{-1} (x_k - \hat{\mu})$$

Finally, to get the multivariate Hotelling's T^2 control chart, we need to highlight the column consists of the values of Hotelling's T^2 statistic and click the Chart menu. The Chart Wizard will appear, choose the line chart. The intended multivariate control chart will be obtained (**Figure 3**).

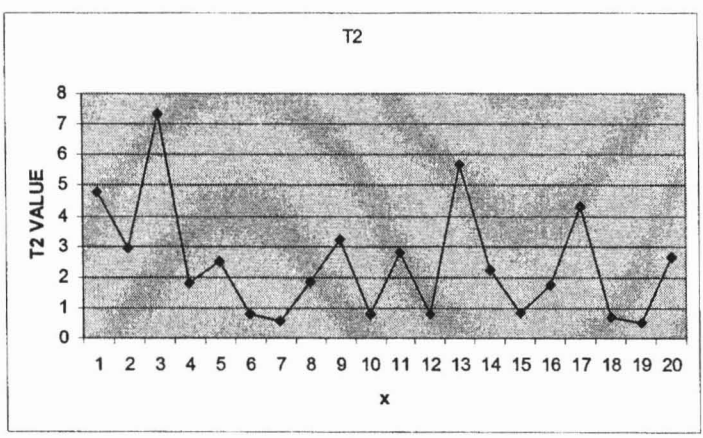


Figure 3: The Hotelling's T^2 Multivariate Control Chart for Monitoring Water Supply Quality.

Conclusion

The use of EXCEL spreadsheet will provide a user with a deep understanding on the process of constructing a control chart mathematically. The other users are encouraged to propose a better and an improved application of EXCEL spreadsheet for constructing MSPC chart. For beginners, this guideline can provide ideas for other application which involving mathematical equations and formulas.

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