

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

BOUND ON THE THIRD ORDER HANKEL
DETERMINANT FOR CERTAIN SUBCLASSES OF
ANALYTIC FUNCTION

P41S18

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Bachelor of Science (Hons.) Mathematics
Faculty of Computer and Mathematical Sciences

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**Report submitted in partial fulfilment of the requirement
for the degree of
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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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TABLE OF CONTENTS

ACKNOWLEDGEMENT	ii
ABSTRACT	iv
CHAPTER 1 : INTRODUCTION	1
1.1 Subclasses of Analytic Function.....	2
1.2 Hankel determinant.....	4
1.3 Problem Statement.....	5
1.4 Objective.....	5
1.5 Significants and Benefit of the Study.....	5
1.6 Scope and Limitation of the Study.....	6
CHAPTER 2 : BACKGROUND THEORY AND LITERATURE REVIEW	7
2.1 Background Theory.....	7
2.2 Literature Review and Related Research.....	8
CHAPTER 3 : METHODOLOGY AND IMPLEMENTATION	12
3.1 Determination of The Function.....	12
3.2 Determination of Third Order Hankel Determinant.....	12
CHAPTER 4 : RESULT AND DISCUSSION	14
4.1 Class of Function $S^*_{\alpha}{}^{\beta}(\delta)$	14
4.2 Class of Function $B^*_{\alpha}{}^{\beta}(\delta)$	27
4.3 Verification of The Result.....	40
4.4 Discussion for The Result of The Third Order Hankel Determinant for Class of Function $S^*_{\alpha}{}^{\beta}(\delta)$ and Class of Function $B^*_{\alpha}{}^{\beta}(\delta)$	44
CHAPTE 5 : CONCLUSION AND RECOMMENDATIONS	45
REFERENCES	46

ABSTRACT

In analytic functions, there have many subclasses of class of function such as class of convex function, class of close-to-convex function and class of starlike function, but this research only focusing on class of starlike function. The class of the analytic function $f(z)$ standardly nonnalized for two classes denoted by $S_{\alpha}^{\beta}(\delta)$ and $B_{\alpha}^{\beta}(\delta)$ satisfied

$$\operatorname{Re} e^{i\delta} \left\{ \frac{zf'(z) + \alpha z^2 f''(z)}{f(z)} \right\} > \beta \text{ and } \operatorname{Re} e^{i\delta} \left\{ \frac{zf'(z) + \alpha z f''(z)}{f(z) + f'(z)} \right\} > \beta$$

respectively where $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, $\cos(\delta) > \beta$ in unit disk $\Delta, \Delta = \{z \in \mathbb{C} : |z| < 1\}$.

Nowadays, many classes of function are developed by some researchers, but some of the properties did not discover yet. It is hard to know whether the methods used are suitable to obtain the wanted properties. In order to derive coefficient bound a few methods are used to get the results such that power series, comparisons of coefficient with positive real part, relative minima and maxima and triangle inequalities. Next, Toeplitz determinant is used to prove the sharp bound for third Hankel determinants. The results obtained for the third Hankel determinant for the class of function $S_{\alpha}^{\beta}(\delta)$ and class of function $B_{\alpha}^{\beta}(\delta)$ are $|H_3(1)| \leq \frac{235}{729}$ and $|H_3(1)| \leq \frac{25}{729}$ respectively. Recommendation for the future research is the researchers must explore more about the other subclasses of the analytic function and use the class of function to find the other properties.