UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

UPPER BOUND OF SECOND HANKEL DETERMINANT FOR GENERALIZED CLASS OF ANALYTIC FUNCTIONS

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

Geometric function theory of a complex functions makes a study of analytic univalent functions and various geometric properties of certain classes of analytic functions. In this area of study, there are many classes of functions that have been introduced. Apart from that, every class of functions have their own properties such as extremal properties, radius properties, representation theorem and coefficient bound. Thus, the purpose of this study are defining new subclasses of analytic functions, $M(\alpha, \delta, t)$ as well as determining the second Hankel determinant for the class of functions.

In this research, we introduced new subclasses of analytic functions and focus on finding upper bounds for the functional $|a_2a_4 - a_3^2|$, that known as second Hankel determinant and it is one of the coefficient inequalities in geometric function theory. For the new subclass of analytic functions, we introduced class of close-to-convex functions, $M(\alpha, \delta, t)$ defined in the unit disk, $E = \{z : |z| < 1\}$, which satisfies the condition $\operatorname{Re}\left\{e^{i\alpha} \frac{zf^i(z)}{g(z)}\right\} > \delta$

where $|\alpha| < \pi$, $\cos(\alpha) > \delta$, $g(z) = \frac{z}{(1-tz)(1-z)}$ and $-1 \le t \le 1$. In order to derive our main result, we use the Lemma of Pommerenke (1975), Lemma of Toeplitz determinants and Lemma of Libera (1983) and Zlotkiewicz (1982). By obtaining the upper bounds for second Hankel determinant $|a_2a_4 - a_3^2|$ for the class of functions $M(\alpha, \delta, t)$, we can reduce to Kaharudin et al.(2011). The result of second Hankel determinant obtained is a keen result and this property lead to development of second Hankel determinant.

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