

# Quest for Research Excellence On Computing, Mathematics and Statistics

**Editors**

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# **Quest for Research Excellence on Computing, Mathematics and Statistics**

## **Chapters in Book**

The 2<sup>nd</sup> International Conference on Computing, Mathematics  
and Statistics (iCMS2015)

Editors:

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## CHAPTER 15

# Characterization of $p$ -Groups with a Maximal Irredundant 10-Covering

Rawdah Adawiyah Tarmizi and Hajar Sulaiman

**Abstract.** A group  $G$  is covered by a collection of its proper subgroups if it is equal to the union of the collection. A covering is called irredundant if there is no proper sub-collection is also a covering of  $G$ . A covering in which all members are maximal subgroups of  $G$  is called maximal. For any integer  $n > 2$ , a covering with  $n$  members is called an  $n$ -covering. We say the covering of  $G$  as  $C_n$ -covering if it is an irredundant maximal core-free  $n$ -covering for  $G$ . In this paper, we characterize 3-groups having a maximal irredundant 10-covering with core-free intersection and we prove that a group  $G$  is a  $p$ -group having  $C_{10}$ -covering if and only if  $G \cong (C_3)^5$  or  $G \cong (C_5)^3$ .

---

**Keywords:** covering groups by subgroups;  $p$ -groups; maximal irredundant covering; core-free intersection.

# 1 Introduction

Let  $G$  be a finite group. If  $G$  is non-cyclic, then  $G$  can be obtained as a union of its proper subgroups. A covering  $C$  of a group  $G$  is a collection of proper subgroups of  $G$  whose union is the whole group  $G$ . We use the term  $n$ -covering for  $C$  with  $n$  members.

A covering  $C$  of  $G$  is irredundant if no proper sub-collection is also a covering for  $G$ , and is called maximal if all its members are maximal subgroups of  $G$ . We denote the intersection of members of maximal covering by  $D$ . A covering  $C$  of  $G$  is called core-free if the intersection  $D = \bigcap_{M \in C} M$  of  $C$  is core-free in  $G$ , i.e.  $D_G = \bigcap_{g \in G} g^{-1} D g$  is the trivial subgroup of  $G$ . The covering  $C$  of  $G$  is called a  $C_n$ -covering whenever  $C$  is an irredundant maximal core-free  $n$ -covering for  $G$ . We say a group  $G$  is a  $C_n$ -group if  $G$  admits  $C_n$ -covering.

It is well known that there is no group can be covered by two proper subgroups. Scorza in [8] was the first to determine the structure of all groups having an irredundant 3-covering with core-free intersection.

**Theorem 1.1.** (See [8]) Let  $\{A_i \mid 1 \leq i \leq m\}$  be an irredundant covering with core-free intersection  $D$  for a group  $G$ . Then,  $D = 1$  and  $G \cong C_2 \times C_2$ .

In [7] Greco listed all groups with an irredundant 4-covering with core-free intersection. He also listed all groups with an irredundant 5-covering in which all pairwise intersection are the same.

Bryce et al. in [6] characterized groups with maximal irredundant core-free intersection 5-covering completely. They proved that  $G$  is a  $p$ -group if and only if  $G$  is elementary of order  $2^4 = 16$ .

Abdollahi et al. in [3] characterized groups with maximal irredundant 6-covering with core-free intersection.

In [4] Abdollahi and Jafarian listed all groups having a maximal irredundant 7-covering with core-free intersection.

Abdollahi et al. in [1] characterized of  $p$ -groups with a maximal irredundant  $n$ -covering with core-free intersection for  $n \in \{7, 8, 9\}$  completely.

Ataei in [5] characterized nilpotent group having a maximal irredundant 8-covering with core-free intersection.

In [2] Ataei and Sajjad characterized 5-groups with a maximal irredundant 10-covering with core-free intersection.

**Theorem 1.2.** (See [2]) Let  $G$  be a 5-groups such that  $|G| \neq 5^4$ . Then  $G$  is a  $C_{10}$ -group, if and only if  $G \cong (C_5)^3$ .

There are problems of similar nature with slightly different aspects have been studied by many authors ([11], [12], [13], [14], [15]).

Here we characterized 3-groups having a maximal irredundant 10-covering with core-free intersection. Then we prove that  $G$  is a  $p$ -group that admits  $C_{10}$ -covering if and only if  $G \cong (C_3)^5$  or  $G \cong (C_5)^3$ .

Also, we use the notation as in [2]; for example,  $C_n$  the cyclic group of order  $n$ ,  $(C_n)^j$  is the direct product of  $j$  copies of  $C_n$ ,  $[n]$  is the set  $\{1, \dots, n\}$ , and  $[n]^m$  is the set of all subsets of  $[n]$  of size  $m$ .

## 2 Preliminaries

We will require the following lemma as an aid tool to obtain our results.

**Lemma 2.1.** (See [9], Proposition 2.5) *Let  $G$  be a finite  $p$ -group for a prime  $p$ , with a maximal irredundant  $n$ -covering. Then either  $n \geq \frac{3(p+1)}{2}$  or  $n = p + 1$ .*

**Lemma 2.2.** (See [1], Theorem 1.3) *Let  $G$  be a finite 2-group and  $n$  be a positive integer. Then  $G$  admits a  $C_{n+1}$ -covering if and only if  $n$  is even and  $G \cong (C_2)^n$ .*

**Lemma 2.3.** (See [6], Lemma 2.2). *Let  $\Gamma = \{A_i \mid 1 \leq i \leq m\}$  be an irredundant covering of a group  $G$  whose intersection of the members is  $D$ .*

- (a) *If  $p$  is a prime,  $x$  a  $p$ -element of  $G$  and  $|\{i \mid x \in A_i\}| = n$ , then either  $x \in D$  or  $p \leq m - n$ .*
- (b)  *$\bigcap_{j \neq i} A_j = D \forall i \in \{1, 2, \dots, m\}$ .*
- (c) *If  $\bigcap_{i \in S} A_i = D$  whenever  $|S| = n$ , then  $|\bigcap_{i \in T} A_i : D| \leq m - n + 1$  whenever  $|T| = n - 1$ .*
- (d) *If  $\Gamma$  is maximal and  $U$  is an abelian minimal normal subgroup of  $G$ , then if  $|\{i \mid U \subseteq A_i\}| = n$ , either  $U \subseteq D$  or  $|U| \leq m - n$ .*

**Lemma 2.4.** (See [1], Lemma 3.2). *Let  $G$  be a finite  $p$ -group having a  $C_n$ -covering  $\{M_i \mid i = 1, \dots, n\}$ . Then*

- (a)  *$p \leq n - 1$ .*
- (b) *If  $s$  the integer such  $1 \leq s \leq n - 2$  and  $p = n - s$ , then  $\bigcap_{i \in S} M_i = 1$  for every subset  $S$  of  $\{1, 2, \dots, n\}$  with  $|S| \geq s + 1$ .*
- (c) *if  $n = p + 1$ , then  $G \cong (C_p)^2$ .*

**Lemma 2.5.** (See [1], Lemma 3.3). *Let  $G = (C_p)^d$  for  $d \geq 2$  and  $p$  is a prime number. Suppose that  $G$  has  $C_n$ -covering  $\{M_i \mid i = 1, \dots, n\}$ . Let  $T \subseteq \{1, 2, \dots, n\}$ .*

- (a) *If  $|T| = n - p$ , then  $|\bigcap_{i \in T} M_i| = 1$  or  $p$ .*

- (b) If  $|T| = 2$ , then  $|\cap_{i \in T} M_i| = p^{d-2}$ .
- (c)  $\cap_{i \in T} M_i = 1$  for some  $T$  of size  $d$ .
- (d) If  $\cap_{i \in S} M_i = 1$  whenever  $|S| = d$ , then  $p \leq |\cap_{i \in T} M_i| \leq n - d + 1$  whenever  $|T| = d - 1$ .

### 3 Result

In this section we characterize  $p$ -groups with a maximal irredundant 10-covering.

**Theorem 3.1.** *Let  $G$  be a 3-group such that  $|G| \neq 3^4$ . Then  $G$  is a  $C_{10}$ -group, if and only if  $G \cong (C_3)^5$ .*

**Proof.** Suppose that  $G$  is a 3-group,  $G = \cup_{i=1}^{10} M_i$  and  $D = \cap_{i=1}^{10} M_i$ . Since the Frattini subgroups of  $G$ ,  $\phi(G) = G'G^3 \leq D$ , we have  $D$  is a normal subgroup of  $G$ . Therefore  $D = 1$  and  $G$  is an elementary abelian 3-group. By Lemma 2.5(b)

$$|G:M_i| = 3 \text{ and } |G:M_i \cap M_j| = 3^2 = 9 \text{ for all distinct } i, j \in [10] \quad (15)$$

and by lemma 2.4(b) implies that

$$\forall S \subseteq [10] \text{ with } |S| \geq 10 - 3 + 1 = 8, \cap_{i \in S} M_i = 1 \quad (2)$$

It follows that  $|G| \leq 3^8$ . Since an elementary abelian group of order  $3^2$  has only four maximal subgroups, we have  $|G| \geq 3^3$ .

Let  $|G| = 3^3$ , so that  $G \cong (C_3)^3$ . We have used the following function run in GAP [16] to proof the claim. The inputs of the function are a group and a number of covering, and the outputs are all combinations of maximal irredundant 10-covering with core free intersection of  $G$ , and if there is no such covering for  $G$ , then the list is empty.

```
f:=function(G,p) local S,M,n,i,C,T,Q,R;
n:=Size(G);
M:=MaximalSubgroups(G);
C:=Combinations(M,p);
S:=[];
for i in [1..Size(C)] do if Size(Union(C[i]))=n then
Add(S,C[i]);
fi;od;
T:=[];
for i in [1..Size(S)] do if
Size(Core(G,Intersection(S[i])))=1 then Add(T,S[i]);
```

```

fi;od;
R:=[];
for i in [1..Size(T)] do Q:=Combinations(T[i],p-1);
if (n in List(Q,i->Size(Union(i))))=false then
Add(R,T[i]);
fi;od;
return R;
end;

```

finally, we found that the list is empty, and therefore  $|G| \neq 3^3$ . Assume that  $|G| = 3^5$ , so that  $G \cong (C_3)^5$ . By GAP in [16], we can check that if  $G = \langle a, b, c, d \rangle$ , then the set

$$C = \{ \langle a, b, c, d \rangle, \langle a, b, c, e \rangle, \langle a, b, d, e \rangle, \langle a, c, d, e \rangle, \langle b, c, d, e \rangle, \langle ac^{-1}, b, d, e \rangle, \langle a, b, ce, d \rangle, \langle a^{-1}b, a^{-1}c, a^{-1}d, a^{-1}e \rangle, \langle a, b, c, de \rangle, \langle ae, a^{-1}b, a^{-1}c, a^{-1}d \rangle \}$$

of maximal subgroups form a  $C_{10}$ -covering for  $G$ .

Let  $|G| = 3^6$ . By part (c) of Lemma 2.5, we assume that there exist  $S \in [10]^6$  such that  $|\cap_{i \in S} M_i| = 1$ . Since the covering is irredundant, therefore there exist  $S \in [10]^2$  such that for all  $T \in [10]^6$ ,  $O = \cap_{i \in T} M_i \not\leq \cap_{i \in S} M_i$ . Therefore,

$$|G| = 3^6 = \left| G : \bigcap_{i \in 1}^8 M_i \right| = \left| G : \bigcap_{i \in 1}^6 M_i \right| \left| G : \bigcap_{i \in 1}^2 M_i \right| = |G : O| 3^2$$

$$|O| = 3^2,$$

$$|G : O| = 3^4$$

which is a contradiction by  $|\cap_{i \in 1}^6 M_i| = 1$ .

If  $|G| = 3^7$ . Then Lemma 2.5 implies that  $\cap_{i \in T} M_i = 1$  for at least  $T \in [10]^7$ . Therefore, we assume that there exist  $S \in [10]^7$  such that  $|\cap_{i \in S} M_i| = 1$ . Since the covering is irredundant, therefore there exist  $j \in [10]$  such that for all  $L \in [10]^7$ ,  $N = \cap_{i \in L} M_i \not\leq M_j$ . Therefore

$$|G| = 3^7 = \left| G : \bigcap_{i \in 1}^8 M_i \right| = |G : N| |G : M_j| = |G : N| 3$$

$$|N| = 3,$$

$$|G : N| = 3^6$$

which is a contradiction by  $|\cap_{i \in 1}^7 M_i| = 1$ .

Now assume that  $|G| = 3^8$ . Then Lemma 2.5(c) implies that

$$|\cap_{i \in T} M_i| = 3 \text{ for every } T \in [10]^7. \quad (3)$$

Then it follows from (1) that for every  $T \in [10]^3$ , we have  $|G: \cap_{i \in K} M_i| = 3^5$  or  $3^6$ . Now we prove that  $|\cap_{i \in K} M_i| = 3^5$  for all  $K \in [10]^3$ . Suppose, for contradiction, that there exist  $L \in [10]^3$  such that  $|\cap_{i \in L} M_i| = 3^6$ . Let  $L' \in [10]^4$  such that  $L \cap L' = \emptyset$ . Then it follows from (1) and (3) that  $|\cap_{i \in L \cup L''} M_i| = |\cap_{i \in L' \cup L''} M_i| = 3$  for every  $L''$  is a proper subgroup of  $L$  of size 2. Since  $|L'' \cup L'| = 6$ , it follows that  $|G| \leq 3^6$ , which is a contradiction. Therefore,

$$|\cap_{i \in K} M_i| = 3^5 \text{ for all } K \in [10]^3. \quad (4)$$

By (1),  $|\cap_{i \in T} M_i| \in \{3^4 \text{ or } 3^5\}$  for all  $T \in [10]^4$ , we prove that  $|\cap_{i \in T} M_i| = 3^4$  for all  $T \in [10]^4$ . Suppose, for a contradiction that there exists  $L \in [10]^4$  such that  $|\cap_{i \in L} M_i| = 3^5$ . Let  $L' \in [10]^3$  such that  $L \cap L' = \emptyset$ . Then (1) and (3) imply that  $|\cap_{i \in L \cup L''} M_i| = |\cap_{i \in L' \cup L''} M_i| = 3$  for every  $L'' \subset L$  of size 3. Since  $|L'' \cup L'| = 6$ , it follows that  $|G| \leq 3^6$ , which is a contradiction. We conclude that,

$$|\cap_{i \in T} M_i| = 3^4 \text{ for all } T \in [10]^4. \quad (5)$$

By a similar argument as in the previous, we can prove that

$$\forall V \in [10]^5, |\cap_{i \in V} M_i| = 3^3 \quad (6)$$

and

$$\forall W \in [10]^6, |\cap_{i \in W} M_i| = 3^2. \quad (7)$$

Now using (1) – (7), it follows from the inclusion-exclusion principle that

$$\left| \bigcup_{i=1}^{10} M_i \right| = \binom{10}{1} 3^7 - \binom{10}{2} 3^6 + \binom{10}{3} 3^5 - \binom{10}{4} 3^4 + \binom{10}{5} 3^3 - \binom{10}{6} 3^2 + \binom{10}{7} 3 - \binom{10}{8} + \binom{10}{9} - \binom{10}{10} = 6453,$$

which is not  $3^8$ , a final contradiction.

**Theorem 3.2.** *Let  $G$  be a  $C_{10}$ -group. Then  $G$  is a  $p$ -group for a prime  $p$  if and only if  $G \cong (C_3)^5$  or  $G \cong (C_5)^3$ .*

**Proof.** Let  $G$  be a  $p$ -group with a  $C_{10}$ -covering  $\{M_i | i \in [10]\}$ . Since the Frattini subgroups of  $G$ ,  $\phi(G) = G'G^p \leq D$ , we have  $D$  is a normal subgroup of  $G$ . Therefore  $D = 1$  and  $G$  is an elementary abelian  $p$ -group. By Lemma 2.1 implies that  $p \leq 5$  and it follows from Lemma 2.2 that  $p \neq 2$ . Therefore  $p = 3$  or  $p = 5$ .

If  $p = 5$ , then by Theorem 1.2 it follows that  $G \cong (C_5)^3$ . If  $p = 3$ , then Theorem 3.1 implies that  $G \cong (C_3)^5$ .

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