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Exploring Two Methods of Partial Fraction Decomposition on Students' Performance

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ABSTRACT

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Different partial fraction decomposition (PFD) methods may drive students to explore and understand partial fractions and thus, improve their mastery in PFD performance. Hence, this study explored the effectiveness of using two different methods, namely the improved version of the Heaviside method and the undetermined coefficients method, in performing PFD of the proper partial fraction. Literature showed that most of the instructors employed the undetermined coefficients method, and little is known about the effectiveness of employing other methods on students' performance. This study used a quasi-experimental approach with a pre-test and post-test interval. Purposive sampling was employed as all the participants are from science stream, have completed Calculus I course, and learnt PFD. A total of 148 undergraduates from two faculties of a Malaysian public university were purposefully chosen for this study. The pre-test and post-test scores of PFD for three categories of factors in the denominator using the two methods were collected. Then, the statistical results of pre-test and post-test were examined using IBM SPSS 21. The mean scores of the tests were analysed using paired sample t-tests and analysis of covariance. The findings revealed that students who used the improved version of the Heaviside method outperformed those who used the undetermined coefficients method in performing the PFD of proper rational functions for distinct linear factor and irreducible quadratic factor in the denominators. However, the performance for both methods was insignificantly different for solving PFD of proper rational functions concerning repeated linear factor in the denominator. This study provides valuable insights into the choice of PFD methods employed by instructors in bringing out the best in students' performance.

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1. INTRODUCTION

Partial fraction decomposition (PFD) is the process of decomposing a complex rational fraction into the sum of simple rational fractions. PFD is thus the reverse of the summation of simple rational fractions. In learning elementary integral calculus, PFD is the initial step of computation before integrating. It is usually easier to integrate simple rational fractions than to integrate complex rational functions. Performing the PFD computation effectively with high numerical accuracy is often the primary concern in PFD learning. Though numerous algorithms or approaches to decompose certain types of rational functions into partial fractions are available, not all of them are suitable for manual calculation. Fundamental knowledge of certain methods used to complete PFD computation is also needed. For instance, the PFD coefficient can be found by using Taylor polynomial computation (Kwang & Xin, 2018) but to the knowledge of the researchers in this study, students must be well-versed in the divide-and-conquer method to perform this computation effectively. In addition, the PFD coefficient can also be found by employing repeated synthetic division (Kim & Lee, 2016). However, to our knowledge and based on our observations, students need to apply repeated synthetic division. Furthermore, the PFD coefficient can be found using the differentiation method (Özyapici & Pintea, 2012) but students must have a strong basic knowledge of differentiation to conduct the computation. Although Wang (2007) proposed a set of PFD formulas that can be used for immediate integration, students must have superior memorization reasoning to memorize the formulae needed to perform the computation.

The theoretical and empirical literature review shows that students are often introduced to use the undetermined PFD method in solving PFD coefficients at schools and even higher learning institutions. (e.g., William, 2018; Manoj, Ashvini, & Hole, 2020; Kwang & Xin, 2018). The undetermined coefficients PFD method largely emphasizes the use of the algebra approach for solving PFD coefficients. However, students who are not proficient in applying basic algebraic concepts for solving PFD coefficients will eventually end up with a poor performance in PFD. Furthermore, it has an impact on integrating proper rational fractions when students make mistakes or use incorrect PFD coefficients. Hence, students tend to lose marks in this whole process of performing PFD and integrating proper rational fractions which will ultimately affect their overall performance.

The concerns and awareness of the limitations discussed above, specifically in employing undetermined coefficients PFD method to solve PFD coefficients have prompted the researchers of this study to look for other PFD methods that can lead to optimal student performance. Therefore, a complete computation of the improved Heaviside PFD method introduced by Man (2012) was explored in this study. This method uses the formulation of simple polynomial division and the substitution concept to obtain PFD coefficients. The fewer steps required in this method help reduce students' errors in computation and prompt them to obtain accurate solutions as compared to using the undetermined coefficients PFD method. To obtain insights into the two PFD methods chosen for this study, we explore the effectiveness of these two PFD methods on students' PFD performance of the proper rational function.

2. LITERATURE REVIEW

Several computation methods of decomposing a rational function into partial fraction have been broadly employed in the application of calculus, differential equations, control theory and some areas of pure or applied mathematics (Kwang & Xin, 2018; Manoj et al., 2020; Kim & Lee, 2016; Ma et al., 2014; Bradley & Cook, 2012; Man, 2012; Özyapici & Pintea, 2012). However, it is observed that two PFD coefficients computation methods that are more commonly used, namely the undetermined coefficients method and the cover-up method. (e.g., William, 2018; Kim & Lee, 2016; Ma et al., 2014; Man, 2012). According to Linner (1974, cited in Ma et al., 2014), the well-known cover-up method always serves as a basis for other PFD methods and provides a compact solution to PFD problems. This, however, has a limitation when it comes to the evaluation of high-order poles in high-order polynomials as it could result in huge numerical errors when the successive differentials procedures increase (Ma et al., 2014; Kwang & Xin, 2018). Another standard PFD method, namely the undetermined coefficients method, requires the construction of a system of equations by matching up the variables after removing the fractions form from the combination of partial fractions and a proper rational function using the least common denominator procedure and resolving of the resultant system of equations to obtain PFD coefficients. It can be a very lengthy, complicated, and inconvenient computation when decomposing more than two partial fractions (Wang, 2007; Gupta, 2011, Man, 2012). Therefore, there is a higher possibility for students to make more arithmetic mistakes in this whole process of computation.

The extant literature shows that many students have difficulties solving questions that are associated with the concepts of fractions and algebraic expressions. Titus' (2010) study reported 35% to 42% of the college students enrolled in development mathematics course committed error patterns in the real number computations because most of the students have an unclear understanding of signed number arithmetic, fractions, distributive property, as well as exponential errors. Moreover, Brown and Quinn's research (2006) discovered that more than half of the 143 ninth graders who enrolled in an elementary algebra course at an upper-middle-class school showed a lack of experience and had low proficiency in both fraction concepts and computations. In addition, Bentley and Bossé's (2018) study supported Gabriel et al.'s (2013) finding that college students committed mistakes in wrongly applying fraction operations, as seen in elementary students' misunderstandings and misconceptions. Hanson and Hogan (2000) who examined the computational estimation skills of 77 college students majoring in a variety of disciplines discovered that many students struggled with the process of obtaining common denominators. They also highlighted that few students in the lower performing groups, added or subtracted the numerators and denominators but failed to find common denominators. Furthermore, Steen (2007) emphasized that even adults were found confused if a problem requires anything in the simplest of fractions. Considering the above findings, students' difficulties with fraction concepts are found to be partly responsible for failure in finding PFD coefficients using undetermined coefficients method computation. Hence, many instructors seek alternative methods that could increase the accessibility of the PFD method for students who are weak in concepts of fractions.

Another error pattern, namely difficulties with algebraic equations and arithmetical computation in schools has also been well-documented. The difficulties are related to the inability to see the algebraic structures of the tasks, inadequate conceptual knowledge of the problem, a lack of manipulative expression skills, calculation mistakes, and technical errors (Taban & Cadorna, 2018). In addition, algebra's structure sense is said to be a part of students' difficulties. The difficulties in structure sense include using arithmetical operations in numerical and algebraic expressions, understanding the notion of variables, algebraic expressions, as well as determining the meanings of the equal sign and mathematization (Jupri et al., 2014; Hoch & Dreyfus, 2010). It is also reported that students with high-performance mathematics in secondary schools also had difficulty with algebraic manipulation. They struggled to formulate equations by manipulating correct algebraic expressions; they had weak arithmetic skills; and they made arithmetic errors which caused them to make algebraic errors (Novotna & Hoch, 2008). Another finding shows that students find it difficult to apply previously learnt algebraic techniques (Matzin & Shahrill, 2015). The in-depth analysis on school-aged students' errors in algebra problem solving conducted by Booth and colleagues (2014) reveals six common errors made: variable errors, negative sign errors, equality or inequality errors, operation errors, mathematical properties errors, and fraction errors. Moreover, Ashlock (2010) in his analysis of error patterns made by students discovered that, school-aged students often have misconceptions and make procedural errors in both mathematical operations and methods of computations. These error analyses highlight the most crucial computational mistakes committed by students prior to obtaining the final PFD coefficients when undetermined coefficients method computation was being carried out.

Concerning the above discussions, many PFD methods were proposed to complement the undetermined coefficients methods commonly and widely employed by instructors. Some of the methods are found to perform better than undetermined coefficients methods under specific conditions. For example, some methods are more suitable for small-scale problems, but they may become complicated when used for large-scale problems. In Man's (2007) research, he proposed a Heaviside's cover-up method, which requires simple substitutions to find partial fraction coefficient with single poles and apply successive differentiation for multiple poles. Man (2012) subsequently proposed an improved version of Heaviside's approach to compute partial fraction coefficients by using simple substitutions and polynomial divisions. This method does not require solving the complex roots of the quadratic polynomial, differentiation, or the solution of a system of linear equations for the PFD of a proper rational function. Its simplicity and applicability in applied and engineering mathematics as recommended by several researchers (e.g., William, 2018; Manoj et al., 2020; Man, 2012) to employ this improved method in teaching integrals of proper rational functions have compelled the researchers of this current study to explore the potential application of this method on teaching undergraduate students as an alternative method to the undetermined coefficients method in finding the PFD coefficients.

To further examine students' understanding in applying partial fraction decomposition method, the effectiveness of applying the improved version of the Heaviside PFD method and the undetermined coefficients PFD method on their PFD performance is explored. Thus, the research question of this study is: Which application of PFD method (the improved version of Heaviside PFD method or the undetermined coefficients PFD method) improve students' performance? The following Null Hypotheses were developed to answer the research question:

1. The improved version of Heaviside method has no effect on students' PFD performance.

2. The undetermined coefficients method has no effect on students' PFD performance.

3. There is no significant difference between students' PFD performance taught with improved version of the Heaviside method and those taught with undetermined coefficients method.

2.1 Partial Fraction Decomposition

A brief description of a partial fraction decomposition is presented in the next page:

Assume that G is a constant field comprises two polynomials, $\mathrm{W}(x)$ and $\mathrm{S}(x)$. A proper rational function is $\mathrm{G}(x)=\frac{\mathrm{W}(x)}{\mathrm{S}(x)}$, where the degree of $\mathrm{W}(x)$ is lower than the degree of $\mathrm{S}(x)$ and $\mathrm{S}(x)=\prod_{i=1,l=1}^{i=m,l=n}\left(x-\mathrm{a}_i\right)^{j_i}\left(x^2+\mathrm{b}_l\,x+\mathrm{c}_l\right)^{k_l}$, $\mathrm{a}_i,\mathrm{b}_l,\mathrm{c}_l$ are constants with $\mathrm{b}_l{}^2-4\mathrm{c}_l<0$, $\mathrm{S}(x)$ never be 0 and belongs to G, and i,j,k,l,n,m are positive integers.

A partial fraction decomposition of G(*x*) is:
$\mathrm{G}(x)=\sum_{i=1}^{m}\sum_{t=1}^{j_i}\frac{\mathrm{A}_{it}}{\left(x-\mathrm{a}_i\right)^t}+\sum_{l=1}^{n}\sum_{t=1}^{k_l}\frac{\mathrm{B}_{lt}\,x+\mathrm{C}_{lt}}{\left(x^2+\mathrm{b}_l\,x+\mathrm{c}_l\right)^t}$, where $\mathrm{A}_{it},\mathrm{B}_{lt},\mathrm{C}_{lt}$ are coefficients constants with *t* representing positive integers.

Two methods of PFD were used in this study to compute the unknown coefficients $\mathrm{A}_{it},\mathrm{B}_{lt},\mathrm{C}_{lt}$ and followed the procedure as shown below:

2.1.1 The Improved Version of Heaviside Method

For distinct and repeated linear polynomial denominator, assume that B_{lt} and C_{lt} are zeros, and multiplying the equation of G(*x*) with $\left(x-\mathrm{a}_i\right)^t$, and replacing *x* with a_i to get coefficient -*i,t* of A, polynomial division, $\mathrm{A}_{it}=\frac{\mathrm{W}(x)}{\mathrm{S}(x)}\left(x-\mathrm{a}_i\right)^t\,|_{x=\mathrm{a}_i}$ is obtained. In order to obtain the next coefficient -*i,t-ji* of A in the polynomial division, the known partial fractions are subtracting from F(*x*), $\mathrm{A}_{i(t-j_i)}=\left(\frac{\mathrm{W}(x)}{\mathrm{S}(x)}-\sum_{k=0}^{j_i-1}\frac{\mathrm{A}_{i,j_i-k}}{\left(x-\mathrm{a}_i\right)^{j_i-k}}\right)\left(x-\mathrm{a}_i\right)^{t-j_i}\,|_{x=\mathrm{a}_i}$ or using the straightforward process, $\mathrm{A}_{i(t-j_i)}=\frac{\mathrm{W}_{j_i}(x)}{\mathrm{S}_{j_i}(x)}\left(x-\mathrm{a}_i\right)^{t-j_i}\,|_{x=\mathrm{a}_i}$. This process would be progressing until all the unknown coefficients A_{it} are found.

For irreducible quadratic polynomial denominator, assume that A_{it} is zero and multiplying the equation of G(*x*) with $\left(x^2+\mathrm{b}_l\,x+\mathrm{c}_l\right)^t$ and modifying the numerator and denominator for the purpose of replacing x^2 with $\mathrm{b}_l\,x+\mathrm{c}_l$ to obtain coefficient -*i,t* of B and C in polynomial division, $\mathrm{B}_{it}\,x+\mathrm{C}_{it}=\frac{\mathrm{W}(x)}{\mathrm{S}(x)}\left(x^2+\mathrm{b}_l\,x+\mathrm{c}_l\right)^t\Big|_{x^2=-\mathrm{b}_l\,x-\mathrm{c}_l}$. Repeat the same process that described above to