Universiti Teknologi MARA Cawangan Kedah Kampus Sungai Petani

2021 iCMS International Conference on Computing, Mathematics and Statistics

e-PROCEEDINGS

of The 5th International Conference on Computing, Mathematics and Statistics (iCMS2021)

4-5 August 2021

Driving Research Towards Excellence

e-Proceedings
of the 5th International Conference on Computing, Mathematics and Statistics (iCMS 2021)

Driving Research Towards Excellence

e-ISBN: 978-967-2948-12-4
DOI

Library of Congress Control Number:

Publication by
Department of Mathematical Sciences
Faculty of Computer & Mathematical Sciences
UiTM Kedah

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A CONSISTENCY TEST OF FUZZY ANALYTIC HIERARCHY PROCESS

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One of the most important steps in Fuzzy Analytic Hierarchy Process (FAHP) is to perform the consistency test of matrix judgment. This test aims at investigating the consistency in decision makers' judgment. It helps to minimize improper decisions made. Nevertheless, this step is more than often opted out by many researchers due to its complexity. This paper, thus intends to introduce a consistency test for matrix judgment in FAHP method. This proposal is obtained by aggregating the decision makers' triangular fuzzy number (TFN), forming a matrix judgment and finalising with matrix determinant. Twenty four different TFN were tested to illustrate the proposed method. Results show that seventeen TFN passed the consistency test, whereas the remaining seven failed. The consistency test offers a significant contribution to check the suitability of TFN in judgement matrix of FAHP.

***Keywords*:** fuzzy sets, multi criteria analysis, matrix judgment, consistency test

1. Introduction

Achieving the level of consistency in FAHP is not a direct process as each of the pair-wise comparison matrix and positive reciprocal matrix is described by fuzzy numbers (Wang and Chen, 2008). Furthermore, when a consistent state of decision makers (DMs) perceptions fails to be achieved, then it has to correct or redo the judgment matrix to obtain acceptable consistency index value, which is less than or equal to 0.1 (Saaty and Vargas, 2012) otherwise, it may lead to some problems, such as time-consuming, or requires a fairly large size in the calculations in decision making (Xu and Xiong, 2017). Thus, consistency of preference relations has become an important issue in decision making with paired comparisons to check and measure the quality of DMs judgments (Wang, 2019). There are some methods focusing on the inconsistency problems where fuzzy numbers in the matrix are manipulated mathematically and regenerated various methods of consistency test. Sahin (2017) for example used a consistency control along with expert consistency prioritization for direct fuzzy inputs as basic events (BEs) assigned to the fault tree analysis (FTA) method. Bulut et al. (2012) proposed a novel FAHP procedure named generic FAHP (GF-AHP) by extending the conventional method with centric consistency index control, prioritization of expert contribution and providing a ranking method for use of numerical inputs without expert consultation. Chang et al. (2009) introduced the maximum value of lambda prior to substituting into typical lambda-max equation in testing the consistency while implementing FAHP in service evaluations during pre-negotiation stages. Wang and Chen (2008) adopted fuzzy linguistic preference relations to construct pair-wise comparison matrices with additive reciprocal properties and consistencies. Liu (2009) utilized a convex combination method; a set of crisp reciprocal comparison matrices with consistency is obtained and aggregated to find the interval weights from consistency of reciprocal comparison.

The researches do take into account pre-existing of TFN in computing consistency level. The TFN are normally arrayed in n x m decision matrix where n and m are representing number of alternatives and number of DMs respectively. In AHP, consistency of judgment is determined using an arithmetic operation of division. It requires only $(n-1)$ comparison judgments to ensure consistency for a level with n alternatives. Consistency test is computed by dividing the result of subtracting the size of matrix from a maximum value of eigenvalue λ_i , (i = 1, 2, 3….n) with size of matrix minus one. The maximum value of eigenvalue λ_i is obtained from the list of possible eigenvalues. It is a very direct process whereby maximum value of eigenvalue is chosen from the maximum value of all possible eigenvalues. This method of identifying maximum eigenvalue might have been much more convincing if more simple and objectively computing method had considered. This unique maximum eigenvalue is called lambda-max. The Lambda-max is a maximum eigenvalue of matrix A. The maximum eigenvalue can be used to estimate consistency in a matrix, as reflected in the proportionality of preferences (Saaty, 1980). Especially, the closer Lambda-max is to the number of elements *n* in the matrix A , the more consistent the matrix will be (Salem, 2010).

In this paper we introduce the concept of matrix determinant as the prime source of obtaining lambda-max. The maximum value of lambda is obtained by taking the aggregated TFN of each evaluation of the pair-wise comparison matrix. The purpose of using aggregated TFN is to determine the average midpoint of the TFN numbers of each evaluation matrix. Therefore, our approach is more geared toward defining a consistency test of FAHP method. The rest of this paper proceeds as follows. The basic definitions of fuzzy sets and its affiliation in FAHP decision-making is presented in Section 2. A TFN preference scale and proposed stepwise of FAHP are discussed in Section 3. Implementation of Consistency Test in FAHP with illustrative examples are developed in Section 4. Finally, a short conclusion is given in Section 5.

2. Preliminaries

This section introduces the basic definitions relating to fuzzy sets, fuzzy numbers, and triangular fuzzy number (TFN) and an overview of consistency ratio in AHP.

2.1 Definition of Fuzzy sets

To deal with the uncertainty of human thought as effects of imprecision and vagueness, Zadeh (1965) introduced fuzzy sets and fuzzy logic theory which are most potent as mathematical tools for modeling uncertainty measurements in the system.

Definition 1: A fuzzy set A in the universe of discourse $\mathrm{X}=\{\mathrm{x}_1,\mathrm{x}_2,...,\mathrm{x}_\mathrm{n}\}$ is defined as:

$$\mathrm{A}=\left\{\left\langle \mathrm{x},\mu_\mathrm{x}\left(\mathrm{x}\right)\right\rangle \middle| \mathrm{x}\in \mathrm{X}\right\} \tag{1}$$

which is characterized by membership function $\mu_\mathrm{x}\left(\mathrm{x}\right):\mathrm{X}\to\left[0,1\right]$, where $\mu_\mathrm{x}\left(\mathrm{x}\right)$ indicates the membership degree of the element x to the set A.

2.2 Definition of Fuzzy numbers and TFN

The fuzzy set theory is designed to deal with the extraction of the primary possible outcome from a multiplicity of vague and imprecise information. Fuzzy set theory treats vague data as possibility distributions in terms of set memberships. TFN are one of the major components. According to the definition of Laarhoven and Pedrycz (1983) TFN should possess the following basic features.

Definition 2: A fuzzy number A on $\Re$ to be a TFN if its membership functions $\mu_{\tilde{\mathrm{A}}}\left(\mathrm{x}\right):\Re\to\left[0,1\right]$ is equal to

$$\mu_{\tilde{A}}\left(x\right)=\begin{cases}\left(x-l\right)/\left(m-l\right), & l\le x\le m\\ \left(u-x\right)/\left(u-m\right), & m\le x\le u\\ 0, & \text{otherwise}\end{cases} \tag{2}$$

where l and u represent the lower and upper bounds of the fuzzy numbers $\tilde{\mathrm{A}}$, respectively, and m is the median value. The TFN is denoted as $\tilde{A}=\left(l,m,u\right)$.

2.3 Consistency Ratio

In order to deal with consistency judgment of DMs measurement, Saaty devised a consistency test to distinguish the consistent from the inconsistent comparison Daji et. al, (2011) as follows:

Definition 3: The pair-wise comparison matrix can pass the consistency test, if the consistency ratio $\mathrm{C.R.} = \frac{\mathrm{C.I.}}{\mathrm{RI}} < 0.1$, where the consistency index, $\mathrm{C.I} = \frac{\lambda_{max} - n}{n-1}$, RI is the average random index based on matrix size, λ_{max} is the maximum eigenvalues of a matrix comparison of judgment and n is the order of the matrix comparison. Matrix comparison is an acceptable consistency if the ratio of $\mathrm{C.R.} < 0.1$. When the *C.R* is more than 0.1, the matrix is said to be an unacceptable consistency which the judgement of comparison matrix should be changed to ensure the rationality of DMs.

2.4 Fuzzy Consistency

Based on Csutora and Buckley (2000), if the matrix that includes the middle number of the TFN is consistent, then the fuzzy matrix is consistent. If the matrix is consistent, it should satisfy the Saaty's standard solution of consistency test.

Let any positive matrix A has a largest eigenvalue λ_{max} that is real and positive. The corresponding eigenvalue problem $Ax = \lambda_{max} x$ has a solution x with $x_i > 0$ for all i which is unique to within multiplication by a scalar. Uniqueness will be obtained by requiring $\sum x_i = 1$. The eigenvalue λ_{max} always satisfies $\lambda_{max} \geq n$ with $\lambda_{max} = n$ if and only if consistency holds. The eigenvector solution was the original solution proposed to the AHP problem Saaty (1980). For any reciprocal matrix, not necessarily consistent, this vector x can be thought of as an estimate of a ''true'' priority vector. In this case, a matrix with (i, j) element given by x_i / x_j is an approximation to A. Only in the consistent case does a priority vector exist that recovers the A matrix exactly.

Saaty (1990) suggested consistency index (*C.I*) and consistency ratio (*C.R*) to verify the consistency of the judgment matrix. Random index (RI) represents the average consistency index over numerous random entries of the same order reciprocal matrices. The value of RI depends on the value of n matrix. This relationship is shown in Table 1.

Table 1: Random indices (RI) of sizes of matrices

N	1-2	3	4	5	6	7	8	9
RI	0.0	.58	.90	1.12	1.24	1.32	1.41	1.45

3. The Consistency Test

The linguistic scales and the relative importance proposed by Chang (1996) and Bozbura et al., (2007) are used in this lambda-max. Details of the linguistic scales, TFN and its reciprocated values are given in Table 2.

Table 2: Triangular Fuzzy Conversion Scale (Chang, (1996); Bozbura et.al, (2007))

Linguistic scale	TFN scale	TFN reciprocal scale
Just equal	(1,1,1)	(1,1,1)
Equally important (EI)	(1/2,1,3/2)	(2/3,1,2)
Weakly more important (WMI)	(1,3/2,2)	(1/2,2/3,1)
Strongly more important (SMI)	(3/2,2,5/2)	(2/5,1/2,2/3)
Very strongly more important (VSMI)	(2,5/2,3)	(1/3,2/5,1/2)
Absolutely more important (AMI)	(5/2,3,7/2)	(2/7,1/3,2/5)

The stepwise of FAHP procedure is given as follows:

Step 1: Constructing a hierarchical diagram of MCDM problem.

Step 2: Scaling the relative data and constructing the pair-wise comparison matrices.
For this step, construct the comparison matrices of each criterion. The scale of the judgement is made based the linguistic in Table 2.

Step 3: Consistency test of the pair-wise comparison matrices.
In this step, we introduce the consistency test of each matrix comparison prior to continuing to the next step of FAHP. The TFN that defined in Table 2 is used to compute lambda-max. The averaging operator is used in the aggregation to yield a crisp number. Lambda-max can be obtained using the concept of matrix determinant. The determinant of a matrix gives the signed volume of the parallelepiped generated by its columns. A *determinant function* assigns to each square matrix A a scalar associated to the matrix, denoted $\det(\mathrm{A})$ or $|\mathrm{A}|$ such that;

1. The determinant of an $\mathrm{n}\times\mathrm{n}$ identity matrix I is 1. $|\mathrm{I}|=1$
2. If the matrix B is identical to the matrix A except the entries in one of the rows B are each equal to the corresponding entries of A multiplied by the same scalar c, then $|\mathrm{B}|=\mathrm{c}|\mathrm{A}|$.
3. There is one and only one determinant function.

Then, the lambda-max is defined as below:

$$\lambda_{\max}=Det\left[\begin{vmatrix} 1 & \frac{(l+m+u)_{12}}{3} & \cdots & \frac{(l+m+u)_{n}}{3} \\ \frac{(l+m+u)_{21}}{3} & 1 & \cdots & \frac{(l+m+u)_{n}}{3} \\ \vdots & \cdots & 1 & \vdots \\ \frac{(l+m+u)_{n}}{3} & \cdots & \cdots & 1 \end{vmatrix}-\lambda I\right]$$

$$\lambda_{\max}=Det\left[\begin{vmatrix} 1-\lambda & \frac{(l+m+u)_{12}}{3} & \cdots & \frac{(l+m+u)_{n}}{3} \\ \frac{(l+m+u)_{21}}{3} & 1-\lambda & \cdots & \frac{(l+m+u)_{n}}{3} \\ \vdots & \cdots & 1-\lambda & \vdots \\ \frac{(l+m+u)_{n}}{3} & \cdots & \cdots & 1-\lambda \end{vmatrix}\right] \tag{3}$$

The typical procedures of FAHP are continued after testing the consistency judgment. In a case where matrix judgment fails to fulfill consistency test, then the DMs need to re-scale the TFN.

4. Implementation of Consistency Test

The lambda-max is used to test the consistency and reliable of the pair-wise comparison matrix decided by DMs. The consistency of the matrix is important to make sure the result of the hierarchical structure is perfectly feasible and concrete. This lambda-max is built based on the aggregated TFN of the matrix and the usage of matrix determinant and tested using TFN provided by Chang (1992), Bozbura et al. (2007) and Kahraman et al. (2004). To ensure the reliability and validity of the lambda-max, the comparative analysis of the consistency ratio are also conducted on other TFN. The MCDM problems retrieved from Metin and Ihsan (2008), Kahraman et al. (2004), and Isaai et al. (2011) is tested. The computations are executed using the eight steps of FAHP method.

Step 1: The hierarchical structure of the problem is given in Figure 1. The first level of hierarchy provides information on the focus of the main MCDM problems. Next, the second level of hierarchy explains the criterion of the problem statement, C_n while the bottom of the hierarchy explains the alternatives of MCDM problems.

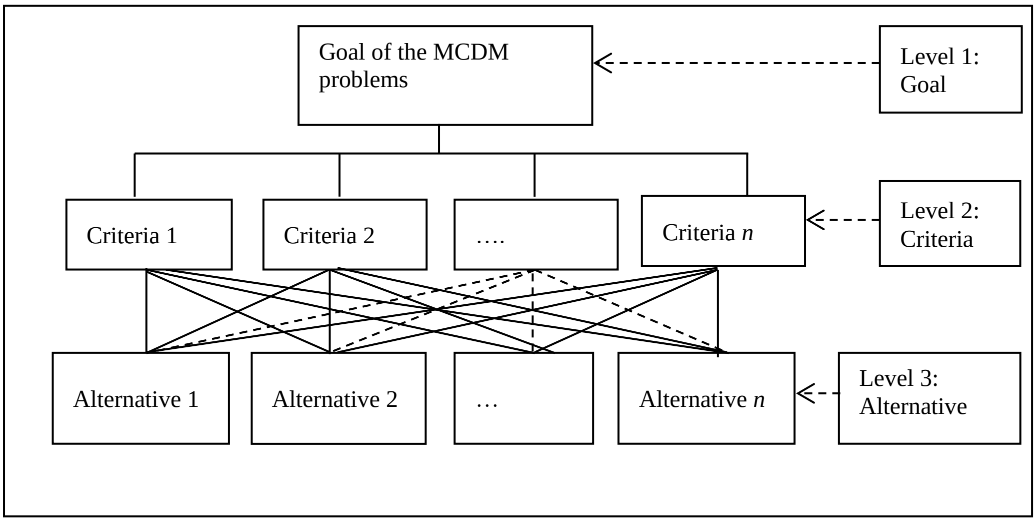

Figure 1: The Hierarchy of the MCDM problem

Step 2: Scaling of the pair-wise comparison scale of FAHP with the preference scale of TFN. Three TFN scales are used to test the feasibility of the proposed lambda-max. The judgment matrices in TFN are presented in Table 3, Table 4 and Table 5 respectively.

Table 3: The Fuzzy Judgement Matrix of Criterion (Kahraman et al., 2004)

	H	QM	QS
H	(1,1,1)	(3/2,2,5/2)	(2/3,1,3/2)
QM	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)
QS	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)

Table 4: The Fuzzy Judgement Matrix of Criterion (Metin and Ihsan, 2008)

	OF	PF	JRF	EF
OF	(1,1,1)	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)	(1, 3/2, 2)
PF	(2/7, 1/3, 2/5)	(1,1,1)	(3/2,2,5/2)	(1, 3/2, 2)
JRF	(5/2, 3, 7/2)	(2/5,1/2,2/3)	(1,1,1)	(2/5, 1/2, 2/3)
EF	(1/2, 2/3, 1)	(1/2, 2/3, 1)	(3/2, 2, 5/2)	(1,1,1)

Table 5: The Fuzzy Judgement Matrix of Criterion (Isaai et al., 2011)

	SWWT	AUWT	MRWJ
SWWT	(1,1,1)	(0.54,0.64,0.79)	(0.47,0.54,0.64)
AUWT	(1.26,1.57,1.86)	(1,1,1)	(0.55,0.66,0.84)
MRWJ	(1.57,1.86,2.14)	(1.19,1.51,1.81)	(1,1,1)

Step 3: Consistency test of the pair-wise comparison matrices. In this step, the lambda-max is tested to define the consistency test for each matrix comparison of criterion. The aggregated fuzzy judgement is computed for each set of TFN. It is shown in Table 6, Table 7 and Table 8.

Table 6: Aggregated Fuzzy Judgement Matrix of Criterion (Kahraman et al., 2004)

	H	QM	QS
DS	1.0	2.0	0.52
IT	0.52	1.0	0.33
FS	2.0	3.0	1.0

Table 7: Aggregated Fuzzy Judgement Matrix of Criterion (Metin and Ihsan, 2008)

	OF	PF	JRF	EF
OF	1.0	3.0	0.34	1.5
PF	0.34	1.0	2.0	1.5
JRF	3.0	0.52	1.0	0.52
EF	0.72	0.72	2.0	1.0

Table 8: Aggregated Fuzzy Judgement Matrix of Criterion (Isaai et al., 2011)

	SWWT	AUWT	MRWJ
SWWT	1.0	0.66	0.55
AUWT	1.56	1.0	0.68
MRWJ	1.86	1.50	1.0

Using the information in Step 1, Step 2, and Step 3, consistency of judgement is computed using (3). The aggregated judgment matrix from Table 6 is used as an example. Determinant of the matrix is computed to obtain the maximum value of lambda. Figure 2 shows part of the computation in obtaining lambda-max.

```
MatrixForm[{{1 - λ, 2.0, 0.5222}, {0.5222, 1 - λ, 0.3397}, {2, 3, 1 - λ}}]

( 1 - λ   2       0.5222 )
( 0.52    1 - λ   0.3397 )
( 2       3       1 - λ  )

Det[( 1 - λ   2       0.5222 )
    ( 0.52    1 - λ   0.3397 )
    ( 2       3       1 - λ  )]

0.069932+ 0.1035 λ + 3 λ^2 - λ^3

Solve[0.069932 + 0.10349999999999993 λ + 3 λ^2 - λ^3 == 0, λ]

{{λ → -0.02079373717616702` - 0.15019840736258405` i},
 {λ → -0.02079373717616702` + 0.15019840736258405` i},
 {λ → 3.041587474352334`}}
```

Figure 2: Example of Computations of lambda-max.

Based on the computation, the value of $\lambda_{max} = 3.04159$. The consistency test for the information in Table 6 is executed as,

$$C.I = \frac{\lambda_{max} - n}{n - 1} = \frac{3.04159 - 3}{2} = 0.02$$

$$\text{C.R.} = \frac{0.02}{0.58} = 0.034$$

With the consistency ratio (*C.R*) less than 0.1, it can be concluded that the matrix judgment in Table 6 is acceptable. Therefore, the TFN in Table 3 is passed the consistency test. The C.R for the aggregated judgment matrix in Table 7 and Table 8 are computed with the similar fashion. It is shown that the consistency ratios for the information in Table 7 and Table 8 are 0.31 and 0.025 respectively. Therefore the matrix judgment in Table 7 is unacceptable to be continued in the next steps of FAHP. In contrast to Table 7, the matrix judgment in Table 8 is acceptable. This acceptable consistency ratio indicates that matrix judgment is appropriate, thereby could be resumed to the next step of FAHP procedures. As to further validate the lambda-max, TFN preference scales from different sources are investigated. With the similar fashion, values of lambda-max and consistency ratios are computed. Table 9 shows the results obtained from testing with different TFN preference scales.

Table 9: Summary of Proposed lambda-max and Consistency Test for different TFN Preference Scale

TFN preference scale	Judgment element (TFN)	λ_{max}	$\lambda_{max} \geq n$	Consistency Ratio, CR	Consistency Test
(Kahraman et al., 2004)	C_1 to C_3	3.0416	Yes	0.03	Passed
	C_1 to C_3	3.0612	Yes	0.05	Passed
	C_1 to C_4	4.3214	Yes	0.09	Passed
	C_1 to C_4	4.3448	Yes	0.10	Passed
	C_1 to A_1... A_3	3.0777	Yes	0.04	Passed
	C_2 to A_1... A_3	3.0729	Yes	0.06	Passed
	C_3 to A_1... A_3	3.1046	Yes	0.09	Passed
	C_1 to A_1... A_3	3.1758	Yes	0.14	Failed
	C_2 to A_1... A_3	3.1758	Yes	0.14	Failed
	C_3 to A_1... A_3	3.2320	Yes	0.20	Failed
	C_4 to A_1... A_3	3.2543	Yes	0.21	Failed
	C_1 to A_1... A_3	3.0165	Yes	0.01	Passed
	C_2 to A_1... A_3	4.2381	Yes	1.06	Failed
	C_3 to A_1... A_3	3.2860	Yes	0.25	Failed
	C_4 to A_1... A_3	3.0165	Yes	0.01	Passed
(Metin and Ihsan, 2008)	C_1 to C_4	5.0568	Yes	0.31	Failed
	C_1 to C_4	4.1428	Yes	0.04	Passed
	C_1 to C_4	5.0567	Yes	0.31	Failed
	C_1 to C_3	3.0659	Yes	0.05	Passed
	C_1 to C_3	3.3445	Yes	0.30	Failed
(Isaai et al., 2011)	C_1 to C_3	3.0293	Yes	0.02	Passed
	C_1 to A_1... A_3	3.0634	Yes	0.05	Passed
	C_2 to A_1... A_3	3.0	Yes	0	Passed
	C_3 to A_1... A_3	3.113	Yes	0.09	Passed

It can be seen that seventeen TFN passed the consistency test whereas nine TFN failed in the consistency test. In cases where the TFN failed in consistency test, it is suggested that the new TFN and matrix judgement should be constructed. The testing on consistency in decision-making is crucial as it provides a gauge not only for DMs but also for the final preference results.

5. Conclusions

Consistency test is one of the important steps in the FAHP. However, this step has been given little attentions by many researchers. Many argue that the FAHP still can be continued even without passing the consistency test and the final decision or preference still can be made. These arguments were totally ignored the definition of $\lambda_{max} \geq n$ proposed by Saaty. In this paper, a simple lambda-max method to check the consistency of TFN in pair-wise comparison of matrix judgement was proposed. Incorporating the concept of matrix determinant in finding lambda-max has contributed a new method in ensuring the FAHP fulfils the consistency test. The feasibility of the proposed method has been tested mathematically by combining the aggregated matrix and matrix determinant using twenty four examples of TFN preference scales. The proposed lambda-max has taken into account the main definition of $\lambda_{max} \geq n$ without loss the generality of the FAHP. Likewise, this simple method of obtaining lambda-max could be easily integrated with the steps of the analytic hierarchy process without neglecting the vital property of the Saaty's analytic hierarchy process and could suitably adapted with any kind of TFN. The consistency test could be re-executed by rescaling of the matrix judgment if the judgment matrix is unacceptable. TFN of the judgement could be redefined as to accommodate the consistency test. The variations in defining TFN based on preference scales have heightened the important contribution of the lambda-max in ensuring consistency of decision in the FAHP. This contribution was evident when the new lambda-max shows its superiority in checking the consistency test of twenty four TFN. The lambda-max has successfully classified the success or the failure of TFN and at the same time preserve the feasibility of general procedures in analytic hierarchy process.

Acknowledgment

We acknowledge the financial support from the Universiti Teknologi MARA (UiTM).

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e ISBN 978-967-2948-12-4

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