

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**FEKETE-SZEGÖ INEQUALITIES
FOR CERTAIN SUBCLASS OF
ANALYTIC BI-UNIVALENT
FUNCTIONS ASSOCIATED WITH
CHEBYSHEV POLYNOMIALS**

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Abstract

Geometric function theory involves in studying the properties of analytic and bi-univalent functions. In the area of geometric function theory, a new subclass of function is constantly introduced and it is in nature for researchers to find its properties. Through Chebyshev polynomial, a new class of function is defined and two of its properties are to be studied. In this project, we introduced a new subclass of analytic and bi-univalent functions $g(z)$ associated with Chebyshev polynomials, $\xi_{\Sigma}(\lambda, \mu, t)$ defined by,

$$(1 - \lambda)g'(z) + \lambda\left(1 + \frac{zg''(z)}{g'(z)}\right) + \mu zg''(z) \prec F(z, t) := \frac{1}{1 - 2tz + z^2}$$

and

$$(1 - \lambda)h'(w) + \lambda\left(1 + \frac{wh''(w)}{h'(w)}\right) + \mu wh''(w) \prec F(w, t) := \frac{1}{1 - 2tw + w^2}.$$

where $\lambda \geq 0$, $\mu \geq 0$ and $t \in (\frac{1}{2}, 1)$.

With the class $\xi_{\Sigma}(\lambda, \mu, t)$, some of the properties is obtained which are coefficient bounds and the sharp bounds of Fekete-Szegö functional, $\xi_{\Sigma}(\lambda, \mu, t)$. Triangle inequality and maximization of function are applied in the process of finding the sharp bound of Fekete-Szegö functional. This project will be a significant endeavor in contributing new results for coefficient bounds and sharp bounds of Fekete-Szegö functional in the field of geometric function theory. Further studies can be done in finding the bounds for second Hankel determinant, third Hankel determinant or other properties for the class $\xi_{\Sigma}(\lambda, \mu, t)$.

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