PRIORITIZING CHAPTER IMPORTANCE FOR INTENSIVE MATHEMATICS USING FUZZY ANALYTICAL HIERARCHY PROCESS

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Abstract

MAT037 is an intensive Mathematics course for pre-diploma students (Pre-Commerce) at Universiti Teknologi MARA. It is crucial for the lecturers to have a proper planning on teaching the course. This course includes six chapters, which are arithmetic and algebra (C1), equation and function (C2), index and logarithm (C3), sequence (C4), introduction to applications of mathematics in business (C5), and introduction to statistics (C6). The objective of this research is to rank the MAT037 chapters based on their importance. The fuzzy analytical hierarchy process (AHP) was used to rank the chapters. The use of fuzzy AHP which to produce the ranking helps the decision makers to make comparison of which chapter is more important than the other and the vagueness of the judgements was handled by the fuzzy basis which cannot be done by the classical AHP. The triangular fuzzy numbers were used to fuzzify the crisp values, and the centre of area defuzzification method was used at the end of the steps. The results showed that C1 is the most important topic meanwhile C4 is the least important among the chapters.

Keyword: Fuzzy Analytical Hierarchy Process, Mathematics Education, Pre-Diploma

Introduction

Decision making is a very powerful activity when solving selection problems or solving problems involving the ranking of alternatives. In decision making, the decision makers are restricted to imperfect decision-relevant information and psychological biases (Aliyev, 2020). Many decision making methods have been developed, such as the technique for order preference by similarity to ideal solution (TOPSIS), analytical hierarchy process (AHP), outranking methods (ELECTRE, PROMETHEE, ORESTE), analytical network process (ANP), and data envelopment analysis (Kahraman, 2008).

The AHP which was developed by Saaty (1980), is an additive weighting method that summarizes the results of a pairwise comparison in the form of a matrix. The use of the matrix helps the decision makers in making preferences, in which an entry a_{ij} denotes the degree to which an i-th criterion is preferred to j-th one (Brunelli, 2018).

On the other hand, the development of fuzzy set theory (Zadeh, 1965) has led to the establishment of fuzzy numbers (Dubois & Prade, 1978), which have better capability in handling data with linguistic terms, as compared to the classical fuzzy sets. The fuzzy knowledge has contributed a lot in pattern recognition, image processing, decision making, operations research, and management (Chaira, 2019).

Meanwhile, Chang (1996) extended the classical AHP method into a fuzzy AHP, in which the preferences in the pairwise comparison matrix are in the form of triangular fuzzy numbers (TFN). Since then, the fuzzy AHP method has been modified and improved extensively. Some applications of fuzzy AHP are personnel selection problems (Güngör et al., 2009), teaching performance evaluation (Chen et al., 2015), and risk assessment (Lyu et al., 2020).

Since the fuzzy AHP method is easy to implement and it helps the decision makers to rank the criteria and alternatives based on the priority weights, this research aims to apply the method in ranking the chapter importance in MAT037 course. Six chapters are considered: arithmetic and algebra (C1); equation and function (C2); index and logarithm (C3); sequence (C4); introduction to applications of mathematics in business (C5); and introduction to statistics (C6). TFNs are used to represent the decision makers' preferences. The final weights are obtained by defuzzifying the fuzzy weights using centre of area defuzzification approach.
The rest of this paper is organized as follows: Section 2 presents some mathematical preliminaries; the methodology is explained in Section 3; Section 4 illustrates the implementation of the fuzzy AHP for prioritizing the MAT037 chapters; the discussion and conclusion are given in Section 5 and Section 6, respectively.

Preliminaries

In this section, some mathematical preliminaries used in the methodology are presented. Firstly, the definition of a triangular fuzzy number (TFN) is reviewed.

Definition 1 *A triangular fuzzy number,* $A=(a_1,a_2,a_3)$ *is characterized by the following membership function*

$$\mu(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & , \quad a_1\le x<a_2\\ 1 & , \quad x=a_2\\ \dfrac{a_3-x}{a_3-a_2} & , \quad a_2<x\le a_3\end{cases} \tag{1}$$

The TFN can be represented as follows:

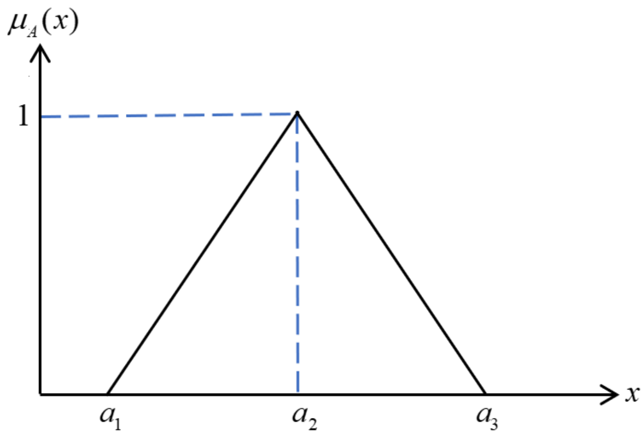

Figure 1 A triangular fuzzy number

Next, the arithmetic operations between TFNs are defined in the following definitions.

Definition 2 *Let* $A_1=(a_{11},a_{12},a_{13})$ *and* $A_2=(a_{21},a_{22},a_{23})$ *be two triangular fuzzy numbers. Then:*

(i) $A_1\oplus A_2=(a_{11}+a_{21},a_{12}+a_{22},a_{13}+a_{23})$;

(ii) $A_1\otimes A_2=(a_{11}\times a_{21},a_{12}\times a_{22},a_{13}\times a_{23})$;

(iii) $kA_1 = (ka_{11}, ka_{12}, ka_{13})$ *if* $k > 0$;

(iv) $\frac{A_1}{A_2} = \left(\frac{a_{11}}{a_{23}}, \frac{a_{12}}{a_{22}}, \frac{a_{13}}{a_{21}}\right)$ *given* a_{21}, a_{22}, a_{23} *are all non-zero; and*

(v) $A_1^{-1} = \left(\frac{1}{a_{13}}, \frac{1}{a_{12}}, \frac{1}{a_{11}}\right)$ *given* a_{11}, a_{12}, a_{13} *are all non-zero.*

Methodology

The fuzzy analytical hierarchy process (AHP) used in this study is presented in the following 8 steps:

Step 1: Determine the goal and attributes (criteria) for the study.

Step 2: Construct pairwise comparison matrices in the form of crisp values given by the decision makers. The following linguistic terms are used to make the judgement.

Table 1 Linguistic terms with corresponding crisp values

Crisp Value	Linguistic Terms
1	Equally important
3	Moderately important
5	Strongly important
7	Very strongly important
9	Extremely important
2,4,6,8	Intermediate value

Step 3: Convert the decision matrices into triangular fuzzy numbers (TFN). The following table is used to convert the crisp values into TFNs.

Table 2 Crisp values with corresponding TFNs

Crisp Value	TFN	Reciprocal Crisp Value	Reciprocal TFN
1	(1,1,1)	1	(1,1,1)
2	(1,2,3)	1/2	(1/3, 1/2, 1)
3	(2,3,4)	1/3	(1/4, 1/3, 1/2)
4	(3,4,5)	1/4	(1/5, 1/4, 1/3)
5	(4,5,6)	1/5	(1/6, 1/5, 1/4)
6	(5,6,7)	1/6	(1/7, 1/6, 1/5)
7	(6,7,8)	1/7	(1/8, 1/7, 1/6)

8	(7,8,9)	1/8	(1/9, 1/8, 1/7)
9	(9,9,9)	1/9	(1/9, 1/9, 1/9)

Step 4: Aggregate the decision matrices of all decision makers using a geometric aggregation operator. Suppose there are n fuzzy numbers, $A_1 = (a_{11}, a_{12}, a_{13})$, $A_2 = (a_{21}, a_{22}, a_{23})$, …, $A_n = (a_{n1}, a_{n2}, a_{n3})$, then the geometric aggregation operator is defined as

$$\Phi(A_1, A_2, ..., A_n) = \left(\left(\prod_{i=1}^{n} a_{i1} \right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} a_{i2} \right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} a_{i3} \right)^{\frac{1}{n}} \right). \tag{2}$$

Step 5: Calculate the fuzzy geometric mean value, R_i using (2) and fuzzy weights by adding all the fuzzy geometric mean using the following formula.

$$P = R_1 \oplus R_2 \oplus ... \oplus R_n = \left(\sum_{i=1}^{n} r_{i1}, \sum_{i=1}^{n} r_{i2}, \sum_{i=1}^{n} r_{i3} \right). \tag{3}$$

Step 6: Multiply R_i with the inverse of P for all $i = 1, 2, ..., n$. Hence, the following fuzzy weights are obtained.

$$W_i = R_i \otimes P^{-1} = (R_{i1}, R_{i2}, R_{i3}) \otimes \left(\frac{1}{p_3}, \frac{1}{p_2}, \frac{1}{p_1} \right). \tag{4}$$

Step 7: Defuzzify the fuzzy weights W_i for all $i = 1, 2, ..., n$ using the centre of area (COA) formula

$$COA(W_i) = \frac{w_{i1} + w_{i2} + w_{i3}}{3}. \tag{5}$$

Step 8: By referring to the crisp values obtained from the previous step, the attributes are ranked. The higher the COA value, the higher the attribute is ranked.

Prioritizing MAT037 Chapter Importance

In MAT037 course, there are six chapters: arithmetic and algebra (C1); equation and function (C2), index and logarithm (C3); sequence (C4), introduction to applications of mathematics in business (C5); and introduction to statistics (C6). Three lecturers who have the experience in teaching the course were chosen as decision makers (DM1, DM2, and DM3) to make judgement on the importance of these chapters.

Step 1: The six chapters are identified (C1, C2, C3, C4, C5, and C6), and the goal is to prioritize the chapters according to their importance.
Step 2: The decision makers give their judgement on the importance of the chapters using pairwise comparison matrices.
Step 3: The decision matrices are converted into triangular fuzzy numbers (TFN). The matrices in the form of TFNs are shown in **Tables 3, 4, and 5**.

Table 3 Judgement matrix from DM1 in the form of TFN

	C1	C2	C3	C4	C5	C6
C1	(1,1,1)	(4,5,6)	(3,4,5)	(6,7,8)	(3,4,5)	(4,5,6)
C2	(1/6, 1/5, 1/4)	(1,1,1)	(1,2,3)	(3,4,5)	(4,5,6)	(3,4,5)
C3	(1/5, 1/4, 1/3)	(1/3, 1/2, 1)	(1,1,1)	(5,6,7)	(4,5,6)	(3,4,5)
C4	(1/8, 1/7, 1/6)	(1/5, 1/4, 1/3)	(1/7, 1/6, 1/5)	(1,1,1)	(1/6, 1/5, 1/4)	(1/5, 1/4, 1/3)
C5	(1/5, 1/4, 1/3)	(1/6, 1/5, 1/4)	(1/6, 1/5, 1/4)	(4,5,6)	(1,1,1)	(2,3,4)
C6	(1/6, 1/5, 1/4)	(1/5, 1/4, 1/3)	(1/5, 1/4, 1/3)	(3,4,5)	(1/4, 1/3, 1/2)	(1,1,1)

Table 4 Judgement matrix from DM2 in the form of TFN

	C1	C2	C3	C4	C5	C6
C1	(1,1,1)	(2,3,4)	(4,5,6)	(7,8,9)	(5,6,7)	(6,7,8)
C2	(1/4, 1/3, 1/2)	(1,1,1)	(3,4,5)	(6,7,8)	(4,5,6)	(5,6,7)
C3	(1/6, 1/5, 1/4)	(1/5, 1/4, 1/3)	(1,1,1)	(4,5,6)	(2,3,4)	(3,4,5)
C4	(1/9, 1/8, 1/7)	(1/8, 1/7, 1/6)	(1/6, 1/5, 1/4)	(1,1,1)	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)
C5	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)	(4,5,6)	(1,1,1)	(2,3,4)
C6	(1/8, 1/7, 1/6)	(1/7, 1/6, 1/5)	(1/5, 1/4, 1/3)	(2,3,4)	(1/4, 1/3, 1/2)	(1,1,1)

Table 5 Judgement matrix from DM3 in the form of TFN

	C1	C2	C3	C4	C5	C6
C1	(1,1,1)	(2,3,4)	(3,4,5)	(6,7,8)	(5,6,7)	(6,7,8)
C2	(1/4, 1/3, 1/2)	(1,1,1)	(3,4,5)	(6,7,8)	(4,5,6)	(5,6,7)
C3	(1/5, 1/4, 1/3)	(1/5, 1/4, 1/3)	(1,1,1)	(4,5,6)	(2,3,4)	(3,4,5)
C4	(1/8, 1/7, 1/6)	(1/8, 1/7, 1/6)	(1/6, 1/5, 1/4)	(1,1,1)	(1/7, 1/6, 1/5)	(1/4, 1/3, 1/2)
C5	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)	(5,6,7)	(1,1,1)	(2,3,4)
C6	(1/8, 1/7, 1/6)	(1/7, 1/6, 1/5)	(1/5, 1/4, 1/3)	(2,3,4)	(1/4, 1/3, 1/2)	(1,1,1)

Step 4: The decision matrices in TFNs from the three decision makers are aggregated using equation (2). Hence, the aggregated decision matrix is presented in **Table 6**.

Table 6 Aggregated decision matrix

	C1	C2	C3	C4	C5	C6
C1	(1.00,1.00,1.00)	(2.52,3.56,4.58)	(3.30,4.31,5.31)	(6.32,7.32,8.32)	(4.22,5.24,6.26)	(5.24,6.26,7.27)
C2	(0.22,0.28,0.40)	(1.00,1.00,1.00)	(2.08,3.17,4.22)	(4.76,5.81,6.84)	(4.00,5.00,6.00)	(4.22,5.24,6.26)
C3	(0.19,0.23,0.30)	(0.24,0.31,0.48)	(1.00,1.00,1.00)	(4.31,5.31,6.32)	(2.52,3.56,4.58)	(3.00,4.00,5.00)
C4	(0.12,0.14,0.16)	(0.15,0.17,0.21)	(0.16,0.19,0.23)	(1.00,1.00,1.00)	(0.16,0.19,0.23)	(0.23,0.30,0.44)
C5	(0.16,0.19,0.24)	(0.17,0.20,0.25)	(0.22,0.28,0.40)	(4.31,5.31,6.32)	(1.00,1.00,1.00)	(2.00,3.00,4.00)
C6	(0.14,0.16,0.19)	(0.16,0.19,0.24)	(0.20,0.25,0.33)	(2.29,3.30,4.31)	(0.25,0.33,0.50)	(1.00,1.00,1.00)

Step 5: The fuzzy geometric mean for each row R_i is calculated. Then, the fuzzy weight is obtained by adding the fuzzy geometric mean.

Table 7 Fuzzy geometric mean, fuzzy weights and defuzzified values

	Fuzzy Geometric Mean	Fuzzy Weights	Defuzzified Values
C1	(3.24,3.93,4.58)	(0.30,0.44,0.63)	0.455
C2	(1.82,2.27,2.75)	(0.17,0.25,0.38)	0.266
C3	(1.06,1.33,1.66)	(0.10,0.15,0.23)	0.158
C4	(0.22,0.25,0.30)	(0.02,0.03,0.04)	0.030
C5	(0.61,0.75,0.92)	(0.06,0.08,0.13)	0.088
C6	(0.37,0.45,0.56)	(0.03,0.05,0.08)	0.054
Sum	(7.32,8.97,10.77)		

Step 6: The obtained sum of the fuzzy geometric mean is $P=(7.32,8.97,10.77)$. Hence, the inverse of P is given by $P^{-1}=(1/10.77,1/8.97,1/7.32)=(0.09,0.11,0.14)$. The fuzzy weights are obtained as shown in Table 7 by multiplying R_i for each row with P^{-1}.

Step 7: The fuzzy weight for each chapter is defuzzified using equation (5). The obtained results are shown in Table 7.

Step 8: Based on the defuzzified values, we obtained the following ranking: $C1 \succ C2 \succ C3 \succ C5 \succ C6 \succ C4$.

Discussion

Based on the judgement from decision makers, the MAT037 chapters are arranged according to their importance as follows: arithmetic and algebra (C1) $\succ$ equation and function (C2) $\succ$ index and logarithm (C3) $\succ$ introduction to applications of mathematics in business (C5) $\succ$ introduction to statistics (C6) $\succ$ sequence (C4). C1 is ranked as the most important chapter, while C4 is the least important one. The obtained ranking seems to be aligned with the current lesson plan of the course.

C1 is critically important because it is perceived as a 'gatekeeper,' not just for other topics in mathematics but is crucially required for advanced studies in many fields such as science and economy. The same goes for the successive chapters positions, C2 and C3. The rank obtained is in line since the understanding of the concepts learned in C1 is essential in mastering the topics for C2 and C3. Meanwhile, C4 is arranged as the least important topic as it only focuses on a few specific calculations. This is because the syllabus outlined for C4 only involves simple problems, not the problems that consist of the applications or case study difficulties discussed in C5 and C6.

Hence, here are few suggestions for the lecturers who teach this course:

- The obtained ranking should be considered in allocating the students' learning time.
- Extra revisions must be done more frequently for C1, C2, and C3 since these three chapters comprise basic mathematical arithmetics. These chapters do not just represent the few substantial domains of mathematics, but conquering those chapters signifies the process to develop a solid mathematics foundation.
- The allocation of assessment marks should be done by emphasizing the chapter's importance weightage.

Conclusion

The MAT037 chapter importance was prioritized using fuzzy AHP method in this research. The fusion of fuzzy knowledge with AHP approach was hoped to be able to handle the vagueness of the information, especially when obtaining the decision makers' judgement. The importance of each chapter was ranked to help lecturers to be well-prepared to teach the course. Proper planning before teaching the course will help to improve students' performance. Besides, lecturers should apply different teaching skills to cater to the different importance of the chapters. The judgement process in this work is limited to three Mathematics lecturers who teach the course, which may lead to biasedness. However, this issue is common in the decision making process. In the future, it is suggested that fuzzy AHP method be used for solving other multi criteria decision making problems such as identifying students' problems during online learning, ranking factors affecting students' performance, and prioritizing lecturers' main issues regarding students' attitude.

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Conflict of Interests

The authors declare that there is no conflict of interest.

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