Quest for Research Excellence On Computing, Mathematics and Statistics

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Content

CHAPTER 13
Visualization of Constrained Data by Rational Cubic Ball Function

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Abstract. The main purpose of this paper is the construction of shape preserving interpolation for visualization of constrained data that will be smooth and pleasant. Three shape parameters are introduced in order to control the shape of the interpolation. The rational cubic Ball function is constructed according to the shape of the data that are constraint between two lines using appropriate conditions on each of the shape parameters. Numerical examples are provided to demonstrate that the anticipated scheme is interactive and smooth.

Keywords: shape preserving interpolation; constrained data; rational cubic Ball function; shape parameters.

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1 Introduction

Shape preserving interpolation is an important tool in Computer Graphics, Computer Aided Geometric Design and Engineering as well as for data visualization. The main goal of data visualization is a graphical representation of information in an effective and clear way [5]. Some shape preserving interpolating scheme arises which not only maintain the shape of the input data but also pay heed to the underlying smoothness of curve. This motivates us to come up with the scheme that can preserve the constrained curve that lies above or below a line.In this work another interpolating scheme is also developed which is not only preserves the constrained curve that lies above or below the line but also between two lines. Many authors have worked in the area of shape preservation, such as curve interpolation, which preserves the shape of data using rational cubic interpolation. Most of them concentrated on the problem of positivity, monotonicity and convexity preserving interpolating scheme [1-2]. Only a few authors considered the problem of shape preserving and constrained data visualization using rational cubic Ball interpolation. While some of them use other different scheme for constrained data visualization [3-4].

In this paper, we have developed a smooth rational cubic Ball function with three shape parameters in its description to preserve the shape of constrained curve with data lying above or below the line and between two lines. The curve is then generated using this modified interpolation function as demonstrated in the numerical demonstration.

2 Rational Cubic Ball Interpolation

Let $\{(x_i, f_i),\ i=1,2,\ldots,n\}$ be a given data points, $x_1 < x_2 < \ldots < x_n$ and $f_1, f_2, \ldots, f_n$ are real numbers. Suppose $h_i = x_{i+1} - x_i$ and $\Delta_i = \dfrac{f_{i+1} - f_i}{h_i}$ for $i = 1,2,\ldots,n-1,$ a piecewise rational cubic Ball function $S(x), x \in [x_i, x_{i+1}],\ i=1,2,\ldots,n-1$ with local variable $\theta = \dfrac{x_{i+1} - x_i}{h_i}$ for $\theta \in [0,1]$ is defined as

$$S(x) \equiv s_i = \frac{p_i(\theta)}{q_i(\theta)} = \frac{(1-\theta)^2 u_i a_i + 2(1-\theta)^2 \theta a_1 + 2(1-\theta)\theta^2 a_2 + \theta^3 a_3}{(1-\theta)^2 u_i + 2(1-\theta)\theta u_i v_i w_i + \theta^2 v_i} \tag{1}$$

With the initial and end conditions,

$$S(x_i) = f_i, \qquad S(x_{i+1}) = f_{i+1} \tag{2}$$
$$S'(x_i) = d_i, \qquad S'(x_{i+1}) = d_{i+1}$$

Here S' denotes the derivative with respect to x and d_i, d_{i+1} denotes the derivative value at x_i and x_{i+1} respectively. We should note that the derivatives can be provided by the users or can be computed using the method proposed by [6] from the data set itself.
From Eqn. (1) and Eqn. (2) a_i, are obtained as follows

$$a_0 = u_i f_i, \qquad a_1 = \frac{1}{2} u_i \left(d_i h_i + 2 v_i w_i f_i \right)$$
$$a_2 = \frac{1}{2} v_i \left(d_{i+1} h_i - 2 u_i w_i f_{i+1} \right), \qquad a_3 = v_i f_{i+1}$$

3 Constrained Curve Visualization

Let $\{(x_i, f_i), i = 1, 2, \ldots, n\}$ be a given data set lying above the straight line $y = mx_i + c$ i.e.

$$f_i > mx_i + c, \quad \forall i = 0, 1, 2, \ldots, n. \tag{3}$$

Where m and c is the slope and the y-intercept of the line respectively.

The curve of the of the given set of data points lies above the straight line if the rational cubic Ball function (1) holds the following condition

$$S(x) > y, \quad \forall x \in [x_0, x_n]. \tag{4}$$

In each subinterval $I_i = [x_i, x_{i+1}]$, the relation (4) can be expressed as:

$$S_i(x) = \frac{p_i(\theta)}{q_i(\theta)} > mx_i + c. \tag{5}$$

The equation of the straight line in parameter θ is defined as:

$$r_i(1-\theta) + s_i\theta, \quad \theta \in [0, 1]. \tag{6}$$

where,
$r_i = mx_i + c$ and $s_i = mx_{i+1} + c.$

The parametric form of the equation (5) is

$$S_i(x) = \frac{p_i(\theta)}{q_i(\theta)} > r_i(1-\theta) + s_i\theta, \quad i = 0,1,2,\ldots,n. \tag{7}$$

or

$$S_i(x) = \frac{p_i(\theta)}{q_i(\theta)} - r_i(1-\theta) + s_i\theta > 0. \tag{8}$$

Multiply both sides of equation (8) by $q_i(\theta)$, we have

$$F_i(x) = \sum_{k=0}^{3} (1-\theta)^{3-k} \theta^k H_{k,i}. \tag{9}$$

where

$$H_{0,i} = u_i\left(f_i - r_i\right)$$
$$H_{1,i} = \frac{1}{3}u_i\left(d_i h_i + 2v_i w_i(f_i - r_i) + f_i - s_i\right)$$
$$H_{2,i} = \frac{1}{3}v_i\left(-d_{i+1}h_i + 2u_i w_i(f_{i+1} - s_i) + f_{i+1} - r_i\right)$$
$$H_{3,i} = v_i\left(f_{i+1} - s_i\right).$$

Necessary conditions derived from equation (3) we have:

$$\begin{cases} (f_i - r_i) > 0 \\ (f_{i+1} - s_i) > 0 \end{cases}. \tag{10}$$

The polynomial $F_i(x) > 0$ if $H_{k,i} > 0, \quad k = 0,1,2,3.$

Since $u_i, v_i > 0$ and from equation (10), we have $H_{0,i} > 0$ and $H_{3,i} > 0.$

$H_{1,i} > 0$ if

$$w_i > -\frac{d_i h_i + f_i - s_i}{2v_i\left(f_i - r_i\right)}. \tag{11}$$

$H_{2,i} > 0$ if

$$w_i > \frac{d_{i+1}h_i - (f_{i+!} - r_i)}{2u_i\left(f_{i+1} - s_i\right)}. \quad \textbf{(12)}$$

The above results can be summarized as:

Theorem 3.1. The rational cubic Ball function $S(x)$, defined over the each subinterval $I_i = [x_i, x_{i+1}]$, in (1), preserves the shape of data that lies above the straight line, if the following sufficient conditions are satisfied.

$$\begin{cases} u_i > 0, v_i > 0 \\ w_i > \max\left\{0, -\dfrac{d_i h_i + f_i - s_i}{2v_i\left(f_i - r_i\right)}, \dfrac{d_{i+1}h_i - (f_{i+1} - r_i)}{2u_i\left(f_{i+1} - s_i\right)}\right\}. \end{cases} \quad \textbf{(13)}$$

The above constraints can be written as:

$$\begin{cases} u_i > 0, v_i > 0 \\ w_i = p_i + \max\left\{0, -\dfrac{d_i h_i + f_i - s_i}{2v_i\left(f_i - r_i\right)}, \dfrac{d_{i+1}h_i - (f_{i+1} - r_i)}{2u_i\left(f_{i+1} - s_i\right)}\right\}, p_i > 0 \end{cases}. \quad \textbf{(14)}$$

In the same manner the same condition given by Equation (14) holds if the line lies above the data points. However if the curve generated has to be constrained between the two lines then the weight has to be the maximum of the two cases.

4 Numerical Demonstration

Consider the sets of data taken at random in Table 1, which lie above the straight line $y = 0.3x$. The data also apply for sets of data that lie below the straight line $y = 1.4x + 60.5$. Fig. 1. drawn with the rational cubic Ball function without any constrained data. On the other hand, Fig. 2.and Fig. 3. when drawn by the constrained rational cubic Ball interpolation preserves the shape of constrained data lying above and below the line respectively. Fig. 4. show more clearly constrained curve obtained by constrained data lie both above and below the straight line in one interpolation. A remarkable difference in smoothness with a visually pleasant view can be seen in these figures due choice on the values of shape parameter given to the designer.

Table 4. A set of data taken by random.

i	x_i	y_i
1	0	5.5
2	2	5.5
3	3	5.5
4	5	5.5
5	6	5.5
6	8	5.5
7	11	9.6
8	12	50
9	14	60
10	15	80
11	16.8	80
12	17	84
13	20	85

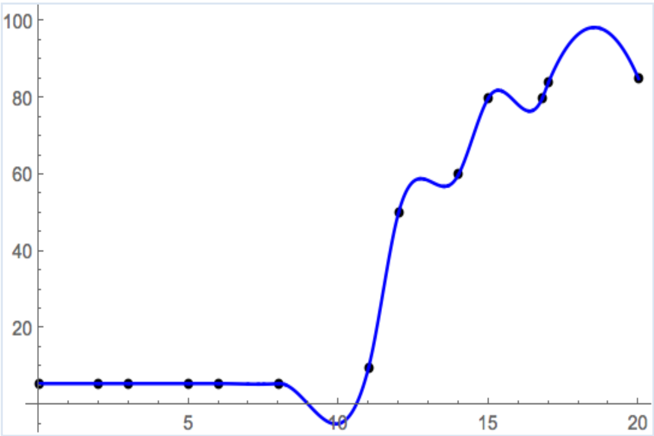

Fig. 2. Rational cubic Ball function without any constrained data.

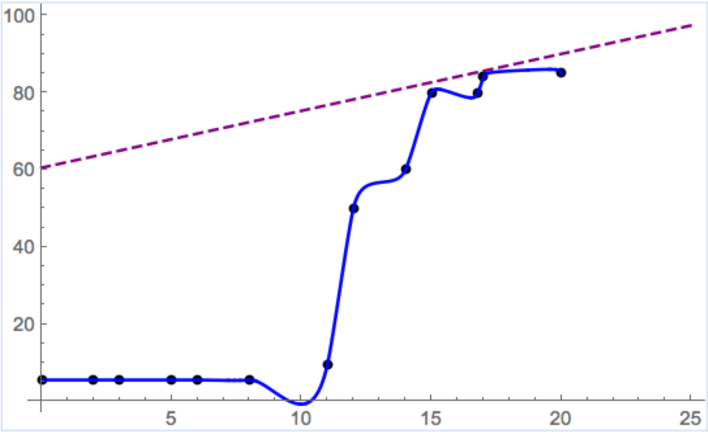

Fig. 3. Constrained data lie above the straight line.

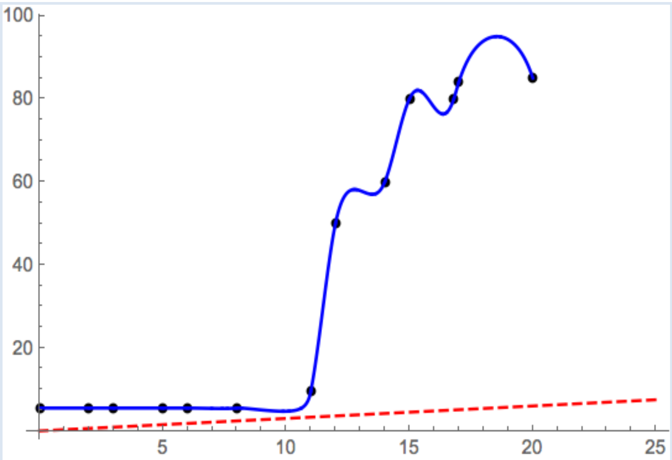

Fig. 4. Constrained data lie below the straight line.

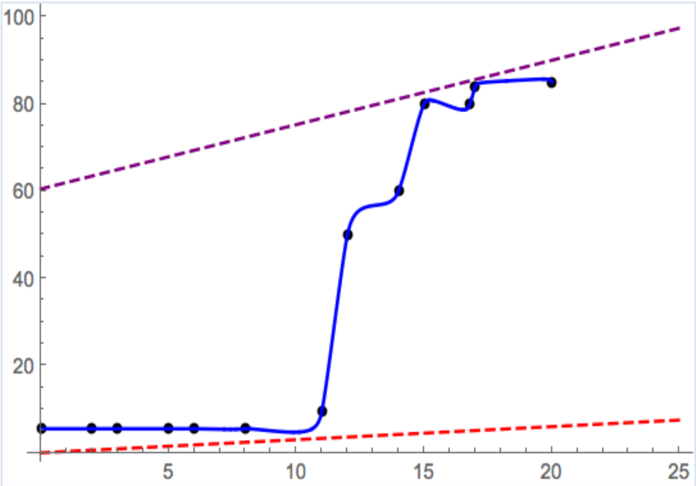

Fig. 5. Combination of Constrained data lie below and above the straight line.

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