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ROTATING FLOW OF A NANOFLUID PAST A NONLINEARLY SHRINKING SURFACE WITH FLUID SUCTION

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An investigation is considered to examine the rotating boundary layer flow and heat transfer past a nonlinear shrinking surface in a nanofluid by taking into consideration the influence of the suction effect at the surface. Three kinds of nanomaterials namely titanium dioxide, copper, and aluminum oxide are considered. The governing equations in the form of partial differential equations (PDEs) for momentum and energy are converted into a system of ordinary differential equations (ODEs). The shooting technique built in the MAPLE program is applied to the resulting system of equations. The impacts of the embedded parameters which include rotation, nanomaterial volume fraction, nonlinear and suction on the velocities, temperature, coefficient of skin friction and the heat transmission rate are plotted graphically and have been discussed further. The outputs showed that the presence of rotation in the flow rises the coefficient of the skin friction as well as the heat transmission rate. It is also noticed that the nonlinear parameter accelerates the boundary layer separation in which the dual solution unites.

Keywords: Rotating flow, Nanofluid, Nonlinear shrinking surface, Permeable surface

1. Introduction

The consideration of fluid flow and thermal energy transport towards stretching and shrinking surfaces has gotten an amount of interest and response from many authors a few decades ago. The first researcher who attempted to study the flow past a moving surface is Sakiadis (1961) where the uniform velocity in an ambient temperature is considered. Following that, there are many articles in the published works for the boundary layer flow past various surfaces in several fluids (Ishak et al., 2009; Pop et al., 2016; Salleh et al., 2019, 2020; Khashi'ie et al., 2020, 2021; Wahid et al., 2021). Nowadays, the rotating fluid phenomena becoming one of the crucial subjects in fluid dynamics due to vital features of certain applications in industrial and engineering purposes. Such applications including, drying and cooling of papers, cooling of the metal in cooling bath, fiber spinning and plastics extrusion process. The problem of the rotating nanofluid adjacent to a linearly stretching plate is conducted by Nadeem et al. (2014). Nadeem et al. (2014) is the first researcher who took an opportunity to consider the rotating flow in a nanofluid. Nanofluid had many applications, for instance, it is used in imaging of cancer diagnosis and deliverance of drugs for cancer treatment, whilst in the engineering field, it has the tendency to productively control the thermal energy in electronic devices by eliminating the high heat flux (Saidur et al., 2011; Huminic and Huminic, 2012; Sajid and Ali, 2019).

Motivated by the work of Nadeem et al. (2014), Salleh et al. (2016) continued to study the rotating nanofluid flow by considering a linearly permeable shrinking sheet. Thereafter, the study of the rotating flow with the influences of partial slip and radiation past a stretched surface immersed in Ag-CuO/H₂O hybrid nanofluid is analyzed by Hayat et al. (2018a). Later on, Nasir et al. (2018) considered the rotational flow of a nanofluid towards a stretched surface containing single-walled carbon nanotubes (SWCNTs) with the impact of radiative heat. It is found from the study that skin friction reduces with a higher nanoparticle volume fraction. Muhammad et al. (2018) proposed a problem of rotational flow of Casson fluids due to a stretched sheet by taking into consideration both single and multi-walled carbon nanotubes (SWCNTs and MWCNTs) as nanomaterials in the presence of heat generation and radiative heat. Just recently, Anuar et al. (2021) addressed the radiative hybrid

nanofluid flow adjacent to a rotating stretched or shrunk surface with suction effect. Their study revealed that the increment of copper nanomaterial volume fractions increases the coefficient of skin friction, while it declines the rate of thermal energy transfer.

All the physical situations described in the above-mentioned papers, however, dealt with the problems of linear stretching and shrinking surface only. In contrast to linear stretching or shrinking case, another important physical phenomenon is the case where the surface is being stretched or shrunk in a nonlinear fashion. The analysis of the flow passing through a nonlinearly stretched sheet in a nanofluid was first discussed by Rana and Bhargava (2012). Later on, Das (2015) studied the boundary layer flow of a nanofluid past a nonlinear permeable stretching surface with the presence of partial slip. Hayat et al. (2016) performed the second-grade nanofluid flow induced by a nonlinearly stretching plate with a magnetic field considering the Buongiorno nanofluid model. Hayat et al. (2018b) considered the boundary layer flow with nonlinear stretching velocity over a curved surface in a nanofluid. They found that the addition of volume fraction of Ag nanomaterials diminishes the fluid velocity. Furthermore, the flow problem with chemical reaction and heat generation or absorption towards a nonlinearly stretched surface is examined by Eid et al. (2020) considering Carreau nanofluid in a porous medium. In the latest published work by Abbas et al. (2021), they analyzed the stagnation point flow near a moving cylinder in a hybrid nanofluid with an inclined magnetic field effect.

However, the flow dynamics caused by a nonlinear shrinking surface in a nanofluid with suction and rotation effects have not been considered yet by any researcher. Therefore, the novelty of this work is to perform the rotational flow towards a nonlinearly shrinking surface in a nanofluid with the influence of suction at the wall. The considered system of equations is facilitated by using the similarity transformations in nonlinear form. The findings of this study are anticipated to give some insight to engineers for designing applications related to the thermal removal process.

2. Problem Modeling

The steady laminar fluid flow and thermal energy transfer adjacent to a permeable nonlinearly shrunk surface in a rotating nanofluid is performed and given as in Figure 1. From the figure, x , y and z are Cartesian coordinates where x and y are measured in the plane $z = 0$ and the fluid occupying the half-space at $z \geq 0$. The surface is presumed to rotate at an angular velocity $\bar{\Omega} = \Omega ax^{n-1}$ in the z -direction. The surface is also being shrunk in the x -direction with velocity $U_s(x) = ax^n$ where $a < 0$ is a shrinking constant. Meanwhile, the constant n is referred to nonlinear parameter such that $n = 1$ for linear case, while $n \neq 1$ for nonlinear case. The fluid motion is three-dimensional due to the appearance of the Coriolis force. The governing equations in this rotating frame are

$$u_x + v_y + w_z = 0, \quad (1)$$

$$\rho_{nf} (uu_x + vv_y + ww_z - 2\bar{\Omega}v) = \mu_{nf}u_{zz}, \quad (2)$$

$$\rho_{nf} (uv_x + vv_y + wv_z + 2\bar{\Omega}u) = \mu_{nf}v_{zz}, \quad (3)$$

$$uT_x + vT_y + wT_z = \alpha_{nf}T_{zz}. \quad (4)$$

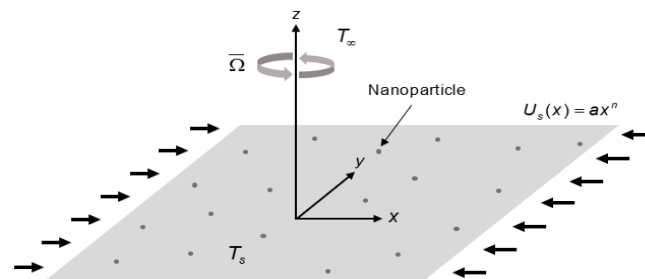


Figure 1: Geometric of the flow.

The boundary restrictions subjected to (1)–(4) are

$$\begin{aligned}
 u &= U_s(x), \quad v = 0, \quad w = -\sqrt{\frac{a\nu_f(n+1)}{2}}x^{\frac{n-1}{2}}s, \quad T = T_s \quad \text{at } z = 0, \\
 u &\rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } z \rightarrow \infty.
 \end{aligned}
 \tag{5}$$

From (1)–(4), u , v and w are velocities in the x , y and z directions, μ_{nf} is the effective dynamic viscosity of nanofluid, ρ_{nf} is nanofluid density, α_{nf} is nanofluid thermal diffusivity and T is the liquid temperature. Meanwhile, in (5), ν_f is the fluid kinematic viscosity, s is the mass flux parameter with $s < 0$ for injection and $s > 0$ for suction, T_s is the surface temperature and T_∞ is ambient temperature. The thermo physical relations of nanofluid are shown below:

$$\begin{aligned}
 \frac{k_{nf}}{k_f} &= \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \frac{\mu_{nf}}{\mu_f} = \frac{1}{(1 - \varphi)^{2.5}}, \\
 \frac{\rho_{nf}}{\rho_f} &= 1 - \varphi + \varphi \left(\frac{\rho_s}{\rho_f} \right), \quad \frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = 1 - \varphi + \varphi \frac{(\rho C_p)_s}{(\rho C_p)_f}.
 \end{aligned}
 \tag{6}$$

In (6), k is the thermal conductivity, ρC_p is the volumetric heat capacity at uniform pressure and φ is the nanomaterial volume fraction parameter. The subscripts 'f' and 's' are denoted by 'fluid' and 'solid nanomaterials', respectively.

The similarity transformation approach is applied to yield the ordinary differential equations. Therefore, the following parameters are proposed:

$$\begin{aligned}
 u &= ax^n f'(\eta), \quad v = ax^n h(\eta), \quad w = -\sqrt{\frac{a\nu_f(n+1)}{2}}x^{\frac{n-1}{2}} \left[f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right], \\
 \eta &= \sqrt{\frac{a(n+1)}{2\nu_f}}x^{\frac{n-1}{2}}z, \quad \theta(\eta) = \frac{T - T_\infty}{T_s - T_\infty},
 \end{aligned}
 \tag{7}$$

in which prime (') is the differentiation in respect of similarity variable η .

By using the above transformations (7), the continuity equation (1) is contented and the momentum and energy equations in (2)–(4) are reduced to:

$$\frac{f'''}{(1 - \varphi)^{2.5} [(1 - \varphi) + \varphi(\rho_s/\rho_f)]} + f f'' - \frac{2n}{n+1} f'^2 + \frac{4\Omega}{(n+1)} h = 0,
 \tag{8}$$

$$\frac{h''}{(1 - \varphi)^{2.5} [(1 - \varphi) + \varphi(\rho_s/\rho_f)]} + f h' - \frac{2n}{n+1} f' h - \frac{4\Omega}{(n+1)} f' = 0,
 \tag{9}$$

$$\frac{(k_{nf}/k_f)}{\text{Pr} [(1 - \varphi) + \varphi(\rho C_p)_s / (\rho C_p)_f]} \theta'' + f \theta' = 0,
 \tag{10}$$

associate with the following conditions

$$\begin{aligned}
 f(0) &= s, \quad f'(0) = -1, \quad h(0) = 0, \quad \theta(0) = 1, \\
 f'(\infty) &\rightarrow 0, \quad h(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0.
 \end{aligned}
 \tag{11}$$

In the above equations (8)–(10), $\Omega = \bar{\Omega}/ax^{n-1}$ is the rotation parameter and $\text{Pr} = \nu_f/\alpha_f$ is the Prandtl number.

The important quantities involved in the current work are the coefficients of skin friction along x and y axes, Cf_x and Cf_y and the local Nusselt number Nu_x which are formulated as:

$$Cf_x = \frac{\tau_{xz}}{\rho_f(ax^n)^2}, \quad Cf_y = \frac{\tau_{yz}}{\rho_f(ax^n)^2}, \quad Nu_x = \frac{xq_s}{k_f(T_s - T_\infty)},
 \tag{12}$$

where τ_{xz} and τ_{yz} are the shear stresses of x - and y -components, and q_s is the heat flux given by

$$\tau_{xz} = \mu_{nf} (u_z) |_{z=0}, \quad \tau_{yz} = \mu_{nf} (v_z) |_{z=0}, \quad q_s = -k_{nf} (T_z) |_{z=0} \quad (13)$$

Substituting (6), (7) and (13) into (12), the following equations are obtained.

$$\begin{aligned} (Re_x)^{\frac{1}{2}} Cf_x &= \frac{f''(0)}{(1-\varphi)^{2.5}} \sqrt{\frac{n+1}{2}}, \quad (Re_x)^{\frac{1}{2}} Cf_y = \frac{h'(0)}{(1-\varphi)^{2.5}} \sqrt{\frac{n+1}{2}}, \\ (Re_x)^{-\frac{1}{2}} Nu_x &= -\frac{k_{nf}}{k_f} \theta'(0) \sqrt{\frac{n+1}{2}}, \end{aligned} \quad (14)$$

where $Re_x = ax^{n-1}/\nu_f$ is the local Reynolds number.

3. Analysis of Findings

The obtained outcomes of the skin friction coefficients, local Nusselt number, velocities and temperature distributions have been provided graphically in Figures 2–5 concerning the relevant parameter of interest, including, rotation Ω , nanomaterial volume fraction φ , nonlinear n and suction s parameters. The physical characteristics of nanomaterials and the fluid used are provided in Table 1. In this study, all the computations are performed for a broad range of values of the embedded parameters and a fixed value of $Pr = 6.2$ (water). Equations (8)–(10) with conditions (11) are executed via a shooting technique in MAPLE program. Since (8)–(11) are in the form of a two-point boundary value problem (BVP), the function of this technique is to convert the BVP to an initial value problem (IVP). The technique is capable to give solutions to the BVP by identifying the proper initial conditions for a related IVP. Details explanation on this technique can be found in the works of Bhattacharyya and Layek (2011) and Bhattacharyya et al. (2011). In applying the method, an appropriate bounded value of η , say η_∞ needs to be chosen which relies on the values of the variables considered. The present results of the local heat flux $|\theta'(0)|$ are compared with the previous works by Rana and Bhargava (2012) and Das (2015) by setting $f(0) = 0$ and $f'(0) = 1$ (stretching case) in the boundary conditions (11) and $\Omega = \varphi = 0$. These comparison values shown excellent agreement, hence the results obtained for the shrinking case are also accurate.

Table 1: Thermo physical features of nanomaterials and base fluid (Oztop and Abu-Nada, 2008).

Properties	C_p (J/kg K)	ρ (kg/m ³)	k (W/mk)
Cu	385	8933	400
Al ₂ O ₃	765	3970	40
TiO ₂	686.2	4250	8.9538
Base fluid	4179	997.1	0.613

The impact of rotation parameter Ω on the shear stress of x - and y -components, $f''(0)$ and $h'(0)$ and the local heat flux $-\theta'(0)$ versus s are given in Figure 2 for Cu nanoparticle. It is observed that the shear stress of both velocity components and the local heat flux increase as the rotation rate enhance. Physically, the presence of rotation in the flow leads to the occurrence of friction at the surface. In addition to that the presence of a high rotation rate also diminishes both momentum and thermal layer thicknesses, and as a consequence, enhancing the friction force and thermal energy flux on the wall. It is noticed that the multiple solutions, namely dual solutions appear when Ω takes the lowest value that is $\Omega = 0.04$. Visibly, dual solutions appear in a certain region of $s_c < s \leq 2.18$ where s_c represents the turning point that connects first and second solutions.

Table 2: Comparison values of the local heat flux $|\theta'(0)|$ when the boundary conditions (11); $f(0) = 0$ and $f'(0) = 1$ and $\Omega = \varphi = 0$.

Pr	n	Rana and Bhargava (2012)	Das (2015)	Present results
1	0.2	0.6113	0.610571	0.610202
	0.5	0.5967	0.595719	0.595201
	1.5	0.5768	0.574525	0.574730
5	0.2	1.5910	1.607130	1.607787
	0.5	1.5839	1.586190	1.586782
	1.5	1.5496	1.557190	1.557695

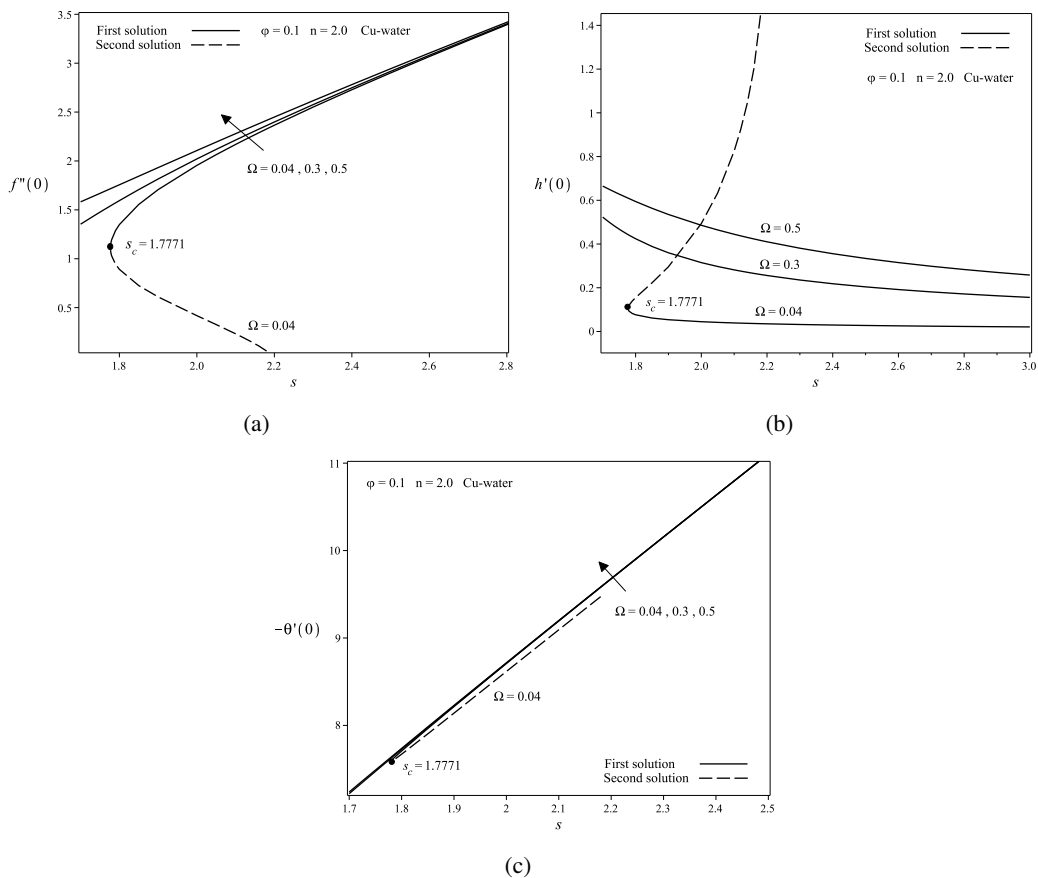


Figure 2: Influence of rotation on (a) $f''(0)$ (b) $h'(0)$ and (c) $-\theta'(0)$ versus s for Cu nanoparticle.

The impact of nonlinear parameter n on the shear stress of x - and y -components and the local heat flux against s are shown in Figure 3 for Cu nanomaterial. It is noticed that when the nonlinear parameter n augments, the numerical values of the shear stress of both velocity components and the heat transfer reduce. Besides, an increase in the parameter n also augment the critical values of s for both solutions meet which is from $s_c = 1.7771$ to $s_c = 1.8666$. This implies that the imposition of a higher value of n faster the boundary layer separation in the flow. Another factor that contributes to this situation is the increment of both momentum and thermal layer thicknesses at the shrinking surface. It is also noted that when $n = 2.0$, the second solution only appears up to $s = 2.18$. This happens due to the considered values for certain parameters used in this work.

The coefficients of skin friction of x - and y -components and the rate of heat transmission versus nanomaterial volume fraction φ for Cu, Al_2O_3 and TiO_2 are plotted as in Figure 4. One can see that

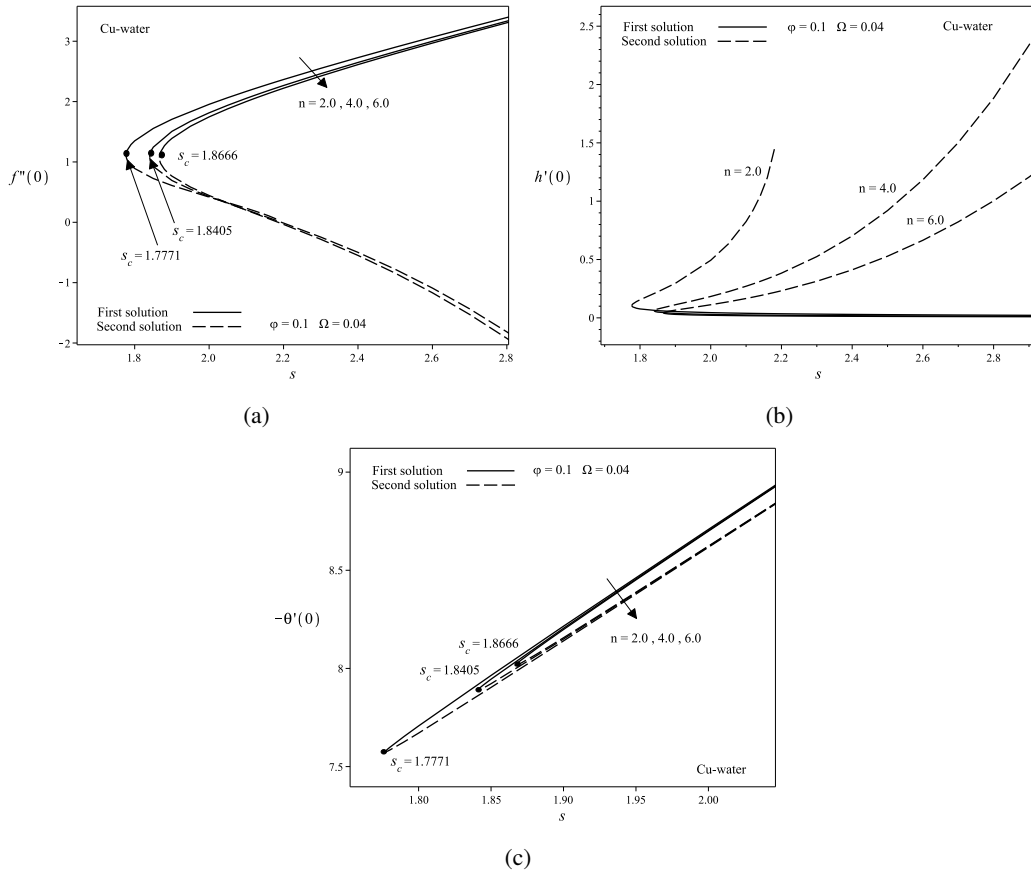


Figure 3: Influence of nonlinear rate on (a) $f''(0)$ (b) $h'(0)$ and (c) $-\theta'(0)$ versus s .

the increment in the parameters Ω and φ enhance the coefficient of the skin friction for both velocity components. The larger value of the skin friction coefficient is because of the increment in the shear stress at the surface when the rotation rate increase (see Figure 2). The imposition of a higher rate of nanoparticle volume fraction causes more collisions between suspended nanomaterials and the base fluid particles that enhance the drag force to occur on the shrinking wall, and as a result, increases the skin friction coefficient for both velocity components. Additionally, Cu has the highest values of skin friction coefficients accompanied by TiO_2 and Al_2O_3 . Other than that, the thermal energy transmission rate seems to maximize with the greater value of the rotation parameter. Such a situation takes place due to the presence of rotation in the flow that helps to speed up the transmission of heat from the shrunk wall to the surrounding fluid. From Figure 4a, it is seen that the thermal energy transmission rate diminishes as the parameter φ enhance. The major reason for this is that when the rate of nanomaterial volume fraction getting higher, the temperature and its thermal layer thickness enhance. The thickening in the thermal layer thickness complicates the transfer of heat into the fluid, thus, decreasing the heat transmission rate. Furthermore, it is noticed in Figure 4(c) that Cu has the greatest values of heat transfer rate compared to Al_2O_3 and TiO_2 . This follows the fact that Cu has greater thermal conductivity compared to others.

The variation of velocities, $f'(\eta)$ and $h(\eta)$ and the temperature $\theta(\eta)$ distributions are displayed in Figure 5 for multiple values of n . It is clearly observed from the figures that the attained profiles satisfied the requirement of the endpoint boundary restrictions (11) asymptotically. Hence, it can be concluded with confidence that the computational outcomes obtained in this research are accurate. Predictably, the thickness of the boundary layer for the first solution is thinner as opposed to the second solution.

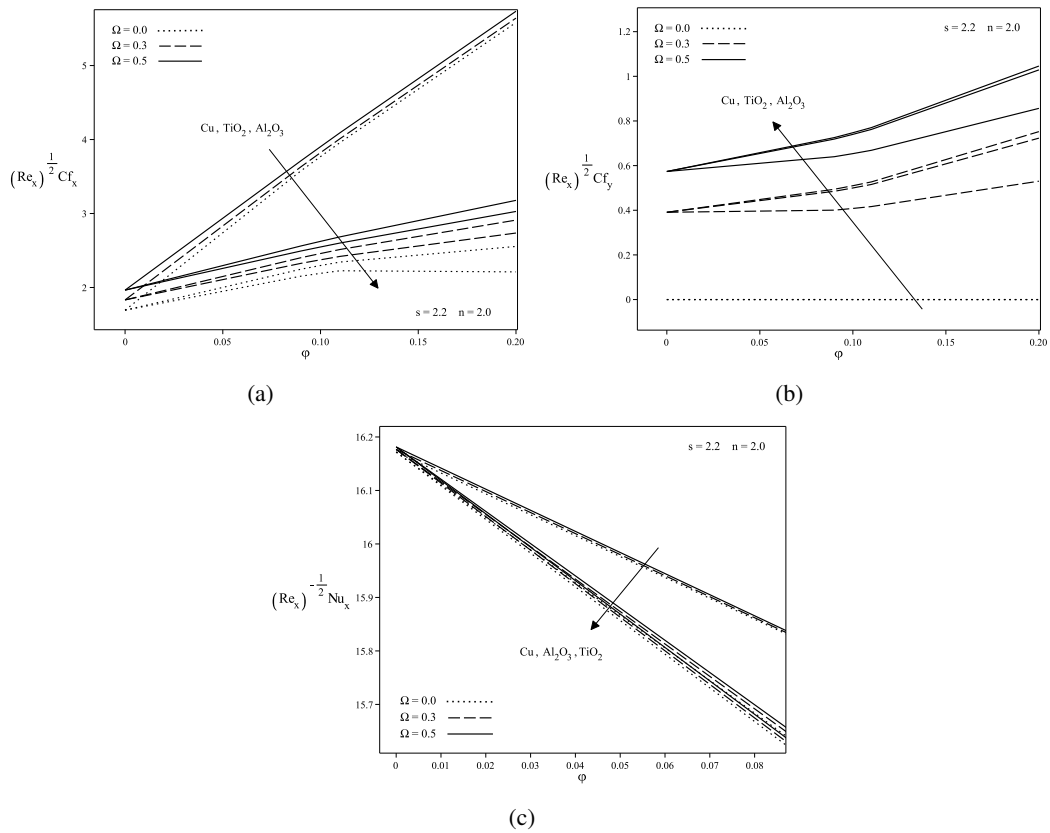


Figure 4: Influence of rotation and nanomaterials on (a) skin friction coefficient of x -component, (b) skin friction coefficient of y -component and (c) heat transmission rate versus φ .

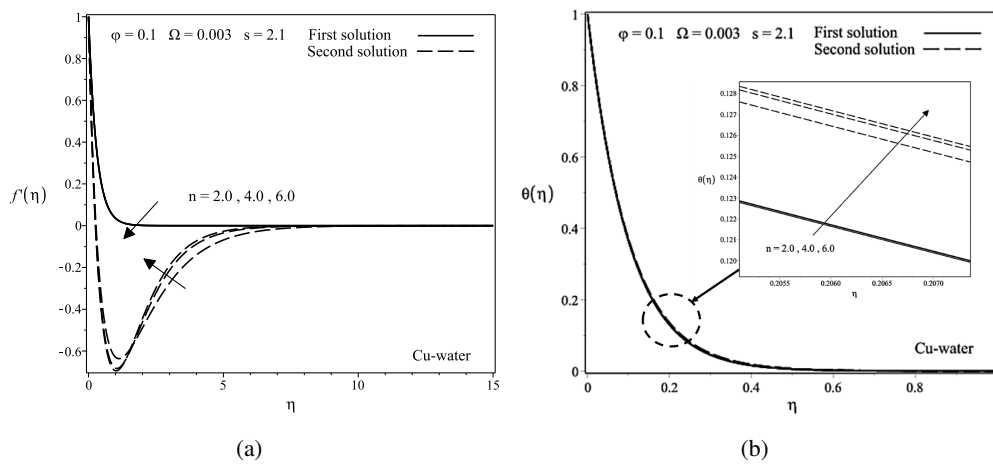


Figure 5: Influence of nonlinear rate on (a) velocity field of x -component and (b) temperature field for Cu-water.

4. Concluding Remarks

A computational study is discussed for the rotational flow of a nanofluid near a nonlinearly shrinking surface with the effect of mass suction at the boundary. The present study is performed to explore the impact of rotation, nanomaterial volume fraction, suction and nonlinear variables on the flow and thermal energy transfer. The following observations are highlighted:

1. The presence of rotation boost the coefficient of skin friction and heat transmission rate.
2. The enhancement of nonlinear rate accelerates the boundary layer separation where the dual solution meets.
3. The imposition of nanoparticles in the flow rises the skin friction coefficients, whilst it reduces the heat transmission rate at the wall.
4. The dual solutions show up when the rotation parameter takes the lowest value that is $\Omega = 0.04$ and when the value of suction exceeds a particular value; $s > 1.7771$.
5. Copper has the highest coefficient of skin friction and heat transmission rate compared to aluminum oxide and titanium oxide.

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