MATHEMATICAL THINKING ENHANCEMENT PROGRAM (MaTh-EP)

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ABSTRACT

The present invention, called "Mathematical Thinking Enhancement Program (MaTh-EP)", generally relates to a program that enhances the development of mathematical thinking. This program is revolutionising from the problem-solving model from Polya (1973) and Schoenfeld (1992). This program integrates the problem-solving stages, heuristics, and metacognitive strategies instruction to guide students in solving non-routine problems that are intellectually challenging. This program is suitable for primary, secondary, and tertiary levels of education. Mathematics instructors, either at the school level or tertiary level, are encouraged to experience this program. The idea of MaTh-EP is to expose the participants to cognitive-metacognitive strategies and heuristics while solving non-routine problems. Non-routine problems are mostly concerned with developing participants' mathematical reasoning power and fostering an understanding that mathematics is a creative endeavour. MaTh-EP could develop participants thinking in Mathematics especially viewing mathematics problems related to the application in daily life. The potential program is suitable for students or instructors interested in booster their way of thinking in Mathematics.

Keywords: mathematical thinking, problem-solving approach, cognitive, metacognitive, heuristics

1. INTRODUCTION

The essence of mathematical thinking is inextricably linked to the cognitive processes which generate mathematical knowledge. One of the ways to ensure that students are involved with mathematical thinking is through non-routine problems. Mathematical thinking requires non-routine or unfamiliar mathematical problems so that students can flexibly include their understanding of mathematics' fundamental concepts and ideas and focus on the problem-solving process (English & Kirshner, 2016; Schoenfeld, 1992). Non-routine problems are the kinds of problems that contribute to students' mathematical problem solving and reasoning skills. According to Hershkowitz et al. (2001), if the students solve a routine problem, they are likely to alternate between recognising and building with previously acquired structures. If they solve a non-routine problem, they can build and reflect on a new (for them) phenomenon, its internal structure, and its external connection to items they already know.

The development of Malaysian students in solving a mathematics problem is still unsatisfactory. Even though the students are studied at the university and college level, they are not able to recognise their final answer or solution is logically incorrect. Hoon et al. (2018) have found that the majority of the participants (university students) were directly applied the procedural knowledge without any reasoning when solving mathematics problems. They could not describe their argument in a meaningful way other than specifying the algorithmic procedure processes. This phenomenon indicates that they are ignorant of their thinking when solving the problem. Singh (2017) has found that students are given the solution based on the direct assumption rather than proven through mathematical and logical reasoning. These difficulties and mistakes are found because many undergraduate students are too focused almost entirely on formal mathematical algorithms, principles, and procedures that appear to be highly distant from conceptual comprehension (Singh et al., 2018; Bowyer & Darlington, 2016). Impact of that, the students are failed to comprehend the fundamental of formulaic structures in their mathematical learning due to being too dependent on formula and procedural knowledge when solving problems (Han et al., 2016; Singh & Hoon, 2017; Hoon et al., 2018). Han et al. (2016) have found that some students can understand the theoretical concepts involved in the problem. However, they do not know how to solve the problem. The students lack alternative strategies even though they struggle to remember the mathematics formula to solve the problem. Hoon et al. (2018) supported these findings, who found that the university students did not show a second attempt or further effort to come out with a solution.

Due to these reasons and facts that have been discussed above, there is important that to develop students with the thinking processes. Generally, research indicates that knowledge and awareness of one's thinking strategies develop relationships with the ability of the students' solving problems (Schoenfeld, 1992; Schoenfeld, 1994; Safari & Meskini, 2016). These thinking strategies relate to students' cognitive self-regulation or monitoring skills when solving the problem. This term "monitoring and control" is referring to the aspect of metacognition known as self-regulation. Schoenfeld (1992) described self-regulation or "monitoring and control" when explaining problem-solving and mathematical thinking. Due to these reasons and facts that have been discussed above, there is important that to develop students with the thinking processes and strategies.

Various researchers have suggested a need program or an initiative to boost students' mathematical thinking, especially at the tertiary level (Hoon et al., 2018; Singh et al., 2016; Schoenfeld, 1992). Therefore, students need to be exposed to the approach they could develop their thinking in mathematics. Based on these issues, we have developed the Mathematical Thinking Enhancement Program (MaTh-EP). MaTh-EP aims to engage students in meaningful learning of mathematics through experience in solving non-routine problems. Specifically, the objectives of the program are: (a) to develop students' cognitive-metacognitive strategies and heuristics in mathematical thinking; (b) to reinforce mathematical processes (reasoning and proof, communication, connections and representation, and problem-solving); and (c) to facilitate the application of mathematical strategies and concepts. This program emphasised the role of cognitive-metacognitive strategies by implementing the six problem-solving stages (read, analyse, exploration, plan, implementation, verification) and heuristics when solving problems. This program encourages students to participate in various exercises, problems, and investigations as they explore mathematics concepts from a problem-solving perspective in an interactive manner. The program emphasised the exploration of various mathematics contexts to learn mathematics, pose problems and problem extensions, solve problems, and communicate mathematical demonstrations.

2. MATERIALS AND METHODS

MaTh-EP refers to the intervention program that integrates the problem-solving stages, heuristics, and metacognitive strategies instruction to guide students in solving non-routine problems that are intellectually challenging. MaTh-EP will guide participants to solve non-routine problems through problem-solving stages and various strategies. This program was developed through the problem-solving six stages model known as reading, analyse, explore, plan, implement and verify introduced by Schoenfeld (1992). Schoenfeld (1992) believed that to solve problems effectively, the students must control, monitor, and self-regulate their thinking. These processes are known as metacognition. Due to that consideration, the participants will be explicitly emphasised to use self-instruction and self-monitoring during the problem-solving processes. Other than that, the usage of heuristics will be emphasised correspondingly to the participants when they are solving non-routine problems as this aspect also involves problem-solving stages (Polya, 1973; Schoenfeld, 1992). This program provides materials, teaching resources, and assessments (formative and summative) for explicit teaching of problem-solving in the environment of deep mathematics content starting secondary level until tertiary level. This program completes basic problem-solving modules which able to develop students' mathematical thinking. This program comprises eight (8) modules that are appropriately implemented to secondary to the tertiary level curriculum. The module materials consist of the following: (a) Program schedule – Modules rules and learning outcomes for each module; (b) Eight (8) two hours lesson plans; (c) Prompting questions as a practical guideline for participants to apply cognitive-metacognitive strategies; (d) A collection of problems (Task and Homework); (e) Slide presentation for each module; and (f) Formative assessment (Pre-Test and Post-Test).

3. RESULTS AND DISCUSSION

MaTH-EP was tested on Mac to May 2021. We have employed a quasi-experimental design focusing on the pre-test and post-test to assess participants' ability in mathematical thinking before and after experienced the program. Two intact groups from first-year undergraduate programs from the same university and program were selected as the experimental and control groups. The experimental group has experienced MaTh-EP for eight weeks, while the control group only received a normal classroom discussion. However, the control group still experienced similar tasks, questions, and answer schemes as the experimental group during the program. The pre-test and post-test are conducted before and after the program. The participant's performance in pre-test and post-test is compared.

In the pre-test, both groups (experimental and control) are seen to have not much difference in mean score and standard deviation with a value of 6.8 (s.d=3.71) and 6.2 (s.d=3.33) out of 40, respectively. However, for the post-test, it was found that there is a huge difference in the mean score of the experimental group (mean=23.28; s.d=9.26) and control group (mean=8.16; s.d=4.52). To determine the significant difference between the mean score of the experimental group and control group, the researchers have conducted an independent sample t-test and paired sample t-test. An independent sample t-test showed that the difference in pre-test score between the experimental group (n=25; M=6.80. s.d=3.71) and the control group (n=25; M=6.20; s.d=3.33) is statistically not significant, $t(48)=0.602, p=0.92$. While for the post-test score, the difference between the experimental group (n=25; M=23.28. s.d=9.26) and the control group (n=25; M=8.16; s.d=4.52) is statistically significant, $t(34.81)=7.34, p=0.00$.

We also have compared the difference of the pair (pre-post tests) according to experimental and control groups by using paired sample t-test. The reason for comparing these pairs of scores was to determine which group have shown a significant improvement before and after the program. For the control group, a paired sample t-test indicated that the difference in mathematical thinking test scores between before and after eight weeks is not statistically significant $t(24)=-2.19, p=0.059$. Meanwhile, for the experimental group, a paired sample t-test showed that the difference in mathematical thinking test scores between before and after eight weeks program is statistically significant $t(24)=-8.804, p=0.000$.

Based on these results, it was indicated that MaTh-EP provides a positive outcome to the participants' performance in mathematical thinking. Furthermore, it was observed that the experimental group showed a significant improvement in their mathematical thinking performance compared to the control group. Besides, the experimental group who have experienced the program for eight weeks became more able to solve the non-routine problem consist in the test and employ successful cognitive-metacognitive strategies. The data also suggested that MaTh-EP has a positive influence on participants' beliefs and working. The percentage of participants who did not attain the question has also reduced drastically if we compared pre-test and post-test among experimental participants. To summarise, MaTh-EP helped developing undergraduates' students' mathematical thinking while adopting more positive beliefs about solving mathematics problems.

4. CONTRIBUTION, USEFULNESS AND COMMERCIALISATION

This program promotes students' consciousness of cognitive-metacognitive strategies and provides an immediate boost in Mathematics strategies and concepts through contextual and meaningful learning. Students are emphasised with logical thinking skills, computation skills, reasoning and problem-solving skills, and mathematics skills (reasoning and proof, communication, connections, and representation) through this program.

This program reinforces students' exploration, thinking, knowledge and understanding in mathematics while concurrently hones a host of generic (teamwork, leadership, and communication) skills via; exploration of ideas; application of concepts; involvement in hands-on activities. Of prime importance, nonetheless, is getting the students excited about, having the interest in and continuously looking forward to learning and exploring mathematics; and doing so in a meaningful manner. The program's unique design has led to successful learning outcomes whereby students reported improving thinking manner, which also reinforced; computation skills and reasoning; problem-solving skills.

It has a high potential in commercialising this product to students all over Malaysian and even the international market. The MaTh-EP is based on modules and instructional materials written on how the mathematics thinking activities are implemented. Seeing the demand from the industry, which places critical thinking and problem-solving skills as the mandate for the job plus with the direction of national curriculum education nowadays, which emphasises higher-order thinking skills, this program is seen to have great potential to be commercialised. Therefore, it will be in high demand among the community. The researchers have recently started collaborating with learning centres, educational organisations, and institutions (schools, universities, and colleges). In the pipeline, the module developers and facilitators of MaTh-EP plan to aggressively market the program to targeted schools, colleges, universities, and communities. Besides, this program has many prospective participants (students, teachers, lecturers, parents). In addition, this program is also a program that does not require a high budget in its implementation. MaTh-EP could be implemented either in face-to-face or hybrid (online) mediums. As such, it makes the program more marketable in the future. Alternatively, the modules can be printed, digitalised, and sold as texts to complement mathematics learning in schools. However, this has to wait since the modules are still in the development of publishing.

5. CONCLUSION

Preparing students and educators to practice thinking processes has become a challenge that should be faced. Plus, to become a good problem solver and good thinker does not just happen. We have believed that students or mathematics instructors must possess sufficient knowledge and skills to play their perspective role. Unfortunately, the mathematics environment in schools or colleges/universities of education focuses almost exclusively on mastery of mathematical content knowledge rather than mathematical thinking or problem-solving. It is undeniable that mathematical thinking requires specialised knowledge and skills related to metacognition and strategies. People should not expect that student-teachers would develop much of this knowledge and skills for themselves.

Based on the experimental study done by the researchers, we have found that MaTh-EP is effective in developing participants' mathematical thinking. Furthermore, there is a significant development among participants in their level of mathematical thinking before and after experience in MaTh-EP. Therefore, we suggest that explicit attention to enhancing mathematical thinking behaviour in solving the problem should be given to teachers and students in the future. This specific program provides direct instruction and training on various cognitive and metacognitive aspects of mathematical behaviour.

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