Quest for Research Excellence On Computing, Mathematics and Statistics

Editors
Kor Liew Kee
Kamarul Ariffin Mansor
Asmahani Nayan
Shahida Farhan Zakaria
Zanariah Idrus

Faculty of Computer and Mathematical Sciences

Quest for Research Excellence on Computing, Mathematics and Statistics

Chapters in Book

The 2nd International Conference on Computing, Mathematics and Statistics (iCMS2015)

Editors:

Kor Liew Lee
Kamarul Ariffin Mansor
Asmahani Nayan
Shahida Farhan Zakaria
Zanariah Idrus

Quest for Research Excellence on Computing, Mathematics and Statistics

Chapters in Book

The 2nd International Conference on Computing, Mathematics and Statistics

(iCMS2015)

4-5 November 2015
Langkawi Lagoon Resort
Langkawi Island, Kedah
Malaysia

Copyright © 2015 Universiti Teknologi MARA Cawangan Kedah

All rights reserved, except for educational purposes with no commercial interests. No part of this publication may be reproduced, copied, stored in any retrieval system or transmitted in any form or any means, electronic or mechanical including photocopying, recording or otherwise, without prior permission from the Rector, Universiti Teknologi MARA Cawangan Kedah, Kampus Merbok, 08400 Merbok, Kedah, Malaysia.

The views and opinions and technical recommendations expressed by the contributors are entirely their own and do not necessarily reflect the views of the editors, the Faculty or the University.

Publication by
Faculty of Computer & Mathematical Sciences
UiTM Kedah

ISBN 978-967-0314-26-6

Content

International Scientific Committee

Preface

CHAPTER 1 1
Towards Ameliorating the Problem of Packet Dropping in IDS using P System Model on GPU
Rufai Kazeem Idowu, Ravie Chandren M., and Zulaiha Ali Othman

CHAPTER 2 11
Analyses of Software Testing Problems in Small and Medium Software Enterprises (SME's) and a Proposed Framework on Exploratory Testing
Murugan Thangiah and Shuib Basri

CHAPTER 3 25
Senior Citizen and Online Form: Hybrid Guideline Form Design
Zanariah Idrus, Nor Hafizah Abdul Razak, and Noor Hasnita Abdul Talib

CHAPTER 4 35
Research Paradigms in Computing Disciplines: A Review
Nor Hafizah Abdul Razak, Noor Hasnita Abdul Talib, and Jasmin Ilyani Ahmad

CHAPTER 5 41
Dijkstra's Algorithm In Product Searching System (Prosearch)
Nur Hasni Nasrudin, Siti Hajar Nasaruddin, Syarifah Syafiqah Wafa Syed Abdul Halim and Rosida Ahmad Junid

CHAPTER 6 49
Developing Waqf Land Computing: A Preliminary Study On The Used Of Web-based Applications And Spatial Database
Siti Nurbaya Ismail, Zanariah Idrus, Nor Hafizah Abdul Razak

CHAPTER 7 ..... **59**
Implementation Of CORDIC Algorithm In Vectoring Mode
Anis Shahida Mokhtar, Abdullah bin Mohd Fadzullah

CHAPTER 8 ..... **71**
A Description of Projective Contractions in the Orlicz-Kantorovich Lattice
Inomjon Ganiev and M. Azram

CHAPTER 9 ..... **83**
The Geometry of the Accessible Sets of Vector Fields
A.Y.Narmanov, and I. Ganiev

CHAPTER 10 ..... **89**
Existence Result of Third Order Functional Random Integro-Differential Inclusion
D. S. Palimkar

CHAPTER 11 ..... **105**
Fourth Order Random Differential Equation
D. S. Palimkar and P.R. Shinde

CHAPTER 12 ..... **115**
New Concept of *e-I*-open and *e-I*-Continuous Functions
W.F. Al-omeri, M.S. Md. Noorani, and A. AL-Omari

CHAPTER 13 ..... **123**
Visualization of Constrained Data by Rational Cubic Ball Function
Wan Zafira Ezza Wan Zakaria, and JamaludinMd Ali

CHAPTER 14 ..... **133**
Octupole Vibrations in Even–Even Isotopes of Dy
A.A. Okhunov, G.I. Turaeva, and M. Jahangir Alam

CHAPTER 15 ..... **141**
Characterization of *p*-Groups with a Maximal Irredundant 10-Covering
Rawdah Adawiyah Tarmizi and Hajar Sulaiman

CHAPTER 16 149
Sensitivity Index of HIV-1 model Parameters with Vertical transmission
Amiru Sule, Mamman Mamuda, Abdullahi Mohammed Baba, Jibril Lawal, and I.G. Usman

CHAPTER 17 163
Derivation of Four-Point Explicit Block Methods for Direct Solution of Initial Value Problems of Third Order Ordinary Differential Equations
Z. Omar, J. O. Kuboye, and Y.A. Abdullah

CHAPTER 18 175
Absolute Translativity of Generalized Nörlund Mean
Amjed Zraiqat

CHAPTER 19 189
Type I Error of the Modified Wilcoxon Signed Rank Test under Leptokurtic Distribution
Nor Aishah Ahad, Sharipah Soaad Syed Yahaya, Suhaida Abdullah, Lim Yai Fung and Zahayu Md Yusof

CHAPTER 20 199
The Combined EWMA-CUSUM Control Chart with Autocorrelation
Abbas Umar Farouk, and Ismail Bin Mohamad

CHAPTER 21 213
Estimating Philippine Dealing System Treasury (PDST) Reference Rate Yield Curves using a State-Space Representation of the Nelson-Siegel Model
Len Patrick Dominic M. Garces, and Ma. Eleanor R. Reserva

CHAPTER 22 225
A Structural Equation Model Analyzing the Relationship Model on Perception Students toward Mathematics
Siti Fairus Mokhtar

CHAPTER 23 233
Partial Least Squares Based Financial Distressed Classifying Model of Small Construction Firms
Amirah-Hazwani Abdul Rahim, Ida-Normaya M. Nasir, Abd-Razak Ahmad, and Nurazlina Abdul Rashid

CHAPTER 24 245
Logit Bankruptcy Model of Industrial Product Firms
Asmahani Nayan, Siti-Shuhada Ishak, and Abd-Razak Ahmad

CHAPTER 25 255
Data Mining in Predicting Firms Failure: A Comparative Study Using Artificial Neural Networks and Classification and Regression Tree
Norashikin Nasaruddin, Wan-Siti-Esah Che-Hussain, Asmahani Nayan, and Abd-Razak Ahmad

CHAPTER 26 265
Risks of Divorce: Comparison between Cox and Parametric Models
Sanizah Ahmad, Norin Rahayu Shamsuddin, Nur Niswah Naslina Azid @ Maarof, and Hasfariza Farizad

CHAPTER 27 277
Reliability and Construct Validity of DASS 21 using Malay Version: A Pilot Study
Kartini Kasim, Norin Rahayu Shamsuddin, Wan Zulkipli Wan Salleh, Kardina Kamaruddin, and Norazan Mohamed Ramli

CHAPTER 28 285
Outlier Detection in Time Series Model
Nurul Sima Mohamad Shariff, Nor Aishah Hamzah, and Karmila Hanim Kamil

CHAPTER 29 297
ROAD Algorithm for Control Charts
Gejza Dohnal

CHAPTER 30 311
Learning Numerals for Down Syndrome by applying Cognitive Principles in 3D Walkthrough
Nor Intan Shafini Nasaruddin, Khairul Nurmazianna Ismail, and Aleena Puspita A.Halim

CHAPTER 31 329
Predicting Currency Crisis: An Analysis on Early Warning System from Different Perspective
Nor Azuana Ramli

CHAPTER 32 341
Using Analytic Hierarchy Process to Rank Takaful Companies based on Health Takaful Product
Noor Hafizah Zainal Aznam, Shahida Farhan Zakaria, and Wan Asma'a Wan Abu Bakar

CHAPTER 33 349
Service Discovery Mechanism for Service Continuity in Heterogeneous Network
Shaifizat Mansor, Nor Shahniza Kamal Basha, Siti Rafidah Muhamat Dawam, Noor Rasidah Ali, and Shamsul Jamel Elias

CHAPTER 34 361
Ranking Islamic Corporate Social Responsibility Activities under Product Development Theme using Analytic Hierarchy Process
Shahida Farhan Zakaria, Wan-Asma' Wan-Abu-Bakar, Roshima Said, Sharifah Nazura Syed-Noh,and Abd-Razak Ahmad

CHAPTER 35 369
A Fuzzy Rule Base System For Mango Ripeness Classification
Ab Razak Mansor, Mahmod Othman, Noor Rasidah Ali , Khairul Adilah Ahmad, and Samsul Jamel Elias

CHAPTER 36 381
Technology Assistance for Kids with Learning Disabilities: Challenges and Opportunities
Suhailah Mohd Yusof , Noor Hasnita Abdul Talib, and Jasmin Ilyani Ahmad

CHAPTER 8
A Description of Projective Contractions in the Orlicz-Kantorovich Lattice

Inomjon Ganiev and M. Azram

Abstract. In the present paper we show that any positive projective contractions $\boldsymbol{Q}$ with $\boldsymbol{Q1 = 1}$ in the Orlicz —Kantorovich lattices $\boldsymbol{L_M(\widehat{\nabla}, \widehat{\mu})}$ can be represented in the form $\boldsymbol{Q(\hat{f})(\omega) = E_\omega(f(\omega)|\nabla^1_\omega)}$ for any $\boldsymbol{\hat{f} \in L_M(\widehat{\nabla}, \widehat{\mu})}$ and for almost all $\boldsymbol{\omega \in \Omega}$, where $\boldsymbol{E_\omega(\cdot\,|\nabla^1_\omega)}$ is conditional expectation operator. Using this result we get abstract characterization conditional expectation operators in the Orlicz-Kantorovich $\boldsymbol{L_M(\widehat{\nabla}, \widehat{\mu})}$ –lattice.

Keywords: Orlicz-Kantorovich lattice; positive projective contraction; conditionally expectation operator.

Inomjon Ganiev (✉) • M. Azram
Department of Science in Engineering
Faculty of Engineering
International Islamic University Malaysia.
e-mail: inam@iium.edu.my, mazram@iium.edu.my

1 Introduction

One of the important problems of positive operator's theory is an abstract characterization of the conditional expectation operators in function spaces. In (Rao, 1976) a characterization of the conditional probability measures as subclasses of vector measures on general Banach function spaces is given. Moreover the following result is proven;

Theorem 1.1. (Rao, 1976). Let (Ω, Σ, μ) be a measurable space with a finite measure μ. If $T: L_p(\mu) \to L_p(\mu), (1 \leq p < \infty)$ is a positive projective contraction with $T1 = 1$, then $Tf = E(f|F)$, $f \in L_p(\mu)$, for a unique σ-subalgebra $F \subset \Sigma$. Where $E(\cdot\,|F)$ is conditional expectation operator relative to F.

In (Rao, 1965) this theorem is proven for Orlicz spaces. In (Rao, 1976), necessary and sufficient conditions for $T: L_1(\mu) \to L_1(\mu)$ to be conditional expectation operator relative to F is obtained. Dodds, Huijsmans and De Pagter in (Dodds et al., 1965) extended these result to the vector lattices. We recall that in the theory of Banach bundles L_0-valued Banach spaces are considered, and such spaces are called Banach–Kantorovich spaces. In (Gutman, 1993), (Gutman, 1995) the theory of Banach–Kantorovich spaces is developed. Analogues of many well-known functional spaces have been defined and studied. For example, in (Ganiev, 2006) Banach-Kantorovich lattice $L_p(\hat{V}, \hat{\mu})$ is represented as a measurable bundle of classical L_p –lattices. In (Zakirov & Chilin, 2009), (Zakirov, 2007) an analogue of the Orlicz spaces has been considered. Naturally, these functional Kantorovich spaces should have many properties similar to the classical ones constructed by the real valued measures.

To investigate the properties of Banach–Kantorovich spaces it is natural to use measurable bundles of such spaces. Since, one has a sufficiently well explored theory of measurable bundles of Banach lattices (Ganiev, 2006), it is an effective tool which gives well opportunity to obtain various properties of Banach–Kantorovich spaces. It is worthy to mention that using this way, weighted ergodic theorems for positive contractions of Banach-Kantorovich lattices $L_p(\hat{V}, \hat{\mu})$, have been established (Chilin & Ganiev (2000))., (Ganiev & Mukhamedov, 2013) .

Definition 1.2. The L_0–linear, L_0–bounded positive operator T from $L_p(\hat{V}, \hat{\mu})$ onto (bo)–closet vector subspace $L_p(\hat{V}^1, \hat{\mu}^1)$ of $L_p(\hat{V}, \hat{\mu})$ is said to be

conditional expectation operator with respect to the regular Boolean sub-algebra $\hat{\nabla}^1$ if $\int T(\hat{f})d\hat{\mu} = \int \hat{f}\, d\hat{\mu}$ and it is denoted by $T = E(\cdot\,|\hat{\nabla}^1)$.

In (Kusraev, 1985), Theorem 4.2.9 it has been proven that there exists conditional expectation operator $E(\cdot\,|\hat{\nabla}^1): L_1(\hat{\nabla}, \hat{\mu}) \to L_1(\hat{\nabla}^1, \hat{\mu}^1)$ satisfying the following conditions:

1) $E(\cdot\,|\hat{\nabla}^1)$ is linear, positive, idempotent operator;

2) $\int E(\hat{f}|\hat{\nabla}^1)d\hat{\mu} = \int \hat{f}\, d\hat{\mu}$;

3) $E(\hat{g}\hat{f}|\hat{\nabla}^1) = \hat{g}E(\hat{f}|\hat{\nabla}^1)$ for any $\hat{g} \in L^{\infty}(\hat{\nabla}^1, \hat{\mu}^1)$ è $\hat{f} \in L_1(\hat{\nabla}, \hat{\mu})$.

It means that $E(\cdot\,|\hat{\nabla}^1)$ is projective contraction in the Banach —Kantorovich lattice $L_1(\hat{\nabla}, \hat{\mu}$. In this case $\| E(\hat{f}|\hat{\nabla}^1) \|_{L_1(\hat{\nabla},\hat{\mu})} \leq \| \hat{f} \|_{L_1(\hat{\nabla},\hat{\mu})}$ for any $\hat{f} \in L_1(\hat{\nabla}, \hat{\mu})$ and $E(\mathbf{1}|\hat{\nabla}^1) = \mathbf{1}$.

Let Banach-Kantorovich lattice $L_p(\hat{\nabla}, \hat{\mu})$ be represented as a measurable bundle of classical $L_p(\nabla_\omega, \mu_\omega)$–lattices. The description of conditional expectation operator $E(\cdot\,|\hat{\nabla}^1): L_1(\hat{\nabla}, \hat{\mu}) \to L_1(\hat{\nabla}^1, \hat{\mu}^1)$ is obtained in (Ganiev, 2006).

Theorem 1.3. Let $E(\cdot\,|\hat{\nabla}^1): L_1(\hat{\nabla}, \hat{\mu}) \to L_1(\hat{\nabla}^1, \hat{\mu}^1)$ be conditional expectation operator. Then for any $\omega \in \Omega$ there exists $E_\omega(\cdot\,|\nabla_\omega^1): L_1(\nabla_\omega, \mu_\omega) \to L_1(\nabla_\omega^1, \mu_\omega^1)$ conditionally expectation operator, such that $E(\hat{f}|\hat{\nabla}^1)(\omega) = E_\omega(f(\omega)|\nabla_\omega^1)$ for any $\hat{f} \in L_1(\hat{\nabla}, \hat{\mu})$ and for almost all $\omega \in \Omega$, where $E_\omega(\cdot\,|\nabla_\omega^1)$ is conditional expectation operator on $L_p(\nabla_\omega, \mu_\omega)$.

Consequences of the development of the general theory, conditional expectation operators in Banach — Kantorovich lattices $L_p(\hat{\nabla}, \hat{\mu})$ over the ring of measurable functions gives rise the problem of an abstract characterization conditional expectation operators in Banach — Kantorovich lattices $L_p(\hat{\nabla}, \hat{\mu})$, which are reasonably solved using the method of measurable bundles. In the present paper we will show that any positive projective contractions Q with $Q\mathbf{1} = \mathbf{1}$ in the Orlicz —Kantorovich lattices $L_M(\hat{\nabla}, \hat{\mu})$ can be represented in the form $Q(\hat{f})(\omega) = E_\omega(f(\omega)|\nabla_\omega^1)$ for any $\hat{f} \in L_M(\hat{\nabla}, \hat{\mu})$ and for almost all $\omega \in \Omega$, where $E_\omega(\cdot\,|\nabla_\omega^1)$ is conditional expectation operator. To prove the main result of this paper we are going to use measurable bundles of Banach — Kantorovich lattices. We note that one of the effective methods to study of Banach —Kantorovich spaces is measurable bundles (Gutman, 1995).

In (Ganiev & Mukhamedov, 2013) prove weighted ergodic theorems and multiparameter weighted ergodic theorems for positive contractions acting on $L_p(\hat{\nabla}, \hat{\mu})$. In (Ganiev & Mukhamedov, 2015) this results generalized for Orlicz-Kantorovich $L_M(\hat{\nabla}, \hat{\mu})$ –lattice.

2 Preliminaries

In this section we recall necessary definitions and results concerning Banach-Kantorovich lattices.

Let (Ω, Σ, μ) be a space with complete finite measure, $L_0 = L_0(\Omega)$ be the algebra of classes of measurable functions on (Ω, Σ, μ). Consider a real vector space E.

A transformation $\|\cdot\|: E \longrightarrow L_0$ is called vector-valued or L_0–valued norm on E, if it satisfies the following conditions:

i) $\| x \| \geq 0$ for all $x \in E$; $\| x \| = 0 \Leftrightarrow x = 0$;

ii) $\| \lambda x \| = |\lambda| \| x \|$ for all $\lambda \in R, x \in E$;

iii) $\| x + y \| \leq \| x \| + \| y \|$ for all $x, y \in E$.

A pair $(E, \|\cdot\|)$ is said to be a lattice-normed space(LNS) over L_0.

An LNS E is disjunctively decomposed or shortly, d — decomposed, if the following axiom is fulfilled :

For any $x \in E$ and disjunct elements $e_1, e_2 \in L_0$, satisfying $\| x \| = e_1 + e_2$, there exist $x_1, x_2 \in E$ such that $x = x_1 + x_2$, $\| x_1 \| = e_1$ and $\| x_2 \| = e_2$.

A net $\{x_\alpha\} \in E$ is (bo)– convergent to $x \in E$, if a net $\{\| x_\alpha - x \|\}$ is (o) – convergent to L_0.

We say that an LNS is *(bo)*– complete, if any *(bo)* – fundamental net $\{x_\alpha\}$ *(bo)* – converges to some element of this space.

Any d —decomposable and *(bo)* – complete LNS over L_0 is said to be a *Banach-Kantorovich space* (BKS) over L_0 (Kusraev, 1985).

If a Banach-Kantorovich space is simultaneously a vector lattice and the norm is monotone, then it becomes a *Banach — Kantorovich lattice.*
Let X be a mapping, which maps every point $\omega \in \Omega$ to some Banach space

$(X(\omega), \|\cdot.\,\|_{X(\omega)})$. In what follows, we assume that $X(\omega) \neq \{0\}$ for all $\omega \in \Omega$. A function u is said to be a section of X, if it is defined almost everywhere in Ω and takes its value $u(\omega) \in X(\omega)$ for $\omega \in dom(u)$, where $\omega \in dom(u)$ is the domain of u.

Let L be some set of sections.

Definition 2. 1. (Gutman, 1995). A pair (X, L) is said to be a measurable bundle of Banach spaces over Ω if

i. $\lambda_1 c_1 + \lambda_2 c_2 \in L$ *for all* $\lambda_1, \lambda_2 \in \mathbb{R}$ *and* $c_1, c_2 \in L$, *where* $\lambda_1 c_1 + \lambda_2 c_2 : \omega \in dom(c_1) \cap dom(c_2) \to \lambda_1 c_1(\omega) + \lambda_2 c_2(\omega)$;

ii. The function $||c||: \omega \in dom(c) \to ||c(\omega)||_{X(\omega)}$ *is measurable for all* $c \in L$;

iii. For *every* $\omega \in \Omega$ *the set* $\{c(\omega): c \in L, \omega \in dom(c)\}$ *is dense in* $X(\omega)$;

A measurable Banach bundle (X, L) is called measurable bundle of Banach lattices (MBBL), if $(X(\omega), \|\cdot\|_{X(\omega)})$ are Banach lattices for all $\omega \in \Omega$ and all $c_1, c_2 \in L$ $c_1 \vee c_2 \in L$, where $c_1 \vee c_2$: $\omega \in \mathrm{dom}\,(c_1) \cap \mathrm{dom}\,(c_2) \to c_1(\omega) \vee c_2(\omega)$.

Henceforth, (X, L) will be denoted just by X.

A section s is a step-section, if there are pairwise disjoint sets $A_1, A_2, \dots, A_n \in \Sigma$ and sections $c_1, c_2, \dots, c_n \in L$ such that $\bigcup_{i=1}^{n} A_i = \Omega$ è $s(\omega) = \sum_{i=1}^{n} \chi_{A_i}(\omega) c_i(\omega)$ for almost all $\omega \in \Omega$.

A section u is measurable, if for any $A \in \Sigma$ there is a sequence s_n of step-sections such that $s_n(\omega) \to u(\omega)$ for almost all $\omega \in A$.

Let $M(\Omega, X)$ be the set of all measurable sections. By symbol $L_0(\Omega, X)$ we denote factorization of the $M(\Omega, X)$ with respect to almost everywhere equality. Usually, by $\hat{u}$ we denote a class from $L_0(\Omega, X)$, containing the

section $u \in M(\Omega, X)$, and by $\| \hat{u} \|$ we denote the element of $L_0(\Omega)$, containing $\| u(\omega) \|_{X(\omega)}$.

Let X be an MBBL. We set $\hat{u} \leq \hat{v}$, if $u(\omega) \leq v(\omega)$ a.e. One can easily show that the relation $\hat{u} \leq \hat{v}$ constitutes a partial order on $L_0(\Omega, X)$.

If X is an MBBL, then $L_0(\Omega, X)$ is a Banach-Kantorovich lattice (Chilin & Ganiev, 2000).

Let $\nabla_\omega, \omega \in \Omega$ be a family of complete Boolean algebras with strictly positive real-valued measures μ_ω. We set $\rho_\omega(e, g) = \mu_\omega(e \triangle g)$, $e, g \in \nabla_\omega$. Then $(\nabla_\omega, \mu_\omega)$are complete metric spaces. Consider the transformation ∇, which assigns some Boolean algebra ∇_ω to every point $\omega \in \Omega$. Let L be a non-empty set of sections ∇.

Definition 2. 2. A pair (∇, L) is called a measurable bundle of boolean algebras over Ω, if

i) (∇, L) is a measurable bundle of metric spaces (Chilin & Ganiev, 2000);

ii) If $e \in L$, then $e^\perp \in L$, where $e^\perp : \omega \in \mathrm{dom}\,(e) \to e^\perp(\omega)$;

iii) If $e_1, e_2 \in L$, then $e_1 \vee e_2 \in L$, where

$$e_1 \vee e_2 : \omega \in \mathrm{dom}\,(e_1) \cap \mathrm{dom}\,(e_2) \to e_1(\omega) \vee e_2(\omega) \qquad (1)$$

Let $M(\Omega, \nabla)$ be the set of measurable sections, $\hat{\nabla}$– factorization of $M(\Omega, \nabla)$ with respect to almost everywhere equality. Define a transformation $\hat{\mu} : \hat{\nabla} \to L_0(\Omega)$ by $\hat{\mu}(\hat{e}) = \hat{f}$, where $\hat{f}$ is a class containing the function $f(\omega) = \mu_\omega(e(\omega))$. Evidently, $\hat{\mu}$ is well defined. It is well known that $(\hat{\nabla}, \hat{\mu})$ is a complete boolean algebra with strictly positive $L_0(\Omega)$ – valued modulated measure $\hat{\mu}$, moreover, the boolean algebra $\nabla(\Omega)$ of all idempotents from $L_0(\Omega)$ is identified with regular sub-algebra in $\hat{\nabla}$ and $\hat{\mu}(g\hat{e}) = g\hat{\mu}(\hat{e})$ for all $g \in \nabla(\Omega)$ and $\hat{e} \in \hat{\nabla}$. By $L_0(\hat{\nabla}, \hat{\mu})$ we denote an order complete vector lattice $C_\infty(Q(\hat{\nabla}))$, where $Q(\hat{\nabla})$ is the Stonian compact associated with complete Boolean algebra $\hat{\nabla}$. Following the well-known scheme of the construction of L_p-spaces, a space $L_p(\hat{\nabla}, \hat{\mu})$ can be defined by

$$L_p(\hat{\nabla}, \hat{\mu}) = \left\{ \hat{f} \in L_0(\hat{\nabla}, \hat{\mu}) : \int |\hat{f}|^p d\hat{\mu} - \text{exist} \right\}, \quad p \geq 1 \qquad (2)$$

where $\hat{\mu}$ is an $L_0(\Omega)$-valued measure on $\hat{\nabla}$.

It is known (Kusraev, 1985) that $L_p(\hat{\nabla},\hat{\mu})$ is a BKS over $L_0(\Omega)$ with respect to the $L_0(\Omega)$-valued norm $\| \hat{f} \|_{L_p(\hat{\nabla},\hat{\mu})} = (\int |\hat{f}|^p d\hat{\mu})^{1/p}$. Moreover, $L_p(\hat{\nabla},\hat{\mu})$ is a Banach-Kantorovich lattice (Kusraev, 1985).

An even continuous convex function $M: R \to [0,\infty)$ is called an N-function, if $\lim_{t\to 0}\frac{M(t)}{t} = 0$ and $\lim_{t\to\infty}\frac{M(t)}{t} = \infty$. An N-function M is said to satisfy Δ_2-condition on $[s_0,\infty), s_0 \geq 0$, if there exists constant k such that $M(2s) \leq kM(s)$ for every $s \geq s_0$ (see (Krasnoselskii et al., 1961)). The set

$$L_M^0 := L_M^0(\hat{\nabla},\hat{\mu}) := \{x \in L_0(\hat{\nabla}): M(x) \in L_1(\hat{\nabla},\hat{\mu})\} \quad (3)$$

is called the Orlicz L_0-class, and the vector space

$$L_M := L_M(\hat{\nabla},\hat{\mu}) := \{x \in L_0(\hat{\nabla},\hat{\mu}): xy \in L_1(\hat{\nabla},\hat{\mu}) for \quad all \quad y \in L_N^0\} \quad (4)$$

is called the Orlicz L_0-space, where N is the complementary N-function to M.

We notice that, $L_M(\hat{\nabla},\hat{\mu}) \subset L_1(\hat{\nabla},\hat{\mu})$.

Define the L_0-valued Orlicz norm on $L_M(\hat{\nabla},\hat{\mu})$ as follows:

$$\| x \|_M := \sup\{|\int x\, y d\hat{\mu}|: y \in A(N)\}, x \in L_M(\hat{\nabla},\hat{\mu}), \quad (5)$$

where $A(N) = \{y \in L_N^0: \int N(y) d\hat{\mu} \leq \mathbf{1}\}$ and $\mathbf{1}$ is identity element of L_0. The pair $(L_M(\hat{\nabla},\hat{\mu}), \|\cdot\|_M)$ is a Banach–Kantorovich lattice which is called the Orlicz–Kantorovich lattice associated with the L_0-valued measure (Zakirov & Chilin, 2009), (Zakirov, 2007).

Theorem 2.3. (Zakirov & Chilin, 2009). If the N-function M meets the Δ_2-condition then the Orlicz-Kantorovich lattice $L_M(\hat{\nabla},\hat{\mu})$ is isometrically and order isomorphic to $L_0(\Omega, X)$, where (X, L) is the measurable Banach bundle over Ω such that $X(\omega) = L_M(\nabla_\omega, \mu_\omega)$ and

$$L = \left\{\sum_{i=1}^n \alpha_i\, e_i: \alpha_i \in \mathbb{R},\ e_i \in M(\Omega,\nabla), i = \overline{1,n}, n \in \mathbb{N}\right\}.$$

3 A Description of Projective Contractions in the contractions in the Orlicz- Kantorovich lattice

In this section we will show that any positive projective contractions Q with $Q\mathbf{1} = \mathbf{1}$ in the Orlicz —Kantorovich lattices $L_M(\hat{\nabla}, \hat{\mu})$ can be represented in the form

$$Q(\hat{f})(\omega) = E_\omega(f(\omega)|\nabla_\omega^1) \tag{7}$$

for any $\hat{f} \in L_M(\hat{\nabla}, \hat{\mu})$ and for almost all $\omega \in \Omega$, where $E_\omega(\cdot\,|\nabla_\omega^1)$ is conditional expectation operator.

Proposition 3.1. Let M be an N-function, and $E(\cdot\,|\hat{\nabla}^1)$: $L_1(\hat{\nabla}, \hat{\mu}) \to L_1(\hat{\nabla}^1, \hat{\mu}^1)$ be conditionally expectation operator. Then

$$E(L_M(\hat{\nabla}, \hat{\mu})|\hat{\nabla}^1) \subset L_M(\hat{\nabla}, \hat{\mu}) \tag{8}$$

and

$$\| E(\cdot\,|\hat{\nabla}^1) \|_{L_M(\hat{\nabla},\hat{\mu})\to L_M(\hat{\nabla},\hat{\mu})} = \mathbf{1}. \tag{9}$$

Proof: Since $\| E(\hat{f}|\hat{\nabla}^1) \|_{L_1(\hat{\nabla},\hat{\mu})} \leq \| \hat{f} \|_{L_1(\hat{\nabla},\hat{\mu})}$ for any $\hat{f} \in L_1(\hat{\nabla}, \hat{\mu})$ and $E(\mathbf{1}|\hat{\nabla}^1) = \mathbf{1}$ by Proposition 3.1 (Zakirov & Chilin, 2009)

$$E(L_M(\hat{\nabla}, \hat{\mu})|\hat{\nabla}^1) \subset L_M(\hat{\nabla}, \hat{\mu}). \tag{10}$$

As

$$\| E\big(\hat{f}\big|\hat{\nabla}^1\big) \|_M\,(\omega) = \| E\big(\hat{f}\big|\hat{\nabla}^1\big)(\omega) \|_{L_M(\nabla_\omega,\mu_\omega)} = \| E_\omega(f(\omega)|\nabla_\omega^1) \|_{L_M(\nabla_\omega,\mu_\omega)} \leq \| f(\omega) \|_{L_M(\nabla_\omega,\mu_\omega)} = \| \hat{f} \|_M\,(\omega) \tag{11}$$

a.e. we get

$$\| E(\hat{f}|\hat{\nabla}^1) \|_M \leq \| \hat{f} \|_M \quad (12)$$

or

$$\| E(\cdot|\hat{\nabla}^1) \|_{L_M(\hat{\nabla},\hat{\mu})\to L_M(\hat{\nabla},\hat{\mu})} \leq \mathbf{1}. \quad (13)$$

As $\| E_\omega(f(\omega)|\nabla_\omega^1) \|_{L_M(\nabla_\omega,m_\omega)} = \| f(\omega) \|_{L_M(\nabla_\omega,\mu_\omega)}$ for almost all $\omega \in \Omega$ and for any $\{f(\omega)\}_{\omega\in\Omega} = \hat{f}$ with $f(\omega) \in L_M(\nabla_\omega^1, \mu_\omega^1)$ we have that

$$\| E(\cdot|\hat{\nabla}^1) \|_{L_M(\hat{\nabla},\hat{\mu})\to L_M(\hat{\nabla},\hat{\mu})} = \mathbf{1}. \quad (14)$$

Let N-function M is said to satisfy Δ_2-condition.

Theorem 3.2. Let $Q: L_M(\hat{\nabla},\hat{\mu}) \to L_M(\hat{\nabla},\hat{\mu})$ be a linear positive operator. If

1. $Q^2 = Q$;
2. $\| Q \|_{L_1(\hat{\nabla},\hat{\mu})\to L_1(\hat{\nabla},\hat{\mu})} \leq 1$;
3. $Q(\mathbf{1}) = \mathbf{1}$;

then

i. $\| Q \|_{L_M(\hat{\nabla},\hat{\mu})\to L_M(\hat{\nabla},\hat{\mu})} \leq \mathbf{1}$;

ii. $Q(\hat{f})(\omega) = E_\omega(f(\omega)|\nabla_\omega^1)$ for any $\hat{f} \in L_M(\hat{\nabla},\hat{\mu})$ and for almost all $\omega \in \Omega$.

Proof:

Let Q_ω be a linear contractions on $L_1(\nabla_\omega,\mu_\omega)$ constructed in Theorem 3.1, such that $Q(\hat{f})(\omega) = Q_\omega(f(\omega))$ for $\hat{f} \in L_1(\hat{\nabla},\hat{\mu})$ and for almost all $\omega \in \Omega$. Since $\| Q_\omega \|_{L_1(\nabla_\omega,\mu_\omega)\to L_1(\nabla_\omega,\mu_\omega)} \leq 1$ and $Q_\omega(\mathbf{1}_\omega) = \mathbf{1}_\omega$ by (Krasnoselskii et al., 1961) (II. sec. 4. Item 6) we have that $\| Q_\omega \|_{L_M(\nabla_\omega,\mu_\omega)\to L_M(\nabla_\omega,\mu_\omega)} \leq 1$. Using Proposition 2.3 (Zakirov & Chilin, 2009) we get that

$$\left(\| Q(\hat{f}) \|_{L_M(\hat{\nabla},\hat{\mu})}\right)(\omega) = \| Q_\omega(f(\omega)) \|_{L_M(\nabla_\omega,\mu_\omega)} \leq \| f(\omega) \|_{L_M(\nabla_\omega,\mu_\omega)} = \| \hat{f} \|_{L_p(\hat{\nabla},\hat{\mu})}(\omega)$$

for almost all $\omega \in \Omega$, i.e.

$$\| Q(\hat{f}) \|_{L_M(\hat{\nabla},\hat{\mu})} \leq \| \hat{f} \|_{L_p(\hat{\nabla},\hat{\mu})} \text{ or } \| Q \|_{L_M(\hat{\nabla},\hat{\mu})\to L_M(\hat{\nabla},\hat{\mu})} \leq \mathbf{1}. \quad (15)$$

As $Q_\omega^2 = Q_\omega$, $\| Q_\omega \|_{L_M(\nabla_\omega,\mu_\omega)\to L_M(\nabla_\omega,\mu_\omega)} \leq 1$, by (Rao, 1965) there exists a unique regular sub-algebra ∇_ω^1 of ∇_ω, such that

$$Q_\omega = E_\omega(\cdot\,|\nabla_\omega^1). \tag{16}$$

Hence $Q(\hat{f})(\omega) = E_\omega(f(\omega)|\nabla_\omega^1)$ for any for any $\hat{f} \in L_M(\hat{\nabla},\hat{\mu})$ and for almost all $\omega \in \Omega$.

Theorem 3.3. *Let* $\hat{f} \in L_1(\hat{\nabla},\hat{\mu})$ *then*

i. $|E(\hat{f}|\hat{\nabla}^1)| \leq E(\widehat{|f|}|\hat{\nabla}^1)$;

ii. *Let* $\hat{f}_n \in L_1(\hat{\nabla},\hat{\mu})$ *such that*

iii.

1) $|\hat{f}_n| \leq \widehat{g} \in L_1(\hat{\nabla},\hat{\mu})$ *and*

2) $\hat{f}_n \overset{(o)}{\to} \hat{f}$ *then*

$$E(\hat{f}_n|\hat{\nabla}^1) \overset{(o)}{\to} E(\hat{f}|\hat{\nabla}^1). \tag{17}$$

Proof:

(i) The numerical case proof is applicable here (see (Doob, 1953)).

(ii) From 1) it follows that $|f_n(\omega)| \leq g(\omega) \in L_1(\nabla_\omega,\mu_\omega)$ then by (Doob, 1953) $E_\omega(f_n(\omega)|\nabla_\omega^1) \overset{(o)}{\to} E_\omega(f(\omega)|\nabla_\omega^1)$ for almost all $\omega \in \Omega$.

Since $\left(E(\hat{f}_n|\hat{\nabla}^1) - E(\hat{f}|\hat{\nabla}^1)\right)(\omega) = E_\omega(f_n(\omega)|\nabla_\omega^1) - E_\omega(f(\omega)|\nabla_\omega^1)$ (18)

we get that $\left(E(\hat{f}_n|\hat{\nabla}^1) - E(\hat{f}|\hat{\nabla}^1)\right)(\omega) \overset{(o)}{\to} 0$ for almost all $\omega \in \Omega$. Then using Theorem 4.1 (Ganiev, 2006) we obtain

$$E(\hat{f}_n|\hat{\nabla}^1) \overset{(o)}{\to} E(\hat{f}|\hat{\nabla}^1). \tag{19}$$

Theorem 3.4. If $T: L_0 \to L_0$ is L_0-linear and L_0-bounded operator and $\hat{f} \in L_1(\hat{\nabla}, \hat{\mu})$, *then*

$$E\left(T(\hat{f})\middle|\hat{\nabla}^1\right) = T(E\left(\hat{f}\middle|\hat{\nabla}^1\right)).$$

4 Conclusion

Any positive projective contractions Q with $Q\mathbf{1} = \mathbf{1}$ in the Orlicz — Kantorovich lattices $L_M(\hat{\nabla}, \hat{\mu})$ can be represented in the form

$$Q(\hat{f})(\omega) = E_\omega(f(\omega)|\nabla_\omega^1)$$

for any $\hat{f} \in L_M(\hat{\nabla}, \hat{\mu})$ and for almost all $\omega \in \Omega$, where $E_\omega(\cdot\,|\nabla_\omega^1)$ is conditional expectation operator.

Acknowledgements.

The first author acknowledges the MOHE Grant FRGS13-071-0312.

References

[1] Rao, M.M. (1976). Two characterizations of conditional probability, Proc. Amer. Math. Soc. 19 75-80.

[2] Rao, M.M. (1965). Conditional expectations and closed projections, Nederl. Akad. Wetensch. Proc. Ser. A 68 = Indag. Math. 27,100-112.

[3] Dodds, P.G., Huijsmans, C.B.& de Pagter B. (1990). Characterizations of conditional expectation-type operators, Pacific J. Math. 141 55–77.

[4] Gutman, A.E. (1993) Banach bundles in the theory of lattice-normed spaces, III. Siberien Adv. Math. 3, n.4, 8-40.

[5] Gutman, A.E. (1995). Banach fiberings in the theory of lattice-normed spaces. Order-compatible linear operators, Trudy Inst. Mat. 29, 63-211. (Russian).

[6] Ganiev I.G. (2006). *Measurable bundles of lattices and their application.* In : Studies on Functional Analysis and its Applications, pp. 9–49. Nauka, Moscow (Russian).

[7] Zakirov, B.S.& Chilin, V.I. (2009). Ergodic theorems for contractions in Orlicz–Kantorovich lattices. Sib. Math. J. 1027–1037.

[8] Zakirov, B.S. (2007). Orlicz–Kantorovich lattices associated with an L_0-valued measure. Uzb. Mat. Zh. No. 4, 18–34 (Russian).

[9] Chilin, V.I.& Ganiev. I.G. (2000). An individual ergodic theorem for contractions in the Banach-Kantorovich lattice $L^p(\nabla, \mu)$. Russian Math. (Iz. VUZ) no. 7, 77–79.

[10] Ganiev, I.G.& Mukhamedov, F.M. (2013). Weighted Ergodic Theorems for Banach–Kantorovich Lattice $L_p(\nabla, \mu)$, Lobachevskii Jour. Math. Vol. 34, No. 1, 1–10.

[11] Ganiev, I.G.& Mukhamedov, F.M. (2015).Weighted ergodic theorems for Orlicz- Kantorovich lattice . Bulletin Malaysian Mathematics Society. 38 (1). 387-397.

[12] Kusraev A.G. 1985. *Vector duality and its applications*. Novosibirsk, Nauka. (Russian).

[13] Krasnoselskii, M. A.& Rutitski, Ya. B. (1961). Convex functions and Orlicz spaces. Translated from the first Russian edition. Groningen.

[14] Doob, J. L. (1953). Stochastic processes. New York: Wiley.

ISBN 978-967-0314-25-6

View publication stats