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Driving Research Towards Excellence

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### **TABLE OF CONTENT**

### **PART 1: MATHEMATICS**

	Page
STATISTICAL ANALYSIS ON THE EFFECTIVENESS OF SHORT-TERM PROGRAMS DURING COVID-19 PANDEMIC: IN THE CASE OF PROGRAM BIJAK SIFIR 2020 Nazihah Safie, Syerrina Zakaria, Siti Madhihah Abdul Malik, Nur Baini Ismail, Azwani Alias Ruwaidiah	1
Idris	
RADIATIVE CASSON FLUID OVER A SLIPPERY VERTICAL RIGA PLATE WITH VISCOUS DISSIPATION AND BUOYANCY EFFECTS Siti Khuzaimah Soid, Khadijah Abdul Hamid, Ma Nuramalina Nasero, NurNajah Nabila Abdul Aziz	10
<b>GAUSSIAN INTEGER SOLUTIONS OF THE DIOPHANTINE EQUATION</b> $x^4 + y^4 = z^3$ <b>FOR</b> $x \neq y$ <i>Shahrina Ismail, Kamel Ariffin Mohd Atan and Diego Sejas Viscarra</i>	19
A SEMI ANALYTICAL ITERATIVE METHOD FOR SOLVING THE EMDEN- FOWLER EQUATIONS Mat Salim Selamat, Mohd Najir Tokachil, Noor Aqila Burhanddin, Ika Suzieana Murad and Nur Farhana Razali	28
<b>ROTATING FLOW OF A NANOFLUID PAST A NONLINEARLY SHRINKING</b> <b>SURFACE WITH FLUID SUCTION</b> <i>Siti Nur Alwani Salleh, Norfifah Bachok and Nor Athirah Mohd Zin</i>	36
MODELING THE EFFECTIVENESS OF TEACHING BASIC NUMBERS THROUGH MINI TENNIS TRAINING USING MARKOV CHAIN Rahela Abdul Rahim, Rahizam Abdul Rahim and Syahrul Ridhwan Morazuk	46
<b>PERFORMANCE OF MORTALITY RATES USING DEEP LEARNING APPROACH</b> Mohamad Hasif Azim and Saiful Izzuan Hussain	53
UNSTEADY MHD CASSON FLUID FLOW IN A VERTICAL CYLINDER WITH POROSITY AND SLIP VELOCITY EFFECTS Wan Faezah Wan Azmi, Ahmad Qushairi Mohamad, Lim Yeou Jiann and Sharidan Shafie	60
<b>DISJUNCTIVE PROGRAMMING - TABU SEARCH FOR JOB SHOP SCHEDULING</b> <b>PROBLEM</b> S. Z. Nordin, K.L. Wong, H.S. Pheng, H. F. S. Saipol and N.A.A. Husain	68
<b>FUZZY AHP AND ITS APPLICATION TO SUSTAINABLE ENERGY PLANNING</b> <b>DECISION PROBLEM</b> <i>Liana Najib and Lazim Abdullah</i>	78
A CONSISTENCY TEST OF FUZZY ANALYTIC HIERARCHY PROCESS Liana Najib and Lazim Abdullah	89
FREE CONVECTION FLOW OF BRINKMAN TYPE FLUID THROUGH AN COSINE OSCILLATING PLATE	98

Siti Noramirah Ibrahim, Ahmad Qushairi Mohamad, Lim Yeou Jiann, Sharidan Shafie and Muhammad Najib Zakaria

### RADIATION EFFECT ON MHD FERROFLUID FLOW WITH RAMPED WALL106TEMPERATURE AND ARBITRARY WALL SHEAR STRESS106

Nor Athirah Mohd Zin, Aaiza Gul, Siti Nur Alwani Salleh, Imran Ullah, Sharena Mohamad Isa, Lim Yeou Jiann and Sharidan Shafie

### **PART 2: STATISTICS**

A REVIEW ON INDIVIDUAL RESERVING FOR NON-LIFE INSURANCE Kelly Chuah Khai Shin and Ang Siew Ling	117
<b>STATISTICAL LEARNING OF AIR PASSENGER TRAFFIC AT THE MURTALA</b> <b>MUHAMMED INTERNATIONAL AIRPORT, NIGERIA</b> <i>Christopher Godwin Udomboso and Gabriel Olugbenga Ojo</i>	123
ANALYSIS ON SMOKING CESSATION RATE AMONG PATIENTS IN HOSPITAL SULTAN ISMAIL, JOHOR Siti Mariam Norrulashikin, Ruzaini Zulhusni Puslan, Nur Arina Bazilah Kamisan and Siti Rohani Mohd Nor	137
<b>EFFECT OF PARAMETERS ON THE COST OF MEMORY TYPE CHART</b> Sakthiseswari Ganasan, You Huay Woon and Zainol Mustafa	146
<b>EVALUATION OF PREDICTORS FOR THE DEVELOPMENT AND PROGRESSION OF DIABETIC RETINOPATHY AMONG DIABETES MELLITUS TYPE 2 PATIENTS</b> <i>Syafawati Ab Saad, Maz Jamilah Masnan, Karniza Khalid and Safwati Ibrahim</i>	152
<b>REGIONAL FREQUENCY ANALYSIS OF EXTREME PRECIPITATION IN</b> <b>PENINSULAR MALAYSIA</b> <i>Iszuanie Syafidza Che Ilias, Wan Zawiah Wan Zin and Abdul Aziz Jemain</i>	160
<b>EXPONENTIAL MODEL FOR SIMULATION DATA VIA MULTIPLE IMPUTATION</b> <b>IN THE PRESENT OF PARTLY INTERVAL-CENSORED DATA</b> <i>Salman Umer and Faiz Elfaki</i>	173
THE FUTURE OF MALAYSIA'S AGRICULTURE SECTOR BY 2030 Thanusha Palmira Thangarajah and Suzilah Ismail	181
<b>MODELLING MALAYSIAN GOLD PRICES USING BOX-JENKINS APPROACH</b> Isnewati Ab Malek, Dewi Nur Farhani Radin Nor Azam, Dinie Syazwani Badrul Aidi and Nur Syafiqah Sharim	186
WATER DEMAND PREDICTION USING MACHINE LEARNING: A REVIEW Norashikin Nasaruddin, Shahida Farhan Zakaria, Afida Ahmad, Ahmad Zia Ul-Saufie and Norazian Mohamaed Noor	192
DETECTION OF DIFFERENTIAL ITEM FUNCTIONING FOR THE NINE- QUESTIONS DEPRESSION RATING SCALE FOR THAI NORTH DIALECT	201

Suttipong Kawilapat, Benchlak Maneeton, Narong Maneeton, Sukon Prasitwattanaseree, Thoranin Kongsuk, Suwanna Arunpongpaisal, Jintana Leejongpermpool, Supattra Sukhawaha and Patrinee Traisathit

# ACCELERATED FAILURE TIME (AFT) MODEL FOR SIMULATION PARTLY 210 INTERVAL-CENSORED DATA

Ibrahim El Feky and Faiz Elfaki

#### MODELING OF INFLUENCE FACTORS PERCENTAGE OF GOVERNMENTS' RICE 217 RECIPIENT FAMILIES BASED ON THE BEST FOURIER SERIES ESTIMATOR 217

Chaerobby Fakhri Fauzaan Purwoko, Ayuning Dwis Cahyasari, Netha Aliffia and M. Fariz Fadillah Mardianto

#### CLUSTERING OF DISTRICTS AND CITIES IN INDONESIA BASED ON POVERTY 225 INDICATORS USING THE K-MEANS METHOD 225

Khoirun Niswatin, Christopher Andreas, Putri Fardha Asa OktaviaHans and M. Fariz Fadilah Mardianto

#### ANALYSIS OF THE EFFECT OF HOAX NEWS DEVELOPMENT IN INDONESIA 233 USING STRUCTURAL EQUATION MODELING-PARTIAL LEAST SQUARE

Christopher Andreas, Sakinah Priandi, Antonio Nikolas Manuel Bonar Simamora and M. Fariz Fadillah Mardianto

#### A COMPARATIVE STUDY OF MOVING AVERAGE AND ARIMA MODEL IN 241 FORECASTING GOLD PRICE

Arif Luqman Bin Khairil Annuar, Hang See Pheng, Siti Rohani Binti Mohd Nor and Thoo Ai Chin

#### CONFIDENCE INTERVAL ESTIMATION USING BOOTSTRAPPING METHODS 249 AND MAXIMUM LIKELIHOOD ESTIMATE

Siti Fairus Mokhtar, Zahayu Md Yusof and Hasimah Sapiri

### DISTANCE-BASED FEATURE SELECTION FOR LOW-LEVEL DATA FUSION OF 256 SENSOR DATA

M. J. Masnan, N. I. Maha3, A. Y. M. Shakaf, A. Zakaria, N. A. Rahim and N. Subari

#### BANKRUPTCY MODEL OF UK PUBLIC SALES AND MAINTENANCE MOTOR 264 VEHICLES FIRMS

Asmahani Nayan, Amirah Hazwani Abd Rahim, Siti Shuhada Ishak, Mohd Rijal Ilias and Abd Razak Ahmad

#### INVESTIGATING THE EFFECT OF DIFFERENT SAMPLING METHODS ON 271 IMBALANCED DATASETS USING BANKRUPTCY PREDICTION MODEL

Amirah Hazwani Abdul Rahim, Nurazlina Abdul Rashid, Abd-Razak Ahmad and Norin Rahayu Shamsuddin

#### INVESTMENT IN MALAYSIA: FORECASTING STOCK MARKET USING TIME 278 SERIES ANALYSIS

Nuzlinda Abdul Rahman, Chen Yi Kit, Kevin Pang, Fauhatuz Zahroh Shaik Abdullah and Nur Sofiah Izani

#### **PART 3: COMPUTER SCIENCE & INFORMATION TECHNOLOGY**

#### ANALYSIS OF THE PASSENGERS' LOYALTY AND SATISFACTION OF AIRASIA 291 PASSENGERS USING CLASSIFICATION 291

Ee Jian Pei, Chong Pui Lin and Nabilah Filzah Mohd Radzuan

#### HARMONY SEARCH HYPER-HEURISTIC WITH DIFFERENT PITCH 299 ADJUSTMENT OPERATOR FOR SCHEDULING PROBLEMS

Khairul Anwar, Mohammed A.Awadallah and Mohammed Azmi Al-Betar

A 1D EYE TISSUE MODEL TO MIMIC RETINAL BLOOD PERFUSION DURING 307 RETINAL IMAGING PHOTOPLETHYSMOGRAPHY (IPPG) ASSESSMENT: A DIFFUSION APPROXIMATION – FINITE ELEMENT METHOD (FEM) APPROACH Harnani Hassan, Sukreen Hana Herman, Zulfakri Mohamad, Sijung Hu and Vincent M. Dwyer

#### INFORMATION SECURITY CULTURE: A QUALITATIVE APPROACH ON 325 MANAGEMENT SUPPORT

Qamarul Nazrin Harun, Mohamad Noorman Masrek, Muhamad Ismail Pahmi and Mohamad Mustaqim Junoh

#### APPLY MACHINE LEARNING TO PREDICT CARDIOVASCULAR RISK IN RURAL 335 CLINICS FROM MEXICO

Misael Zambrano-de la Torre, Maximiliano Guzmán-Fernández, Claudia Sifuentes-Gallardo, Hamurabi Gamboa-Rosales, Huizilopoztli Luna-García, Ernesto Sandoval-García, Ramiro Esquivel-Felix and Héctor Durán-Muñoz

ASSESSING THE RELATIONSHIP BETWEEN STUDENTS' LEARNING STYLES 343 AND MATHEMATICS CRITICAL THINKING ABILITY IN A 'CLUSTER SCHOOL' Salimah Ahmad, Asyura Abd Nassir, Nor Habibah Tarmuji, Khairul Firhan Yusob and Nor Azizah Yacob

#### STUDENTS' LEISURE WEEKEND ACTIVITIES DURING MOVEMENT CONTROL 351 ORDER: UITM PAHANG SHARING EXPERIENCE

Syafiza Saila Samsudin, Noor Izyan Mohamad Adnan, Nik Muhammad Farhan Hakim Nik Badrul Alam, Siti Rosiah Mohamed and Nazihah Ismail

#### DYNAMICS SIMULATION APPROACH IN MODEL DEVELOPMENT OF UNSOLD 363 NEW RESIDENTIAL HOUSING IN JOHOR

Lok Lee Wen and Hasimah Sapiri

#### WORD PROBLEM SOLVING SKILLS AS DETERMINANT OF MATHEMATICS 371 PERFORMANCE FOR NON-MATH MAJOR STUDENTS 371

Shahida Farhan Zakaria, Norashikin Nasaruddin, Mas Aida Abd Rahim, Fazillah Bosli and Kor Liew Kee

### ANALYSIS REVIEW ON CHALLENGES AND SOLUTIONS TO COMPUTER 378 PROGRAMMING TEACHING AND LEARNING

Noor Hasnita Abdul Talib and Jasmin Ilyani Ahmad

#### **PART 4: OTHERS**

#### ANALYSIS OF CLAIM RATIO, RISK-BASED CAPITAL AND VALUE-ADDED 387 INTELLECTUAL CAPITAL: A COMPARISON BETWEEN FAMILY AND GENERAL TAKAFUL OPERATORS IN MALAYSIA Nur Amalina Syafiga Kamaruddin, Norizarina Ishak, Siti Raihana Hamzah, Nurfadhlina Abdul Halim and Ahmad Fadhly Nurullah Rasade THE IMPACT OF GEOMAGNETIC STORMS ON THE OCCURRENCES OF 396 EARTHOUAKES FROM 1994 TO 2017 USING THE GENERALIZED LINEAR MIXED MODELS N. A. Mohamed, N. H. Ismail, N. S. Majid and N. Ahmad **BIBLIOMETRIC ANALYSIS ON BITCOIN 2015-2020** 405 Nurazlina Abdul Rashid, Fazillah Bosli, Amirah Hazwani Abdul Rahim, Kartini Kasim and Fathiyah Ahmad@Ahmad Jali GENDER DIFFERENCE IN EATING AND DIETARY HABITS AMONG UNIVERSITY 413 **STUDENTS** Fazillah Bosli, Siti Fairus Mokhtar, Noor Hafizah Zainal Aznam, Juaini Jamaludin and Wan Siti Esah Che Hussain MATHEMATICS ANXIETY: A BIBLIOMETRIX ANALYSIS 420 Kartini Kasim, Hamidah Muhd Irpan, Noorazilah Ibrahim, Nurazlina Abdul Rashid and Anis Mardiana Ahmad

#### PREDICTION OF BIOCHEMICAL OXYGEN DEMAND IN MEXICAN SURFACE 428 WATERS USING MACHINE LEARNING 428

Maximiliano Guzmán-Fernández, Misael Zambrano-de la Torre, Claudia Sifuentes-Gallardo, Oscar Cruz-Dominguez, Carlos Bautista-Capetillo, Juan Badillo-de Loera, Efrén González Ramírez and Héctor Durán-Muñoz

#### RADIATION EFFECT ON MHD FERROFLUID FLOW WITH RAMPED WALL TEMPERATURE AND ARBITRARY WALL SHEAR STRESS

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This research investigates the effect of thermal radiation on unsteady magnetohydrodynamics free convection flow of ferrofluid over a vertical flat plate with ramped wall temperature and arbitrary wall shear stress. A comparison study between the magnetic nanoparticles iron oxide ( $Fe_2O_3$ ) and non-magnetic nanoparticles alumina oxide ( $Al_2O_3$ ) with water-based fluid are constructed. An appropriate dimensionless variables are introduced to the governing equations and solved analytically by using the Laplace transform technique. The variations of the profile for various values of the pertinent parameters are graphically presented and discussed. The results reveal that the velocity and temperature in the ramped wall condition are always lower than in the isothermal situation. Further, the liquid with the non-magnetic nanoparticles has a higher velocity and temperature in either the ramped plate or isothermal plate.

Keywords: Ferrofluid, MHD, Radiation, Ramped Temperature, Exact Solution.

#### 1. Introduction

In 1995, Choi and Eastman experimentally reported in their innovative work that when a small volume of solid nanoparticles was added to a common host fluid, it increased excessively the effective thermal conductivity of the base fluids. The mixtures were called nanofluids. Recent development in nanotechnology shows that it is possible to create new devices and materials with varieties of potential applications, such as biomaterial, Nano-electronics, transportation, and nuclear reactors (Tlili, 2021).

Joseph et al. (2019) investigated the optimisation of thermo-optical properties of nanofluid for direct absorption solar collectors. They considered nanofluid with suspended spherical nanoparticles  $SiO_2/Ag$ -CuO in the water-base fluid. Kalantari et al. (2019) and Tahmooressi et al. (2021) acknowledged the enhancement of thermal conductivity of a size-controlled silver nanofluid and simulated the nanoparticles' size/aspect ratio effect on the thermal conductivity of the nanofluids by using the lattice Boltzmann method. Rashid et al. (2021) studied the characteristic of two different types of hybrid nanofluids with different shapes of nanoparticles on the heat transfer and fluid flow toward a stretching/shrinking horizontal cylinder. Nanoparticles of Ag and TiO<sub>2</sub> in the water-based nanofluid were considered in their work at the range of volume fraction  $0 \le \varphi \le 0.04$ . An interesting investigation was carried out by Anuar et al. (2021) to determine the numerical computation of dusty hybrid nanofluid flow and heat transfer over a deformable sheet with slip effect.

Saqib et al. (2021) deliberated the shape effect on the MHD flow of a time-fractional Ferro-Brinkman type nanofluid under the influence of ramped heating. They have solved the fluid problem analytically via the Laplace transformed method. They considered water as the conventional base fluids which have been suspended by spherical shape magnetic nanoparticles. Another study related hybrid nanofluid, unsteady hybrid nanofluid flow over a radially permeable shrinking/stretching surface was done by Umair et al. (2021). Anwar et al. (2021) applied the Laplace transform technique to solve the fractional  $\text{Fe}_3\text{O}_4 - \text{MoS}_2$ -water hybrid nanofluid flow over an inclined surface with the ramped heating and ramped boundary motion.

Of recent, the heat transfer and fluid flow analysis of nanofluid with the interaction of magnetic field have increased enormously. Such kinds of nanofluid have numerous industrial and engineering applications. The slip and radiative effects on the water-based magnetic Fe<sub>3</sub>O<sub>4</sub> nanofluid flow over a nonlinearly stretching sheet in a porous media with Soret and Dufour diffusion were numerically examined by Bhatti et al., (2020). Krishna et al. (2021) analyzed the radiation absorption of an MHD convective flow of nanofluids through a vertical movement absorbent plate.

Free convection flows are also of great interest in industrial applications such as granular insulation, fiber, geothermal systems, nuclear reactors, filtration processes, design of spaceship, etc. Soundalgekar (1977) was the first found the exact solution for the free convection flow of a viscous incompressible fluid past an impulsively started infinite vertical plate. Khan and Alzahrani (2021) analyzed the radiation effects on a free convection nanofluid flow (silicon dioxide and molybdenum disulfide) by considering the influence of second-order velocity slip, entropy generation, and Darcy-Forchheimer porous medium. The numerical solution of free convection heat transfer from a concave hemispherical surface was studied by Behera et al. (2021).

The above studies show that no attempt has been made to analyze the radiation effect on the MHD ferrofluid flow with ramped wall temperature and arbitrary wall shear stress. Thus, the present study aims to investigate the behavior of a free convection water-based nanofluid that flows over a vertical plate with ramped wall temperature under the influence of thermal radiation. Besides, the effect of a magnetic field on fluid flow and heat transport is also taken into the consideration. A comparison of non-magnetic nanoparticles with magnetic nanoparticles is conducted. Analytical solutions are obtained through the Laplace transform methods. The interaction of the magnetic field with the magnetic in the free convection nanofluid flow is presented graphically and discussed.

#### 2. Mathematical Formulation

We consider the unsteady MHD free convection flow of an incompressible ferrofluid over an infinite vertical flat plate. The flow being confined to y > 0, where y is the coordinate measured in the normal direction to the plate. The fluid is assumed to be electrically conducting with a uniform magnetic field of strength  $B_0$ , that is applied in a direction perpendicular to the plate. The magnetic

Reynolds number is assumed to be small enough to neglect the effects of the induced magnetic field. Based on the approximation of Boussinesq and taking into consideration the above assumptions, the governing equations of momentum and energy are obtained as follows:

$$\rho_{nf} \frac{\partial u}{\partial t} = \mu_{nf} \frac{\partial^2 u}{\partial y^2} + g \left(\rho\beta\right)_{nf} \left(T - T_{\infty}\right) - \sigma_{nf} B_0^2 u, \qquad (1)$$

$$\left(\rho c_{p}\right)_{nf}\frac{\partial T}{\partial t} = k_{nf}\frac{\partial^{2}T}{\partial y^{2}} - \frac{\partial q_{r}}{\partial y},$$
(2)

subjected to initial and boundary conditions

$$u(y,0) = 0; \quad T(y,0) = T_{\infty}; \quad \forall y \ge 0,$$
 (3)

$$\frac{\partial u(0,t)}{\partial y} = \frac{f(t)}{\mu_{nf}}; \quad t > 0, \quad T(0,t) = \begin{cases} T_{\infty} + (T_{w} - T_{\infty})\frac{t}{t_{0}}, & 0 < t \le t_{0} \\ T_{w}, & t > t_{0} \end{cases}, \tag{4}$$

$$u(\infty,t) = 0; \quad T(\infty,t) = T_{\infty}; \quad t > 0, \tag{5}$$

where u(y,t) is velocity component of the fluid and dust particle phases along the x-direction,  $\rho_{nf}$  is the density of the nanofluid,  $\mu_{nf}$  is the coefficient of viscosity of the nanofluid, g is the acceleration due to gravity,  $(\rho\beta)_{nf}$  is the thermal expansion coefficient of the nanofluid,  $\sigma_{nf}$  is the electrical conductivity of the nanofluid,  $(c_p)_{nf}$  is the specific heat of the nanofluid at constant pressure, T is the temperature of the fluid and  $k_{nf}$  is the thermal conductivity of the nanofluid.

By using Rosseland approximation for radiation, the radiation heat flux (Magyari and Pantokratoras, 2011; Muhammad et al., 2020) can be simplified as

$$q_r = -\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y},\tag{6}$$

where  $\sigma^*$  and  $k^*$  are Stefan-Boltzman constant and the mean absorption coefficient, respectively. Assuming that the temperature difference within the flow is such that  $T^4$  and may be expanded in Taylor's series. Expanding  $T^4$  about  $T_{\infty}$  and neglecting higher-order, we obtain

$$T^4 \approx 4T_{\infty}^3 T - 3T_{\infty}^4. \tag{7}$$

Substituting (6) and (7) into (2), yields

$$\left(\rho c_{p}\right)_{nf}\frac{\partial T}{\partial t} = k_{nf}\frac{\partial^{2}T}{\partial y^{2}} + \frac{16\sigma^{*}T_{\infty}^{3}}{3k^{*}}\frac{\partial^{2}T}{\partial y^{2}}.$$
(8)

The relations of density, dynamic viscosity and thermal conductivity of the nanofluid (Nandkeolyar et al., 2013; Qasim et al., 2014; Devi and Devi, 2016) with the corresponding base fluid are given as

$$(\rho \sigma_{p})_{nf} = (1 - \phi)(\rho \sigma_{p})_{f} + \phi(\rho \sigma_{p})_{s}, \quad \rho_{nf} = (1 - \phi)\rho_{f} + \phi\rho_{s};$$

$$(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_{f} + \phi(\rho \beta)_{s}, \quad \mu_{nf} = \frac{\mu_{f}}{(1 - \phi)^{2.5}}, \quad \frac{k_{nf}}{k_{f}} = \frac{(k_{s} + 2k_{f}) - 2\phi(k_{f} - k_{s})}{(k_{s} - 2k_{f}) + \phi(k_{f} - k_{s})}, \quad (9)$$

where  $\phi$  is the volume fraction of the nanoparticles,  $\rho_f$  and  $\rho_s$  are the densities of the base fluid and solid nanoparticles,  $(c_p)_f$  and  $(c_p)_s$  are the thermal conductivities of the base fluid and solid nanoparticles,  $\beta_f$  and  $\beta_f$  are the volumetric coefficients of thermal expansions of the base fluid and solid nanoparticles,  $\mu_f$  is the dynamic viscosity of the base fluid,  $k_f$  and  $k_s$  are the thermal conductivities of the base fluid and solid nanoparticles. The thermophysical properties of base fluid and nanoparticles (Loganathan et al., 2013; Qasim et al., 2014; Oztop and Abu-Nada, 2008; Hussanan et al., 2017) are listed in Table 1.

 $\beta \times 10^{-5} \left( K^{-1} \right)$  $\rho(kgm^{-3})$  $k\left(Wm^{-1}K^{-1}\right)$  $C_{p} kg^{-1}K^{-1}$ Model  $H_{2}O$ 4179 997.1 0.613 21 765 3970 40 0.85  $Al_2O_2$  $Fe_3O_4$ 670 5180 9.7 0.5

Table 1: Thermophysical properties of base fluid (water) and nanoparticles (iron oxide and alumina oxide)

Now using following dimensionless variables (Khan et al., 2014; Khan et al., 2020; Nandkeolyar et al., 2013)

$$u^{*} = u \sqrt{\frac{t_{0}}{\nu_{f}}}, \quad y^{*} = \frac{y}{\sqrt{\nu_{f}t_{0}}}, \quad t^{*} = \frac{t}{t_{0}}, \quad T^{*} = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \quad f^{*}(t^{*}) = \frac{t_{0}}{\mu_{f}} f(t_{0}t^{*}), \quad t_{0} = \frac{\nu_{f}}{U_{0}^{2}}, \quad (10)$$

so that equations (1) and (8) are reduced to (\* notations are dropped for simplicity)

$$\frac{\partial u}{\partial t} = \frac{1}{\phi_1} \frac{\partial^2 u}{\partial y^2} + GrT\phi_3 - \frac{M}{\phi_2}u,\tag{11}$$

$$\Pr\frac{\partial T}{\partial t} = \frac{\lambda}{\phi_4} \left[ 1 + \frac{Nr}{\lambda} \right] \frac{\partial^2 T}{\partial y^2}.$$
 (12)

The corresponding dimensionless initial and boundary conditions are

 $u(y,0) = 0; \quad T(y,0) = 0; \quad \text{for} \quad y \ge 0,$  (13)

$$\frac{\partial u(0,t)}{\partial y} = (1-\phi)^{2.5} f(t); \quad t > 0, \quad T(0,t) = \begin{cases} t & \text{for} \quad 0 < t \le 1\\ 1 & \text{for} \quad t > 1 \end{cases}, \tag{14}$$

$$u(\infty,t) = 0; \quad T(\infty,t) = 0; \quad t > 0.$$
<sup>(15)</sup>

Denoting

$$\phi_{1} = (1 - \phi)^{2.5} \left[ (1 - \phi) + \phi \frac{\rho_{s}}{\rho_{f}} \right], \quad \phi_{2} = (1 - \phi) + \phi \frac{\rho_{s}}{\rho_{f}},$$

$$\phi_{3} = \frac{(1 - \phi)\rho_{f} + \phi\rho_{s} \left(\frac{\beta_{s}}{\beta_{f}}\right)}{\rho_{nf}}, \quad \phi_{4} = (1 - \phi) + \frac{\phi(\rho c_{p})_{s}}{(\rho c_{p})_{f}}.$$
(16)

where M is the magnetic parameter, Gr is the Grashof number, Pr is the Prandtl number and Nr is the radiation parameter which are defined as

$$M = \frac{\sigma_{nf} B_0^2 \upsilon_f}{\rho_f U_0^2}, \quad Gr = \frac{g \beta_f \upsilon_f \left( T_w - T_w \right)}{U_0^3}, \quad \Pr = \frac{\upsilon_f \left( \rho c_p \right)_f}{k_f}, \quad \lambda = \frac{k_{nf}}{k_f}, \quad Nr = \frac{16\sigma^* T_w^3}{3k_f k^*}.$$
(17)

#### 3. Solution of the Problem

Applying the Laplace transform technique to (11)-(15) and make use of the transformed initial conditions, then we obtained the following differential equations in (y,q)-plane

$$\frac{1}{\phi_1} \frac{d^2 \overline{u}(y,q)}{dy^2} - \frac{M}{\phi_2} \overline{u}(y,q) - q \overline{u}(y,q) + Gr \phi_3 \overline{T}(y,q) = 0$$
(18)

$$\frac{\lambda}{\phi_4} \left[ 1 + \frac{Nr}{\lambda} \right] \frac{d^2 \overline{T}(y,q)}{dy^2} - \Pr q \overline{T}(y,q) = 0, \tag{19}$$

with the boundary conditions

$$\frac{d\overline{u}(0,q)}{dy} = \left(1 - \phi\right)^{2.5} F(q), \tag{20}$$

$$\overline{u}(\infty,q) = 0, \tag{21}$$

$$\bar{T}(0,q) = \frac{1 - e^{-q}}{q^2},$$
(22)

$$\bar{T}(\infty,q) = 0, \tag{23}$$

where  $\overline{u}(y,q)$  and  $\overline{T}(y,q)$  are Laplace transform of u(y,t) and T(y,t). Equation (19) subject to the boundary conditions (22) and (23) has the following solution

$$\overline{T}(y,q) = \left(1 - e^{-q}\right) \frac{e^{-y\sqrt{a_0q}}}{q^2}.$$
(24)

Here,

$$a_0 = \frac{\Pr \phi_4}{\lambda \left(1 + \frac{Nr}{\lambda}\right)}.$$

By taking the inverse Laplace transform of (24) and and using the second shift property, then we obtain

$$T(y,t) = T_{R}(y,t) - T_{R}(y,t-1)H(t-1).$$
(25)

Denoting

$$T_{R}\left(y,t\right) = \left(t + \frac{a_{0}y^{2}}{2}\right) \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a_{0}}{t}}\right) - y\sqrt{\frac{a_{0}t}{\pi}}e^{-\frac{a_{0}y^{2}}{4t}},$$
(26)

where H(t) is the Heaviside unit step function and erfc(.) is the complimentary error function. Then, substituting (24) into (18) and make use of (20) and (21) leads to the following expression

$$\overline{u}(y,q) = -\overline{u}_{1}(q).\overline{u}_{4}(y,q) - (1 - e^{-q})\overline{u}_{2}(q).\overline{u}_{4}(y,q) + (1 - e^{-q})\overline{u}_{3}(q).\overline{u}_{5}(y,q) 
= -\overline{u}_{1}(q).\overline{u}_{4}(y,q) + (1 - e^{-q})[-\overline{u}_{2}(q).\overline{u}_{4}(y,q) + \overline{u}_{3}(q).\overline{u}_{5}(y,q)],$$
(27)

where

$$\overline{u}_{1}(q) = (1-\phi)^{2.5} F(q), \quad \overline{u}_{2}(q) = \frac{b_{1}\sqrt{a_{0}q}}{q^{2}(q-b_{2})}, \quad \overline{u}_{3}(q) = \frac{b_{1}}{q-b_{2}},$$

$$\overline{u}_{4}(y,q) = \frac{e^{-y\sqrt{a_{1}+\phi_{1}q}}}{\sqrt{a_{1}+\phi_{1}q}}, \quad \overline{u}_{5}(y,q) = \frac{e^{-y\sqrt{a_{0}q}}}{q^{2}},$$
(28)

and the arbitrary constants are

$$a_1 = \frac{M\phi_1}{\phi_2}, \quad b_1 = -\frac{Gr\phi_1\phi_3}{a_0 - \phi_1}, \quad b_2 = \frac{a_1}{a_0 - \phi_1}.$$

Then, the inverse Laplace transform of (28) are given by

$$u_{1}(t) = (1-\phi)^{2.5} f(t), \quad u_{2}(t) = b_{1}\sqrt{a_{0}} \left[ -\frac{2}{b_{2}}\sqrt{\frac{t}{\pi}} + \frac{e^{b_{2}t}\operatorname{erf}\left(\sqrt{b_{2}t}\right)}{b_{2}^{\frac{3}{2}}} \right], \quad (29)$$

$$u_{3}(t) = b_{1}e^{b_{2}t}, \quad u_{4}(y,t) = \frac{e^{-\left(m_{1}t + \frac{y^{2}\phi_{1}}{4t}\right)}}{\sqrt{\phi_{1}\pi t}}, \quad u_{5}(y,t) = \left(\frac{a_{0}y^{2}}{2} + t\right)\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a_{0}}{t}}\right) - y\sqrt{\frac{a_{0}t}{\pi}}e^{-\frac{y^{2}a_{0}}{4t}}.$$

Here, erf(.) is the error function and

$$m_1 = \frac{a_1}{\phi_1}.$$

Finally, by using the second shift property in (27), the velocity distribution u(y,t) can be written as

$$u(y,t) = u_a(y,t) + u_R(y,t) - u_R(y,t)H(t-1).$$
(30)

In which  $u_a(y,t)$  and  $u_R(y,t)$  can be found by using the convolution theorem

$$u_{a}(y,t) = -(u_{1} \otimes u_{4})(t)$$
  
=  $-\int_{0}^{t} u_{1}(t-s)u_{4}(y,s)ds$   
=  $-\frac{(1-\phi)^{2.5}}{\sqrt{\phi_{1}\pi}}\int_{0}^{t} \frac{f(t-s)}{\sqrt{s}}e^{-\left(m_{1}s+\frac{y^{2}\phi_{1}}{4s}\right)}ds,$  (31)

$$u_{R}(y,t) = -(u_{2} \otimes u_{4})(t) + (u_{3} \otimes u_{5})(t)$$

$$= -\int_{0}^{t} u_{2}(t-s)u_{4}(y,s)ds + \int_{0}^{t} u_{3}(t-s)u_{5}(y,s)ds$$

$$= -b_{1}\sqrt{\frac{a_{0}}{\pi\phi_{1}}}\int_{0}^{t} \frac{1}{\sqrt{s}}e^{-\left(m_{1}s+\frac{y^{2}\phi_{1}}{4s}\right)} \left[ -\frac{2}{b_{2}}\sqrt{\frac{t-s}{\pi}} + \frac{e^{b_{2}(t-s)}\text{erf}\left(\sqrt{b_{2}(t-s)}\right)}{b_{2}^{\frac{3}{2}}} \right] ds$$

$$+ \int_{0}^{t} b_{1}e^{b_{2}(t-s)} \left[ \left(\frac{y^{2}a_{0}}{2} + s\right)\text{erfc}\left(\frac{y}{2}\sqrt{\frac{a_{0}}{s}}\right) - y\sqrt{\frac{a_{0}s}{\pi}}e^{-\frac{y^{2}a_{0}}{4s}} \right] ds.$$
(32)

#### **3.1** Solution for an Isothermal Plate

It is worth that to compare the obtained results in (25) and (30) with an isothermal case or constant wall temperature. Therefore, the solution of velocity and temperature distributions for an isothermal plate are

$$u(y,t) = -\frac{(1-\phi)^{2.5}}{\sqrt{\phi_{1}\pi}} \int_{0}^{t} \frac{f(t-s)}{\sqrt{s}} e^{-\left(m_{1}s+\frac{y^{2}\phi_{1}}{4s}\right)} ds$$

$$-b_{1}\sqrt{\frac{a_{0}}{b_{2}\phi_{1}\pi}} \int_{0}^{t} \frac{1}{\sqrt{s}} e^{b_{2}(t-s)-\left(m_{1}s+\frac{y^{2}\phi_{1}}{4s}\right)} \operatorname{erf}\left(\sqrt{b_{2}(t-s)}\right) ds$$

$$+b_{1}\int_{0}^{t} e^{b_{2}(t-s)} \operatorname{erfc}\left(\frac{y}{s}\sqrt{\frac{a_{0}}{s}}\right) ds,$$

$$T(y,t) = \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a_{0}}{t}}\right).$$
(27)
(28)

#### 4. Result and Discussion

The combined effects of radiation and magnetic field on Ferrofluid flow through a vertical plate with ramped wall temperature have been studied. The governing partial differential equations are solved analytically by the Laplace transform technique. The effects of the involved parameters on the fluid field and heat transfer are analyzed graphically and presented in Figures 1-6.

Figure 1 illustrates the influence of nanoparticle volume fraction  $\phi$  on the velocity profile of the

 $Fe_3O_4$ -water nanofluid. It is found that the velocity of the nanofluid decreases with the increment of the volume fraction. This is because the increase of volume fraction leads to a larger viscosity of the nanofluid result in more resistance to the fluid flow. This nature is observed either in the situation of the ramped wall or isothermal plate. Furthermore, the ramped wall velocity is smaller as compared to the isothermal case.

Figure 2 shows the effect of magnetite and non-magnetite nanoparticles on the velocity of the ferrofluid. It is observed that the velocity of  $Al_2O_3$ -water-based nanofluid is greater than the  $Fe_3O_4$ -water-based nanofluid. This implies that the  $Fe_3O_4$ -water-based nanofluid is more viscous than the  $Al_2O_3$ -water-based nanofluid. It is also noticed that the thermal conductivity of the  $Fe_3O_4$ -water-based nanofluid is greater than  $Al_2O_3$ -water-based nanofluid. Besides, the isothermal case is also compared

with the ramped wall case and a greater velocity is detected in the condition of ramped wall either for  $Al_2O_3$  or  $Fe_3O_4$  water-based nanofluid.

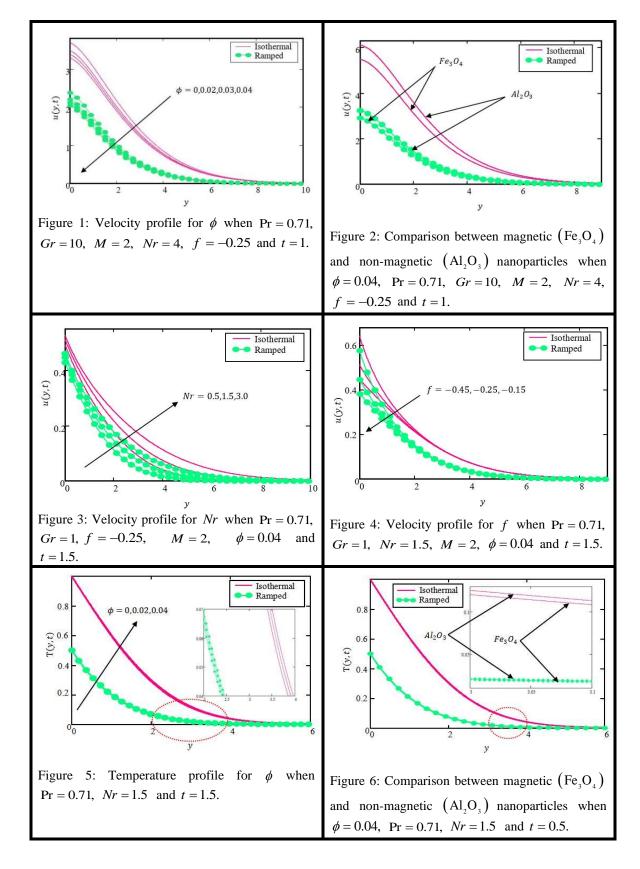


Figure 3 is plotted to show the effect of radiation parameter Nr on the velocity of magnetite nanofluid. It is noticed that the velocity of the Ferrofluid is enhanced with the increase of Nr. This is because increasing the value of radiation parameter will increase the heat conduction which in turn increases the rate of energy transport in the fluids. Thus, decreases the viscosity and thermal conductivity of the Ferrofluid. This will make the fluid move faster and increase the velocity of the Ferrofluid. Meanwhile, Figure 4 depicts the effect of wall shear stress parameter f on the velocity of the Ferrofluid. The figure displays that the velocity decline when the number of f is increased for both isothermal and ramped situations.

Figure 5 demonstrates the temperature distribution for different values of the volume fractions  $\phi$  for isothermal and ramped conditions. The temperature has shown a positive effect on the volume fraction. Physically, the increment of the volume fraction tends to increase the thermal conductivity. Thus, the enhancement of the thermal conductivity has increased the rate of heat transfer and then increase the distribution of the temperature in the fluid. The influence of the volume fraction on the temperature profile either in the isothermal or ramped wall is the same but the temperature is larger in the isothermal situation as compared with the ramped temperature.

Lastly, the graphical results of the temperature profiles for the magnetite and non-magnetite nanoparticles are presented in Figure 6, for both isothermal and ramped plate conditions. The variation of the magnetite and non-magnetite nanoparticles with isothermal and ramped temperatures is very small, almost negligible in ramp temperature. The temperature of non-magnetite nanoparticles is greater than the magnetite nanoparticles. The thermal conductivity and viscosity of the Ferrofluid are temperature dependent. The thermal conductivity increases with the temperature whereas viscosity decreases with the temperature. Therefore, the magnetite nanofluid is more viscous and has a lower temperature as compared to the non-magnetite nanofluid.

#### 5. Conclusion

The present study discusses the combined effects of radiation and magnetic field on the boundary layer flow of a Ferrofluid over a ramped wall temperature vertical plate. Magnetite nanoparticles  $Fe_3O_4$  are suspended inside the conventional water-based fluid. Non-magnetite,  $Al_2O_3$  nanoparticles are chosen for comparison with the magnetite nanoparticles,  $Fe_3O_4$ . Moreover, spherical-shaped nanoparticles are considered in this study. It is found that an increase in the volume fraction  $\phi$  of the nanoparticle has increased the viscosity of the nanofluid, which leads to a decrease in the velocity of the magnetite nanofluid. An opposite tendency is observed for the temperature profile with the volume fraction  $\phi$  of the nanoparticles are more viscous as compared to the non-magnetite nanofluid due to the greater velocity and temperature that have been detected in the fluid containing  $(Al_2O_3)$ . Lastly, it is noticed that the radiation parameter, Nr is an increasing function of the velocity. The influence of this parameter is found similar in the case of temperature. Furthermore, it is recommended that to consider mass transfer with other relevant effects such as porous medium, inclined magnetic field or Newtonian heating in order to expand the possibilities of this study.

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