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PERFORMANCE OF MORTALITY RATES USING DEEP LEARNING APPROACH

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Mortality has a vital role in population dynamics and is critical in a wide variety of fields, including demography, economics, and social sciences. This study aims to model and compare the mortality rate using two different models; the Lee-Carter model and Deep Neural Network (DNN). The sample data used is the case of the United Kingdom population. Mortality rates were modeled with the Lee-Carter model and deviance goodness of fit were used to test the model's suitability of the data. Next, mortality rates are modeled with the Deep Neural Network (DNN) and both models are compared based on the mean square error (MSE) values. The results showed that the DNN model fits the best. Overall, we conclude that DNN approach appears to be a potential model to model and forecast population mortality.

***Keywords*:** Mortality, Deep Neural Network

1. Introduction

The mortality rate is the ratio of deaths that occur in general due to certain factors in a population with the total population. In this 20th century, the mortality rate for each age group has decreased and at the same time increased life expectancy. The rate of mortality can provide a rough estimate of the probability of death of a person in the population in the future as well as the average future life of a person. Estimates and forecasts of future mortality rates are crucial in insurance companies in determining life insurance premiums. Insurance is a premium-based agreement in which the insurer commits to compensate a specified sum to the policyholder in the event of a loss (Anderson and Brown, 2005). The purpose of insurance is to shield the policyholder financially from any losses. The insurance companies bear the risk covered by the policy while the policy buyer will pay a sum of money known as a premium to the insurance companies for the risk coverage. Estimation of mortality rate plays an important role in determining the premium for insurance products. The entire number of policies experienced will change if the setting of premiums in the insurance policy does not accurately measure (Shinde and Raut ,2018).

Several methods have been proposed in estimating mortality rates and among the model that has been widely applied in predicting mortality rates is Lee and Carter (1992). The Lee-Carter model is the most extensively used worldwide, most likely due to its robustness. The initial version employs singular-value decomposition (SVD) to derive three latent parameters from the log-force of mortality: a constant age component and a temporal component representing the mortality trend multiplied by an age-specific function. Nowadays, the rapid and widespread development in the field of computer science has changed the environment and life in all aspects such as research and the world of work. Numerous studies conducted on the impact of data science on the insurance industry show how the use of high technology can improve risk management by reducing estimates in the rate of losses, claims, reserves and at the same time can increase profits. In this research, we employed deep neural network learning techniques to enhance the mortality rates model's prediction performance and compared with Lee Carter model.

2. Literature Review

Various previous studies have been conducted in estimating and modeling mortality rates. The Lee-Carter model was first introduced by Lee and Carter in 1992 to model and predict the immortality of the United States. Since then, many researchers have introduced several adjustments to the Lee-Carter model to develop forecasting and estimation models on more specific features (Hyndman and Ullah,2007). Another key aspect of the Lee-Carter model is it permits uncertainty in forecasting which is called longevity risk (Kamaruddin and Ismail, 2018). The continuity and development of the Lee-Carter model have been widely used in mortality forecasting and still applied today. More studies regarding Lee Carter could be found in Basnayake and Nawarathna (2017), Chavhan and Shinde (2016) and Taruvinga et al. (2017).

Deprez et al. (2017) have shown that the use of machine learning can improve the estimation of mortality rates in mortality stochastic models such as the Lee-Carter model. They apply the regression tree boosting method in analyzing the weaknesses to estimate the mortality rate on both models and help to improve estimation and modeling based on the factors present in each individual. The application of deep learning in mortality rates also not left behind. Deep learning is part of machine learning. Deep learning is a representation learning technique that constructs complex models using deep hierarchies of learned covariates (Richman, 2018). Richman and Wüthrich (2018) used neural networks to extend the Lee-Carter model to multiple populations. Hainaut (2018) employed neural networks to identify the latent variables of mortality and forecast them using a random walk with drift.

3. Methodology

3.1 Data Sample

The sample data used in this paper is the mortality table for the United Kingdom. This dataset is public dataset and obtained from the human mortality database on the website www.mortality.org (Human Mortality Database). This data covers the mortality rate for each age group and for each year from 1950 to 2016. The age limit is set at the age of 99 years for each year. Our study focus on male dataset from 1950 to 2016. Dataset from 1950 to 2019 is used as training dataset. While dataset from 2000 to 2016 is used as validation dataset.

3.2 Lee-Carter Model

The Lee-Carter model can be defined as the log force of mortality:

$$log(\mu_{x,t}) = \alpha_x + \beta_x \kappa_t \tag{1}$$

with,

$\mu_{x,t}$	= Mortality rate age x in period t
α_x	= Average mortality rate in age x
β_x	= Rate of change in mortality at age x
κ_t	= Mortality rate index in year t

This model is in the form of multiplication component in the equation, $\beta x\ \kappa t$. The accuracy of this model's match with the data used will be determined by the deviance goodness of fit test. The Lee-

Carter model will also be used as a benchmark in comparison to the DNN model to identify the best model choice based on the mean square error (MSE).

3.3 Deviance goodness of fit test

The deviance statistic can also be used to assess how well data is fitted with the model. Deviance, *D* can be defined as follows:

$$D = -2\,(log\,L_f - log\,L_s) \tag{2}$$

$$D \sim \chi^2_{n-p}$$

with,

L_f = the likelihood function of model formed

Ls = the likelihood under the "saturated model"

n = number of data matched

p = number of estimated parameters

Deviance measure the difference between a model estimate and a given data. Deviance has a chi-square distribution with *n , p* degree of freedom. The hypotheses for this match accuracy test are as follows:

H_0 : Model is fit to the data

H_1 : Model is not fit with the data

3.4 Deep Neural Network (DNN)

The neural network model used is the Deep Neural Network (DNN) which consists of several layers of nonlinear functions in producing optimal predictions. In this model, there are two categories of variables that are used as input to the neural network; year and age. Both categories are modeled using the embedding layer which maps each category into the input of the matrix. The structure of the neural network model is briefly described as follows:

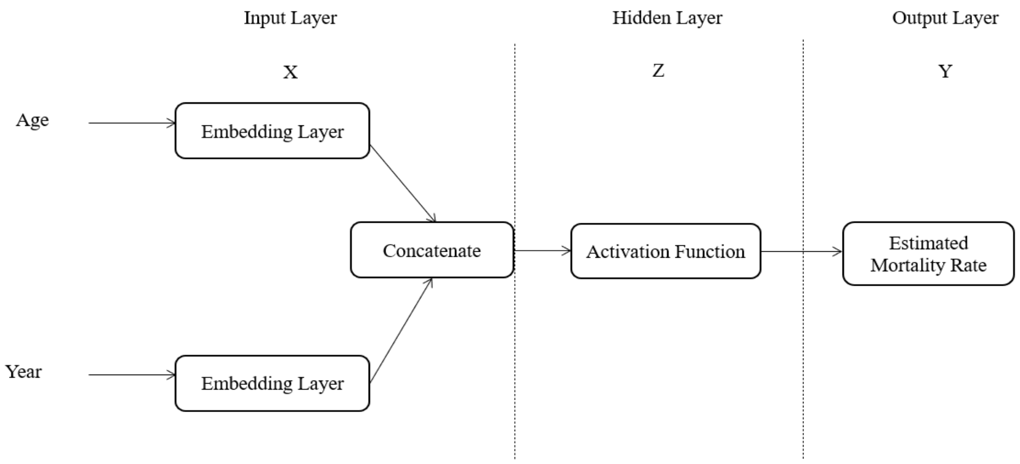

Figure 1 Structure of Deep Neural Network (DNN)

After each category is included in the embedding layer, the vectors of the two variables will be combined into a matrix with the input to the neural network model in predicting mortality rate in year *t*, and age *x*. In detail, the measurement of output will go through several parts in the hidden layer that has activation functions. The number of hidden layers used in this study is three. While activation functions used in this study is two; Rectified Linear Unit (ReLu) for the first layer and second layer and Sigmoid function for the last layer. The equations of the functions found in the hidden layer can be shown as

a. Rectified Linear Unit (Relu)

$$\sigma(x) = max\{0, x\} \tag{3}$$

b. Sigmoid Function

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{4}$$

In general, the process of producing a *Y* output can be written as below:

Suppose $Z^{(i)}$ = matrix in hidden layer-i and $X = Z^{(0)}$ which the input. Therefore,

$$Z^{(1)} = \sigma_1(W^{(0)}X + b^{(0)}) \tag{5}$$

$$Z^{(2)} = \sigma_2(W^{(1)}Z^{(1)} + b^{(1)})$$

$$\dots$$

$$Z^{(i)} = \sigma_i(W^{(i-1)}Z^{(i-1)} + b^{(i-1)})$$

$$\hat{Y}(X) = W^{(i)}Z^{(i)} + b^{(i)} \tag{6}$$

with,

σ_i = Activation Function in i-hidden layer

$W^{(i)}$ = Weighted Matrix

$b^{(i)}$ = Bias

$\hat{Y}(X)$ = Output

There are also methods used in the neural network model that based on the back-propagation approach. Back-propagation is an algorithm for estimating the weights found in neurons in a neural network. This algorithm is used to produce minimal errors in output Y. In the early stages of the learning process, weighting values are given at random, and errors are calculated through subtraction of output results with actual values. Next, the weight of each neuron will be changed based on the given error value.

3.5 Mean Square Error (MSE)

MSE value is used to evaluate the forecasting performance in the mortality model. This value can be described using Equation (7)

$$MSE = \frac{1}{n}\sum_{n=1}^{n}(\mu_{x,t} - \hat{\mu}_{x,t})^2 \tag{7}$$

With,

n = Sample size

$\mu_{x,t}$ = Actual mortality rate

$\hat{\mu}_{x,t}$ = Estimated mortality rate

In this study, the selection of the best models in forecasting mortality rates is based on the lower mean square error (MSE) between the models, Lee-Carter and DNN model.

4. Results and Discussion

4.1 Deviance goodness of fit test

Table 2 Deviance for Lee Carter model

	Deviance	Degree of freedom	P-value
United Kingdom	1.4727	4752	1

Table 2 shows the deviance of the Lee-Carter model from 1950 to 1999. The deviance value is intended to measure the fit of the data match with the model. The smaller the deviance value means the better the data fit with the model. The accuracy test of the deviance match conducted with the deviance is a chi-squared distribution with a degree of freedom 4752. The results of the test performed show that the p-value is greater than the significance level of 0.05 which proves the Lee-Carter model matches the data used.

4.2 Forecasting Mortality

Figure 2 illustrated the estimated rate of mortality for male in 2000. The values for the DNN model seem to be approximately near to the actual value for the male population. The estimation values for the Lee-Carter model seem slightly far for the age between 40 and 50 years old.

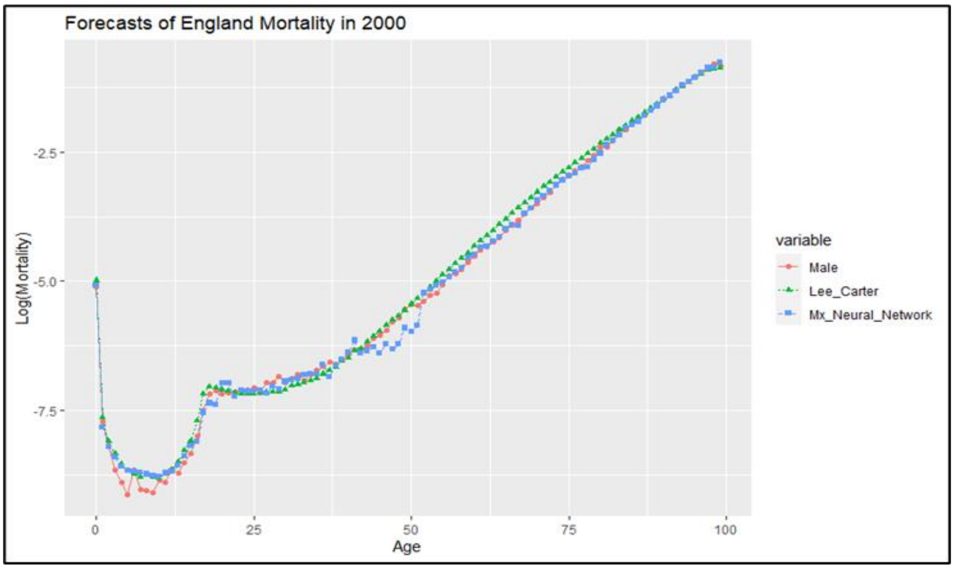

Figure 2: Mortality prediction graph for the United Kingdom in 2000

4.3 Mean Square error

Table 3: Mean Square Error

	Lee-Carter	Deep Neural Network (DNN)
United Kingdom	0.2624	0.0982

The table above shows the results of the mean square error demonstrated by both models on the mortality force for the year from 2000. United Kingdom's mortality rates show the low mean square error value on the DNN model compared to the Lee-Carter model and can be concluded that the DNN model is the best.

5. Conclusions

The objective of mortality study is expected to be more on improving socio-economics through data analysis and advanced methodology for accurate results. In this research, mortality modeling and forecasting were performed using Lee-Carter and DNN models. The results show that the DNN model successfully generates the lower value of mean square errors and can accurately project mortality in United Kingdom This can conclude DNN has a good potential to be developed and employed for mortality rates modeling in the future.Our future study is to apply and compare Lee-Carter and DNN models for another countries.

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