Quest for Research Excellence On Computing, Mathematics and Statistics

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Faculty of Computer and Mathematical Sciences

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Chapters in Book

The 2nd International Conference on Computing, Mathematics and Statistics (iCMS2015)

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Content

CHAPTER 12
New Concept of *e*-*I*-open and *e*-*I*-Continuous Functions

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Abstract. In this paper, new classes of functions are introduced and studied by making use of *e*-*I*-open sets and *e*-*I*-closed sets. Relationship between the new classes and other classes of functions are established besides giving examples, counter examples, properties and characterizations.

Keywords: *e*-*I*-open; *e*-*I*-closed; somewhat; *e*-*I*-continuous; ideal topological space; *e*-*I*-dense; *e*-*I*-separable.

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1 Introduction

The subject of ideals in topological spaces has been studied by Kuratowski [9] and Vaidyanathaswamy [15]. Jankovic and Hamlett [8] investigated further prop- erties of ideal space. The importance of continuity and generalized continuity is significant in various areas of mathematics and related sciences. One of them, which has been in recent years of interest to general topologists, is its decomposition. The decomposition of continuity has been studied by many authors. The class of e-open sets is contains all δ-preopen [12] sets and δ-semiopen [11] sets. In 1992, Jankovic and Hamlett [7] introduced the notion of I-open sets in topological spaces. Abd El-Monsef et al. [1] investigated I-open sets and I-continuous functions. In this paper, using the notion of e-I-open sets, the concepts of somewhat e-I-continuous functions and somewhat e-I-open functions are introduced and studied. Also char- acterizations for somewhat e-I-continuity is obtained besides giving examples and counterexamples.An ideal I on a topological space (X, I) is a nonempty collection of subsets of X which satisfies the following conditions:
$A \in I$ and $B \subset A$ implies $B \in I$; $A \in I$ and $B \in I$ implies $A \cup B \in I$. Applications to various fields were further investigated by Jankovic and Hamlett [8] Dontchev et al. [4]; Mukherjee et al. [10]; Arenas et al. [3]; et al. Nasef and Mahmoud [13], etc. Given a topological space (X, I) with an ideal I on X and if $p(X)$ is the set of all subsets of X, a set operator $(.)^*$: $p(X) \to p(X)$, called a-local function [14, 8] of A with respect to τ and I is defined as follows: for $A \subseteq X$,

$$A^*(I,\tau) = \{x \in X \mid U \cap A \notin I \text{ for every } U \in \tau(x)\}$$

where $\tau(x = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $Cl^*(x)= A \cup A^*(I, \tau)$. When there is no chance for confusion, we will simply write A^* for $A^*(I, \tau)$. X^* is often a proper subset of X.

Definition 1.1. A subset A of an ideal topological space (X, τ, I) is said to be:
(1) I-open [1] if $A \subset Int(A^*)$.
(2) semi*-I-open [6] if $A \subset Cl(\delta IntI(A))$.
(3) e-I-open if [2] $A \subset Cl(\delta IntI(A)) \cup Int(\delta ClI(A))$.

Definition 1.2. [5] A function $f : X \to Y$ is said to be somewhat-continuous function if for $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$, there exists an open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Definition 1.3. [5] A function $f : X \to Y$ is said to be somewhat-continuous function provided that for $U \in \tau$ and $U \neq \emptyset$ there exists an open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Definition 1.4. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat-I-continuous function if for $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$ there exists an I-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Definition 1.5. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat semi*- I-continuous function if for $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$ there exists an semi*-I-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Definition 1.6. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be semi*-I-continuous [2], if the inverse image of each open set is semi*-I-open.

Definition 1.7. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be I-continuous [1], if the inverse image of each open set is I-open.

Definition 1.8. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be I-open (resp. semi*-I-open) function if the image of each open set U in (X, τ) is I-open (resp. semi*-I-open) in (Y, σ, I).

Definition 1.9. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat-I-open function provided that for $U \in \tau$ and $U \neq \emptyset$ there exists an open set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$.

Definition 1.10. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat semi*- I-open function provided that for $U \in \tau$ and $U \neq \emptyset$ there exists an semi*-I-open set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$

2 Somewhat *e-I*-continuous functions

Definition 2.1. Let (X, τ, I) be ideal topological spaces and (Y, σ) be any topological space. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat *e*-

I-continuous function if for every $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$ there exists an e-I-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Example 2.2. Let $X = \{a, b\}$ with a topology $\tau = \{\emptyset, X\}$, $I = \{\emptyset, \{a\}\}$, $Y = \{a, b\}$, $\sigma = \{\emptyset, X, \{a\}\}$. Now defined a function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ as follows: $f(a) = a$ *and* $f(b) = a$. Then f is somewhat e-I-continuous function Because $Cl(\delta IntI(A)) \cup Int(\delta ClI(A)) = Cl(\{b\}) \cup Int(X) = X$, if $U = \{a\}$, $V = \{b\}$. Then V is e-I-open and $V \subset f^{-1}(U)$.

Theorem 2.3. Every somewhat semi*-I-continuous function is somewhat e-I- continuous function.

Proof. Let $f : X \rightarrow Y$ be somewhat semi*-I-continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \emptyset$. Since f is somewhat semi*-I-continuous function, there exists a semi*-I-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$. Since every semi*-I-open set is e-I-open, there exist a e-I-open set V such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$, which implies that f is somewhat e-I-continuous function. □

Remark 2.4. Converse of the above theorem need not be true in general which follows from the following example.

Example 2.5. Let $X = Y = \{a, b, c\}$ with a topology $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, $I = \{\emptyset, \{a\}\}$, $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. Defined a function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ as follows: $f(a) = a$, $f(c) = c$ and $f(b) = b$. Then f is somewhat e-I-continuous function but not somewhat semi*-I-continuous. Since the inverse image of $\{c\}$ in (Y, σ) is $\{c\}$ in (X, τ, I) which is not semi*-I-open set.

Theorem 2.6. Every somewhat I-continuous function is somewhat semi*-I-continuous function.

Theorem 2.7. Every somewhat I-continuous function is somewhat e-I-continuous unction.

Proof. Theorem follows from Theorem 2.3 and Theorem 2.6.

Remark 2.8. Converse of the above theorem need not be true in general which follows from the following example.

Example 2.9. In example 2.5, the function $f : X \rightarrow Y$ defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ is somewhat *e-I*-continuous but not somewhat-*I*-continuous since the inverse image of $\{c\}$ is $\{c\}$ which is not *I*-open set.

Theorem 2.10. *Let* $f : (X, \tau, I) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \zeta)$ be any two functions. If f is somewhat *e-I*-continuous function and g is continuous function, then $g \circ f$ is somewhat *e-I*-continuous function.

Proof. *Let* $U \in \zeta$. Suppose that $g^{-1}(U) \neq \emptyset$. Since $U \in \zeta$ and g is continuous function $g^{-1}(U) \in \zeta$. Suppose that $f^{-1}(g^{-1}(U)) \neq \emptyset$. Since by hypothesis f is somewhat *e-I*-continuous function, there exists a *e-I*-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(g^{-1}(U))$. But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subset (g \circ f)^{-1}(U)$. Then $g \circ f$ is somewhat *e-I*-continuous function. □

Remark 2.11. In the above Theorem 2.10, if f is continuous function and g is somewhat *e-I*–continuous function, then it is not necessarily true that $g \circ f$ is somewhat *e-I*-continuous function. The following example serves this purpose.

Example 2.12. Let X = Y = {a, b, c} with a topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$, $I = \{\phi, \{b\}\}$, $\sigma = \{\phi, X, \{a\}, \{b, c\}\}$, $\zeta = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $J = \{\phi, \{a\}\}$. defined a function $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows: f (a) = a, f (c) = c and f (b) = b and defined a function $g : (Y, \sigma, J) \rightarrow (Z, \zeta)$ as follows: g(a) = a, g(c) = c and g(b) = b. Then clearly f is continuous function and g is somewhat e-I-continuous function but $g \circ f$ is not somewhat e-I-continuous function. Since $(g \circ f)^{-1}(U) = (g \circ f)^{-1}(\{c\}) = \{c\}$ which is subset of $f^{-1}(U)$, but not somewhat e-I-open.

Definition 2.13. A subset S of an ideal topological space (X, τ, I) is called *e-I*-dense if $Cl^*(S) = X$. In other words if there is no proper *e-I*-closed set M in X such that $S \subset M \subset X$.

Theorem 2.14. Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be a function. Then the following are equivalent:
(1) f is somewhat *e-I*-continuous function.
(2) If M is a closed subset of Y such that $f^{-1}(M) = X$, then there is a proper

e-I-closed subset D of X such that $D \subset f^{-1}(M)$.

(3) If S is a e-I-dense subset of X then $f(S)$ is a dense subset of Y.

Proof. (1)⇒(2): Let M be a closed subset of Y such that $f^{-1}(M) = X$. Then $Y - M$ is an open set in Y such that $f^{-1}(Y - M) = X - f^{-1}(M) \neq \emptyset$. By hypothesis (1) there exists a e-I-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(Y - M) = X - f^{-1}(M)$. This means that $X - V \supset f^{-1}(M)$ and $X - V = D$ is a e-I-closed set in X. This proves (2).

(2)⇒(3):Let S be a e-I-dense set in X. We have to show that $f(S)$ is dense in Y. Suppose not, then there exists a proper closed set M in Y such that $f(S) \subset M \subset Y$. Clearly $f^{-1}(M) = X$. Hence by (2) there exists a proper e-I-closed set D such that $S \subset f^{-1}(M) \subset D \subset X$. This contradicts fact that S is e-I-dense in X.

(3)⇒(2): Suppose that (2) is not true. This means there exists a closed set M in Y such that $f^{-1}(M) = X$. But there is no proper e-I-closed set D in X such that $f^{-1}(M) \subset D$. This means that f^{-1} (M) is e-I-dense in X. But by (3) $f(f^{-1}(M)) = M$ must be dense in Y, which is contradiction to the choice of M .

(2)⇒(1): Let $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$. Then $Y-U$ is closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq \emptyset$. By hypothesis of (2) there exists a proper e-I-closed set $D \supset f^{-1}(Y - U)$. This implies that $X - D \subset f^{-1}(U)$ and $X - D$ is e-I-open and $X - D \neq \emptyset$.

Theorem 2.15. Let (X, τ, I) be any ideal topological space. Let (Y, σ) be any topological space. A be an open set in X and $f : (A, \tau /A) \to (Y, \sigma)$ be somewhat e-I-continuous function such that $f(A)$ is dense in Y. Then any extension F of f is somewhat e-I-continuous function.

Proof. Let U be any open set in (Y, σ, J) such that $F^{-1}(U) \neq \emptyset$. Since $f(A) \subset Y$ is dense in Y and $U \cap f(A) \neq \emptyset$ it follows that $F -1 (U) \cap A \neq \emptyset$. That is $f -1 (U) \cap A \neq \emptyset$. . Hence by hypothesis on f , there exists a e-I-open set V in A such that $V \neq \emptyset$. and $V \subset f^{-1}(U) \subset F^{-1}(U)$ which implies F is somewhat e-I-continuous function. □

Theorem 2.16. Let (X, τ, I) and (Y, σ, J) be any two ideal topological spaces, $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau, I) \to (Y,$

σ, J) be a function such that f/A and f/B are somewhat *e-I*-continuous function. Then f is somewhat *e-I*-continuous function.

Proof. Let U be any open set in (Y, σ, J) such that $f^{-1}(U) \neq \emptyset$. Then $(f/A)^{-1}(U) \neq \emptyset$ or $(f/B)^{-1}(U) \neq \emptyset$. or both $(f/A)^{-1}(U) \neq \emptyset$. and $(f/B)^{-1}(U) \neq \emptyset$.

Case(1) Suppose $(f/A)^{-1}(U) \neq \emptyset$.

Since f /A is somewhat *e-I*-continuous, there exists a *e-I*-open set V in A such that $V \neq \emptyset$ and $V \subset (f/A)^{-1}(U) \subset f^{-1}(U)$. Since V is *e-I*-open in A and A is open in X, V is *e-I*-open in X. Thus f is somewhat *e-I*-continuous function.

Case(2) Suppose $(f/B)^{-1}(U) \neq \emptyset$.

Since f/B is somewhat *e-I*-continuous, there exists a *e-I*-open set V in B such that $V \neq \emptyset$. and $V \subset (f/B)^{-1}(U) \subset f^{-1}(U)$. Since V is *e-I*-open in B and B is open in X, V is *e-I*-open in X. Thus f is somewhat *e-I*-continuous function.

Case(3) Suppose $(f/A)^{-1}(U) \neq \emptyset$.and $(f/B)^{-1}(U) \neq \emptyset$.

This follows from both the cases (1) and (2). Thus f is somewhat *e-I*-continuous function. □

Definition 2.17. A topological space X is said to be *e-I*-separable if there exists a countable subset B of X which is *e-I*-dense in X.

Theorem 2.18. If $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is somewhat *e-I*-continuous function from X on to Y and if X is *e-I*-separable, then Y is separable.

Proof. Let $f : X \to Y$ be somewhat *e-I*-continuous function such that X is *e-I*-separable. Then by definition there exists a countable subset B of X which is *e-I*-dense in X. Then by Theorem 2.14, $f(B)$ is dense in Y. Since B is countable $f(B)$ is also countable which is dense in Y, which indicates that Y is separable. □

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