

Examination Timetabling for Undergraduate Programme using Graph Coloring Approach

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Abstract: One of the most common academic scheduling issues that can be seen in any educational system is the generation of examination time tables. Traditionally, the system of managing a timetable was done manually by paper work and could create havoc if there were changes to be made. Nowadays with the advent of advanced computer softwares, it is feasible to develop a good quality timetable management system. Issues such as having a large number of students as well as courses offered could be tackled easily within a short period of time. Any clashes of timetabling that arise can be detected and remedied immediately. An algorithm based on Graph coloring technique is one of the feasible solutions that can address these issues of managing timetables. In this study, a systematic model was developed by using a graph coloring technique to generate an examination timetable based on the database obtained from the Examination Unit in UiTM Cawangan Kelantan, Machang Campus. In the problem domain, types of constraints are defined as hard and soft in order to accommodate certain decisions. Major part of solving is focused on the degree of constraint satisfaction. Workflow of the system is described by using a case study and efficient output was generated. The chromatic and clique numbers are also obtained in this paper.

Keywords: Examination timetable, Graph coloring, Chromatic number, Clique number

1 Introduction

Graph coloring has many applications in real time, of which include the map coloring, scheduling problem, parallel computing, network architecture, SUDOKU, allocation of registers and bipartisan graph detection. On top of that it relates considerably to a wide range of dynamic problems including optimization. Conflict resolutions or the optimal partitioning of mutually exclusive occurrences, can also be accomplished by graph coloring as well. Examples of these issues are timetables for classes or exams.

Voloshin [1] defined the basic principle of coloring began with the problem of coloring the map of some countries in such a way that no two countries with a common border got the same colour. In general, the principle of coloring is known as the conflict theory; adjacent vertices in a graph must always have distinct colors.

Earlier there were other researches that used the same graph coloring concept. First, Hussin et al. [2] stated that if all the soft constraints under consideration are reduced, the review schedule shall be regarded as good enough. The use of clustering heuristic is very useful in decomposing the subject based on its characteristic and this filtering strategy is very effective when the researcher uses the conflict matrix to identify the subject that is clashing. It allows the researcher to recheck the clashing of topics based on cluster color with the conflict matrix table. Ayob et al. [3] also used a graph coloring approach to design and implement a constructive heuristic to solve the Universiti Kebangsaan Malaysia (UKM) timetabling problem.

Ganguli and Roy [4] also stated that the complexity of a problem with scheduling is directly proportional to the number of constraints involved. Although all solutions can be said to be optimal when

solved using a minimum number of colors, a better schedule is one that maximizes soft constraints satisfaction among its alternative solutions. Nandhini et al. [5] did the research about subject scheduling problem and exam timetable scheduling problem using graph coloring method by taking the list of subjects number of faculties available for the exams and the number of rooms available for the exams. Burke and Bykov [6] said that the quest is intended to take place one time at a time, first locating exams for that period, then fitting them in the rooms so that any leftover can then be rescheduled to a new period as mentioned. The user or the solutions of the algorithms, amended to match, can impose or lift constraints at each step.

Besides, Malkawi et al. [7] presented a graph-coloring-based algorithm for the examination scheduling application which achieves the goals of fairness, accuracy and optimum examination time. Furthermore, there were also other researches using algorithm techniques. Akbulut and Yılmaz [8] combined two approaches which were graph coloring and algorithm, mentioned that, many approaches to artificial intelligence are used to investigate the timetable issue. Lastly, Pattillo and Butenko [9] studied the relationship between maximum clique, maximum independent set, graph coloring and minimum clique partitioning problems.

This paper applies a new real-world capacitated examination timetabling data set at the Universiti Teknologi MARA (UiTM) Cawangan Kelantan, Kampus Machang that has more practical constraints as compare to existing benchmark examination data sets. It focused on graph coloring technique to create examination timetable.

2 Methodology

In this paper, there are four steps that are used in order to create examination timetabling. The data set was collected from Examination Unit in UiTM Cawangan Kelantan, Kampus Machang.

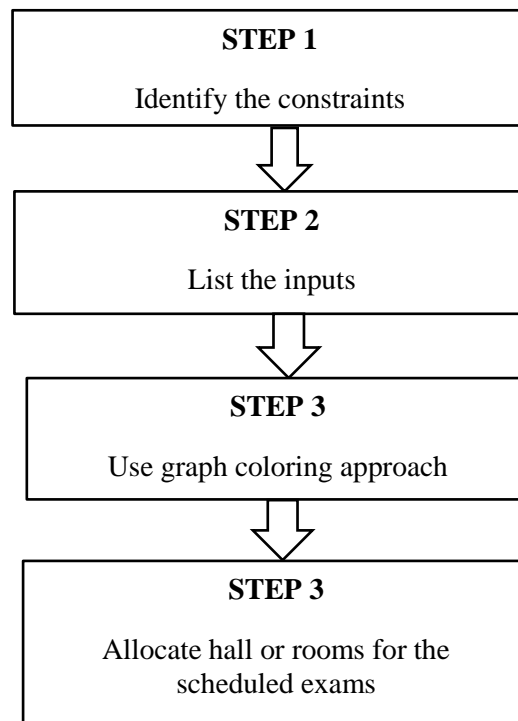


Figure 1: Steps to create an examination timetable using graph coloring technique

STEP 1: Identify the constraints

Constraints of the exam timetabling problem must be determined before the input details are being examined. These are the constraints that are classified as "hard" and "soft" for the problem:

A. Hard constraint is a constraint that must be satisfied by any feasible solution to the model.

- A student cannot take more than one exam in the same time period.
- A student should have not more than three exams on the same day.
- The number of students who sit for exams must not exceed the available hall or room capacity in the same period.
- Exams that take place within the same period of time must be of the same duration.

B. Soft constraints may be violated but if fulfilled it will make a good result.

- The most crowded exams must be put in the timetable primarily.
- A student is predicted to have only one exam in one day according to the given ratio.
- There must not be more than four exams at the same time.

STEP 2: List the inputs

This paper also needs essential inputs to generate the exam timetable. The inputs are as follows:

1. The time and date of the examinations.
2. The list of subjects or examination papers offered in each semester.
3. The number of students registered in each paper.
4. The capacity of the hall and each room must be recorded.

Table 1 showed the subjects taken by the students for Part 4, 5 and 6. The columns represent the groups that take the same subjects which are 13 groups for Part 4, 18 groups for Part 5 and 17 groups for Part 6. Number "1" means that the subject taken by each group. There are 245 total students in Part 4, 5 and 6. Numbers of students were separated by each group in the second row. The last row showed the total number of subjects taken by each group. Last column stated the total number of students take that subject. Degree showed the frequency of the subject taken by each group. Highest degree means that many groups take that particular subject and vice versa. Knowing the highest degree until lowest degree is very important to start the graph coloring.

STEP 3: Use Graph Coloring Approach

There are four steps to follow in order to obtain final graph coloring.

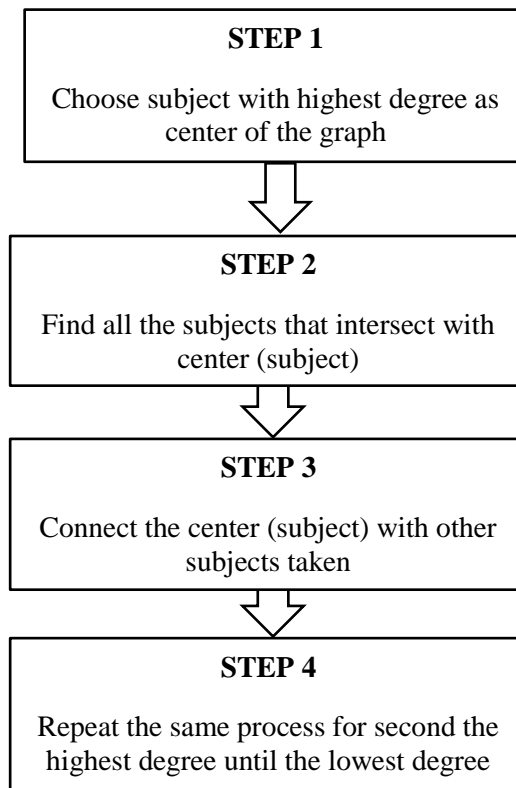


Figure 2: Flow in Graph Coloring Approach

By using the degree from Table 1, the graph is sketched. From the observation, the highest degree is MAT612 with 23 degrees, so let MAT612 be the center of the graph and color it with yellow. Next, find all the subjects that intersect with the MAT612. Intersect means the group that takes MAT612 also takes other subjects. Data shows that MAT612 intersects with MAT578, MAT631, ACC516, STA470, ISP565, MAT668, DSC551, MAT652, ACC466, MAT530, MAT571, MAT560, and QMT634. It indicates that MAT612 and other subjects listed above cannot have an exam at the same time because if it happens, the exam will be clashed which means the students have to take more than one subject in the same period. Then, MAT612 will adjacent to the other subjects. The adjacent vertices cannot have the same color of the vertex. The same process will be repeated for another 19 subjects according to the next highest degree until lowest degree.

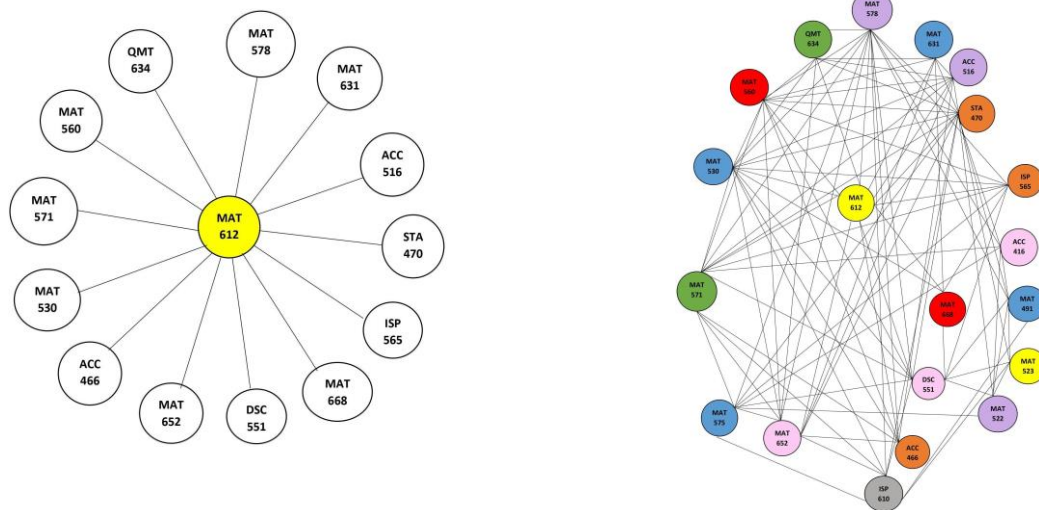


Figure 3: First Step of Graph Coloring

Lastly, MAT523 has the least degree which is one. MAT523 is intersected with MAT571, STA470, DSC551, MAT522 and ISP610. MAT523 needed to be colored with new color but remember to minimize the color used. Starting to check with first color that used in the graph which is yellow. Noticed that MAT523 does not adjacent with MAT612. This means that MAT523 can be colored as yellow. Thus, this is the final step of graph coloring that all the 20 vertices are colored.

STEP 4: Allocate hall or rooms for the scheduled exams

The examination halls or rooms are listed according to their sizes. The size of an examination indicates the numbers of students sitting for the examinations while the size of a hall or room indicates the capacity of the hall or room that will be the venue to conduct the examination. After finishing the graph coloring method, takes the smallest exam and proceeds through the list. The exam will be put in the smallest room it suits. If the size of the exam is more than the capacity of the hall or rooms, this exam will replace by the minimum number of students, and the exam will be proceed to the next larger room. Therefore, the combined size of the remaining exams is smaller than the size of the room, repeated with the next larger room and others.

There are 3 rooms provided in a day to conduct the examination. A room can have maximum 60 students. Therefore, the combined size of the remaining exams is smaller than the size of the room, repeated with the next larger room and others.

Next, the chromatic and the clique numbers can be established based on the graph coloring.

i. The Chromatic Number, $\chi(\Gamma)$

The graph coloring is k -edge-coloring if its edges can be coloured with k colours so that no two adjacent edges have the same colour. From graph coloring, there is 8-edge-coloring. If graph is 8-edge-coloring, chromatic index of graph,

$$\chi(\Gamma) = 8.$$

where chromatic index denoted by minimum colour in graph.

ii. The Clique Number of Graph Coloring, $\omega(\Gamma)$

A clique, $\omega(\Gamma)$ in an undirected graph $\Gamma = (V, E)$ is a subset of the vertices such that every two distinct vertices are adjacent while a clique number of a graph Γ , $\omega(\Gamma)$, is the number of vertices in a maximum clique in Γ . Let

$$\begin{aligned} V_1 &= \text{MAT 560}, V_2 = \text{QMT 634}, V_3 = \text{MAT 578}, V_4 = \text{MAT 631}, \\ V_5 &= \text{ACC516}, V_6 = \text{STA470}, V_7 = \text{ACC416}, V_8 = \text{MAT 522}, \\ V_9 &= \text{MAT 491}, V_{10} = \text{ISP565}, V_{11} = \text{MAT 523}, V_{12} = \text{DSC551}, \\ V_{13} &= \text{MAT 688}, V_{14} = \text{MAT 652}, V_{15} = \text{ISP610}, V_{16} = \text{MAT 575}, \\ V_{17} &= \text{ACC466}, V_{18} = \text{MAT 530}, V_{19} = \text{MAT 571}, V_{20} = \text{MAT 612}. \end{aligned}$$

From the graph coloring in Figure 4, an adjacency matrix is created. Adjacency matrix is a square matrix used to represent a finite graph. It is (0,1)-matrix with zeros on its diagonal. ‘1’ indicates the pairs of vertices that are adjacent while ‘0’ indicates the pairs of vertices that are not adjacent. After the adjacency matrix is made, the total is calculated based on the total number of connected vertices that are adjacent.

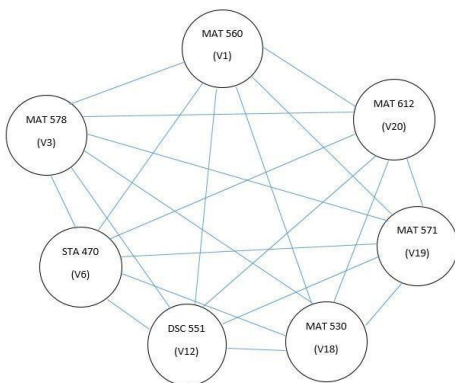
First, to find the clique in a simple way, list all the vertices with its elements that have the total number of adjacent vertices above 10. This is because the vertices have the tendency to be included in the maximum clique of the graph coloring.

$$\begin{aligned} V_1 &= V_2, V_3, V_4, V_5, V_6, V_{10}, V_{12}, V_{14}, V_{17}, V_{18}, V_{19}, V_{20} = 12 \\ V_3 &= V_1, V_2, V_6, V_7, V_9, V_{10}, V_{12}, V_{14}, V_{15}, V_{16}, V_{17}, V_{18}, V_{19}, V_{20} = 14 \\ V_6 &= V_1, V_2, V_3, V_4, V_5, V_7, V_8, V_9, V_{11}, V_{12}, V_{14}, V_{15}, V_{16}, V_{18}, V_{19}, V_{20} = 16 \\ V_{12} &= V_1, V_3, V_6, V_8, V_9, V_{10}, V_{11}, V_{13}, V_{15}, V_{16}, V_{18}, V_{19}, V_{20} = 13 \\ V_{14} &= V_1, V_2, V_3, V_4, V_5, V_6, V_5, V_{16}, V_{18}, V_{19}, V_{20} = 12 \\ V_{18} &= V_1, V_2, V_3, V_5, V_6, V_{10}, V_{12}, V_{13}, V_{14}, V_{15}, V_{17}, V_{19}, V_{20} = 13 \\ V_{19} &= V_1, V_3, V_5, V_6, V_7, V_{10}, V_{12}, V_{14}, V_{15}, V_{16}, V_{17}, V_{18}, V_{20} = 13 \\ V_{20} &= V_1, V_2, V_3, V_4, V_5, V_6, V_{10}, V_{12}, V_{13}, V_{14}, V_{17}, V_{18}, V_{19} = 13 \end{aligned}$$

Next, find the intersection between all the vertices. Then, the intersection with the largest possible number of vertices will form a clique that represent as a maximum clique. Hence, the maximum clique of the graph coloring is clique with size seven. There are two cliques with size seven.

Clique 1, $\omega_1(\Gamma) = V_1, V_3, V_6, V_{12}, V_{18}, V_{19}, V_{20}$

Intersection of vertices for $\omega_1(\Gamma)$



$$\begin{aligned} V_1 &= V_3, V_6, V_{12}, V_{18}, V_{19}, V_{20} \\ V_3 &= V_1, V_6, V_{12}, V_{18}, V_{19}, V_{20} \\ V_6 &= V_1, V_3, V_{12}, V_{18}, V_{19}, V_{20} \\ V_{12} &= V_1, V_3, V_6, V_{18}, V_{19}, V_{20} \\ V_{18} &= V_1, V_3, V_6, V_{12}, V_{19}, V_{20} \\ V_{19} &= V_1, V_3, V_6, V_{12}, V_{18}, V_{20} \\ V_{20} &= V_1, V_3, V_6, V_{12}, V_{18}, V_{19} \end{aligned}$$

Figure 5 : Image of $\omega_1(\Gamma)$

Clique 2, $\omega_2(\Gamma)$, = $V_1, V_3, V_6, V_{14}, V_{18}, V_{19}, V_{20}$

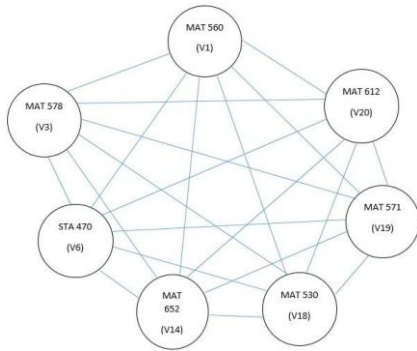


Figure 6 : Image of $\omega_2(\Gamma)$

Intersection of vertices for $\omega_2(\Gamma)$

$$V_1 = V_3, V_6, V_{14}, V_{18}, V_{19}, V_{20}$$

$$V_3 = V_1, V_6, V_{14}, V_{18}, V_{19}, V_{20}$$

$$V_6 = V_1, V_3, V_{14}, V_{18}, V_{19}, V_{20}$$

$$V_{14} = V_1, V_3, V_6, V_{18}, V_{19}, V_{20}$$

$$V_{18} = V_1, V_3, V_6, V_{14}, V_{19}, V_{20}$$

$$V_{19} = V_1, V_3, V_6, V_{14}, V_{18}, V_{20}$$

$$V_{20} = V_1, V_3, V_6, V_{14}, V_{18}, V_{19}$$

Table 2: Adjacency Matrix (Γ)

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20
V1	0	1	1	1	1	1	0	0	0	1	0	1	0	1	0	0	1	1	1	1
V2	1	0	1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	1
V3	1	1	0	0	0	1	1	0	1	1	0	1	0	1	1	1	1	1	1	1
V4	1	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	1
V5	1	1	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1	1	1	1
V6	1	1	1	1	1	0	1	1	1	0	1	1	0	1	1	1	0	1	1	1
V7	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	1	0	0	1	0
V8	0	0	0	0	0	1	1	0	1	0	1	1	0	0	0	1	0	0	0	0
V9	0	0	1	0	0	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0
V10	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	1	1
V11	0	0	0	0	0	1	0	1	1	0	0	1	0	0	1	0	0	0	0	0
V12	1	0	1	0	0	1	0	1	1	1	1	0	1	0	1	1	0	1	1	1
V13	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	1
V14	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1
V15	0	0	1	0	0	1	0	0	1	0	1	1	0	1	0	1	0	1	1	0
V16	0	0	1	0	0	1	1	1	0	1	0	1	0	1	1	0	0	0	1	0
V17	1	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1
V18	1	1	1	0	1	1	0	0	0	1	0	1	1	1	1	0	1	0	1	1
V19	1	0	1	0	1	1	1	0	0	1	0	1	0	1	1	1	1	1	0	1
V20	1	1	1	1	1	1	0	0	0	1	0	1	1	1	0	0	1	1	1	0
TOTAL	12	8	14	7	9	16	6	6	7	8	5	13	5	12	9	9	7	13	13	13

3 Results and Discussions

As a result of implementing a graph coloring technique, a systematic examination timetable is developed. There are three classes with the maximum capacity of 60 students at one time and two time period for a day, which are morning and evening session. In the graph, the vertices that have the same colour can have the exam at the same time period. There are eight chromatic numbers which representing the number of colour of vertices in the graph coloring. It means that there are eight days for examination.

Table 3: An Examination Schedule for CS249 students for Part 4, 5 and 6.

DAY	TIME	CLASS A	CLASS B	CLASS C
1	AM	MAT612	MAT612, MAT523	-
	PM	-	-	-
2	AM	MAT560	MAT560, MAT668	-
	PM	-	-	-
3	AM	MAT631	MAT631, MAT491	-
	PM	MAT530	MAT575	MAT530, MAT575
4	AM	MAT571	QMT634	MAT571, QMT634
	PM	-	-	-
5	AM	MAT578	ACC516	MAT578, ACC516, MAT522
	PM	-	-	-
6	AM	DSC551	DSC551, MAT652	ACC416
	PM	-	-	-
7	AM	ISP610	-	-
	PM	-	-	-
8	AM	STA470	ISP565	STA470, ISP565, ACC466
	PM	-	-	-

Starting with Day 1, the yellow color vertices indicates MAT612 and MAT523. Since number of student in MAT612 exceeds the capacity of room which is only 60 students in a room, balance of 60

students will move to next class. Hence, the number of student in MAT523 is small, so balance of MAT612 and MAT523 can have the exam in the same room to minimize the usage of rooms and the number of lecturers. In Day 2, by highlighting the vertices with red color, they indicates MAT560 and MAT668. Similar with the situation in Day 1, number of student in MAT560 exceeds the capacity of room. So, balance of student in MAT560 can have an exam with MAT668 since the combination of student of these two subjects does not exceed the room capacity.

Day 3 proceeds with the vertices with blue color which are MAT631, MAT491, MAT530 and MAT575. There are three subjects with number of students exceeded the room capacity which are MAT631, MAT530 and MAT575. So, these three subjects need to fill each class and the balances need to combine with other subject. For Day 4, the vertices in green color shows MAT571 and QMT634. It also have the same condition with Day 3. Number of student in MAT571 and QMT634 exceed the room capacity. So, the balance from these two subjects need to combine in the other room.

In Day 5, the purple color of vertices shows MAT578, ACC516 and MAT522. Same with other situation before, number of student in MAT578 and ACC516 exceeded the room capacity and the balance must be combined in the same room. But, since there are more spaces in the room and number of students in MAT522 is small. So, these three subjects can have an exam in the same room since it not exceeds the room capacity. Next, chose the pink color vertices for Day 6 which are DSC551, MAT652 and ACC416. Different with Day 5 situation. Student in MAT652, ACC416 and balance of student in DSC551 cannot have an exam in the same room because the total number of student exceeds the room capacity. So, ACC416 will attend the exam in other room.

In Day 7, grey color is used in the graph coloring which only has one vertex that is ISP610. So, only one subject runs in a day.

Last day, which is Day 8, the color of vertices in the graph coloring is orange which are STA470, ISP565 and ACC466. The situation is the same in Day 5. Number of student in STA470 and ISP565 exceed the room capacity and the balance of the student must combine in other room. But, since total number of student in ACC466 is small, so these three subjects can have an exam in the same room because total number of students does not exceed the room capacity.

Therefore, the chromatic number of the graph coloring is eight depending on the minimum color of the vertices. The colors of the vertices in the graph coloring show the number of day for examination. There were eight colours of vertices so that there were eight days for examination for part 4, 5 and 6.

Lastly, the clique number can be established based on the graph coloring. The clique number which is the number of vertices in the maximum size of clique is seven. There were two cliques with size seven that represent seven subjects. The intersection between the vertices from the clique number means that a subject contains all the elements that belong to another subject. So, the intersection between vertices for $\omega_1 (\Gamma)$ contains MAT560 (V_1), MAT578 (V_3), STA470 (V_6), DSC551 (V_{12}), MAT530 (V_{18}), MAT571 (V_{19}) and MAT612 (V_{20}) while for $\omega_2 (\Gamma)$ contains MAT560 (V_1), MAT578 (V_3), STA470 (V_6), MAT652 (V_{14}), MAT530 (V_{18}), MAT571 (V_{19}) and MAT612 (V_{20}).

4 Conclusion

This paper presented a real world examination timetabling problem and a graph coloring that was implemented in the university exam timetabling of UiTM Cawangan Kelantan, Kampus Machang. It

focuses on the fourth semester to final year students from Faculty of Computer and Mathematical Sciences (FSKM) for Bachelor of Science (Hons.) Mathematics (CS249). In this paper, the examination time table is generated by using graph coloring approach by considering the essential inputs given. From the results, the students' final examination schedule is manageable within eight days of examination time with seven courses taken. Those students will take the exam based on the predetermined venue and the time period with no hassle.

Acknowledgements

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