

Misconception As Barrier in Understanding Index and Logarithm – The Case of Pre-Tertiary Education Students

Wan Norliza Wan Bakar^{1*}, Siti Farah Haryatie Mohd Kanafiah ²

^{1,2}Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Kelantan, Bukit Ilmu, Machang, Kelantan, Malaysia
wannorliza67@gmail.com
*Corresponding author

Abstract: Misconception is an opinion that is wrong because it has been failure to understand the concept. Index and logarithm are the basic mathematical concept that is very important in advanced mathematics. It is being taught in the subject of additional mathematics at school level and refreshed again at university level. In Universiti Teknologi Mara Cawangan Kelantan (UiTMCK), index and logarithm is one of the topics in the subject of essential mathematics (MAT 037) . This research aims to explore and identify the misconception in understanding index and logarithm among pre diploma students. There were 120 pre diploma students taking Essential Mathematics on Semester September 2019 until January 2020 at UiTMCK involved in the diagnostic test. A set of diagnostic tests on the topic of index and logarithms developed by the experts were given to pre diploma students. The results revealed that the students do understand the topic of index and index equation very well. Unfortunately, when it comes to the topic of logarithm students became confused and the arrangement of BODMAS is not in appropriate order. Most of the students confused and they have their own interpretation in determine the meaning of variable.

Keywords: Index and Logarithms, Essential Mathematics, Misconception

1 Introduction

Index and Logarithm is one of the topic in the subject of Essential Mathematics. Many students are facing problem when comes to this topic. They felt that the topic is hard to understand. There are many documents have been reported in literature discussing about the misconception of student in understanding the mathematical concept such as index and logarithms topic. A study initiated by Liang and Wood [1] in a research on secondary school at Singapore found that the student can do the basic part of simple calculation on logarithms but unable to do the solution at higher level. Even the students have developed an instrumental understanding of logarithms, they still make several mistakes due to over-generalization of previously learned rules. There have been a number of studies shown that student lack understanding of logarithms as we can see in (Kenney [2]; Watters and Watters [3]; Weber [4]).

As stated in Williams [5], logarithms were divided into four parts which are logarithm as object, as process, as function and in contextual issues. That study create a research-based framework of what it means to understand logarithms. With regard to logarithm laws, Weber [6] wrote, "In general, as time goes by, one 's knowledge of symbolic laws will decay. If one has a clear understanding of the principles involved, it is possible to recreate certain rules. If not, the rules cannot be enforced." This statement means if the students misremember rules and fail to verify the validity of rules, then it could indicate a lack of understanding.

Besides, other study by Kastberg [7] found that students who were good with computational logarithm cannot remembered rules a few weeks later. It is not enough for students to simply memorize the rules even they being skilled in understanding logarithms as objects. If students don't understand logarithm rules they probably won't remember them or restore them if they forget exactly what the rules are. The students did not realize that logarithms could be used to solve such problems, so they prefer to

solve them by comparing the problems to exponents and succeeded which is not efficient in solving the problems.

In analysing students' interpretation of logarithms and logarithmic expressions as numbers, Berezovski and Zazkis [8] expressed doubt that problems with calculating logarithmic expressions involving only numbers either using a calculator or manually calculation interpretes an understanding of logarithms as numbers. The students may have learnt a procedure where certain types of problems are presented but do not understand that the logarithmic expression is a number. This is in line with study from Kastberg's [7] which reported that students who can solve problems do not necessarily perceive logarithmic expressions as numbers.

In a study conducted by Liang and Wood [9] observed that students have difficulties to understand when it relates to logarithms. This could result from a lack of real application problems, and he suggested some applications in the real world that teachers could use to help students have better understanding relate to logarithms. The students always got the wrong idea in applying definition, determining logarithmic properties to solve problems, applying logarithmic properties and applying the prerequisite material. According to Aziah [10] on the understanding level of matriculation students on the topic of logarithms revealed that students have made four major errors which were in algebra, the generalization of logarithmic law, misconception of the index concept and do not follow the instructions.

Other than that, based on the study of Dintarini [11], the lack understanding of logarithmic definitions, ability to see the facts relating to problems, over-focus on facts of rote and technical procedures, relying on improper intuition, and inconsistencies in symbolic writing and inaccuracy will lead the student misunderstand the concept. Another case study of Bukari and Yakubu [12] in Kalpohin Senior High School reported that most of the students having conceptual problem in solving logarithmic functions. In enhancing the performance of students to logarithmic functions, they introduced Constructivist Based Instructional Approach which will increase interest and motivation of the students' retention and self-confidence of their understanding and ability to solve more questions on logarithm functions.

Motivated by the literature above, the present study aims to analyse the possible cases and errors always made by the students. The misconception of students' understanding in index and logarithms were examined from the diagnostic test experienced by the pre diploma students in UiTM Cawangan Kelantan.

2 Methodology

The study was conducted using a diagnostic test on the topic of index and logarithms which consists of 5 questions on index and 5 questions on logarithms. The diagnostic test were given to 120 students of pre diploma taken Essential Mathematics subject in UiTM Cawangan Kelantan semester September 2019 to January 2020.

A Rules of Index and logarithms

The basic rules of index and logarithms are shown in Table 1 and Table 2 as follows:

Table 1: Rules of Index

| No | Rules |
|----|-----------------------------|
| 1 | $a^m \times a^n = a^{m+n}$ |
| 2 | $\frac{a^m}{a^n} = a^{m-n}$ |
| 3 | $(a^m)^n = a^{mn}$ |
| 4 | $(ab)^m = a^m \times b^m$ |

| | |
|---|--|
| 5 | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ |
| 6 | $a^{-n} = \frac{1}{a^n}$ |

Table 2: Rules of Logarithms

| No | Rules |
|----|--|
| 1 | $\log a + \log b = \log ab$ |
| 2 | $\log a - \log b = \log\left(\frac{a}{b}\right)$ |
| 3 | $\log x^n = n \log x$ |
| 4 | $\log a = 1$ |
| 5 | $\log 1 = 0$ |

B Instruments

The diagnostic test consists of 10 questions which is 5 questions on index and 5 questions on logarithms. The first three questions were the combination of the first and the sixth rule of index topics. The fourth and fifth questions were on the index equation. The sixth to eighth questions were the combination of first and second rule of logarithmic equations. The ninth and tenth questions were on the logarithmic equations. The distribution of question in diagnostic test are shown in the following Table 3.

Table 3: Distribution of Diagnostic Test

| Category | Items |
|----------|---|
| Q1 - Q3 | Combination of first and sixth rule of Index |
| Q4 - Q5 | Index equations |
| Q6 – Q8 | Combination of first and second rule of Logarithm |
| Q9 – Q10 | Logarithmic equations |

C Data Analysis

The diagnostic test was administered to 120 students from pre diploma students in UiTM Cawangan Kelantan who taken Essential Mathematics. The resulting script were collected for data analysis. Each script were marking by the lecturer and the details by each items was carried out in examining the students' responses. The data analysis are shown in the table 4 below.

Table 4: Item Analysis based on Types of Concept and its Misconceptions

| QUESTION | SAMPLE QUESTION | CONCEPT | MISCONCEPTION | FREQUENCY |
|----------|---|--|---------------------------|-----------|
| Q1 - Q3 | $\frac{\frac{1}{4^2} \times 8^3}{16^{\frac{1}{4}}}$ | Combination of first and sixth rule of index | No misconception happened | 0% |

| | | | | |
|----------|----------------------------------|---|---|-----|
| Q4 - Q5 | $9^{2x-2} = 81$ | Index equations | No misconception happened | 0% |
| Q6 - Q8 | $2 \log 10 - 2 \log 5 + \log 25$ | Combination of first and second rule of logarithmic equations | There is a problem on BODMAS. The students have their own perceptions | 65% |
| Q9 - Q10 | $\log(x + 2) + \log(x - 1) = 1$ | Logarithmic equations | The students misinterpret the variable. | 65% |

It can be analyzed that the students do understand the topic of index and index equation very well. Unfortunately, when it comes to the topic of logarithm, about 65% of the students became confused and the arrangement of BODMAS is not in the appropriate order. Most of the students confused and they have their own interpretation. When they answered the logarithmic equations problem their mistakes happened while interpreting the meaning of the variable.

3 Result and Discussion

Due to the diagnostic test that had been given to 120 students of Pre-Diploma semester September 2019 to January 2020 shown that 70 out 120 students or 65% of the students' performance is not satisfied. This can be discussed on the common errors that have been made by the students in the topic of index and logarithm. Due to the topic of index which were the combination of first rule and sixth rule, students can attain a clear interpretation on it. The first rule is sometimes known as the product rule which states that if two numbers have the same base, but possibly different indices are multiplied together, their indices are added. The students did very well where 90% of them answered the question correctly. Table 5 shows an example of students' answer to be compared with answer scheme.

Table 5: Sample of Answer Scheme and Students' answer

| Question | Answer Scheme | Students Answer |
|---|---|--|
| 3. Evaluate $\frac{4^{\frac{1}{2}} \times 8^{\frac{2}{3}}}{16^{\frac{1}{4}}}$ | $= \frac{4^{\frac{1}{2}} \times 8^{\frac{2}{3}}}{16^{\frac{1}{4}}}$ $= \frac{(2^2)^{\frac{1}{2}} \times (2^3)^{\frac{2}{3}}}{(2^4)^{\frac{1}{4}}}$ $= \frac{2^1 \times 2^2}{2^1}$ $= 2^{1+2-1}$ | $= \frac{4^{\frac{1}{2}} \times 8^{\frac{2}{3}}}{16^{\frac{1}{4}}}$ $= \frac{(2^2)^{\frac{1}{2}} \times (2^3)^{\frac{2}{3}}}{(2^4)^{\frac{1}{4}}}$ $= 2^{1+2-1}$ $= 2^2$ |

| | | |
|--|--|--|
| | $= 2^2$ | $= 4$ |
| 4. Find the value of x if , $9^{2x-2} = 81$ | $9^{2x-2} = 81$ $9^{2x-2} = 9^2$ $2x - 2 = 2$ $2x = 4$ $x = 2$ | $9^{2x-2} = 81$ $9^{2x-2} = 9^2$ $2x - 2 = 2$ $2x = 2 + 2$ $x = \frac{4}{2}$ $x = 2$ |
| 8. Evaluate $2 \log 10 - 2 \log 5 + \log 25$ | $= 2 \log 10 - 2 \log 5 + \log 25$ $= 2 \log(10)^2 - 2 \log(5)^2 + \log 25$ $= \log \frac{100 \times 25}{25}$ $= \log 100$ $= 2$ | $= 2 \log 10 - 2 \log 5 + \log 25$ $= 2 \log(10)^2 - 2 \log(5)^2 + \log 25$ $= \log 100 \log 25 \log 25$ $= \log \frac{625}{100}$ $= \log 6.25$ $= 0.795880017$ |
| 10. Solve $\log 2 + \log 5 + \log x - \log 3 = 2$ | $\log 2 + \log 5 + \log x - \log 3 = 2$ $\log \frac{2 \times 5 \times x}{3} = \log 100$ $10x = 300$ $x = 30$ | $\log\left(\frac{2 \times 5 \times x}{3}\right) = \log 2$ $\frac{10x}{3} = 2$ $10x = 2 \times 3$ $x = \frac{6}{10}$ $x = \frac{3}{5}$ |

Based on Table 5, Under index equation, the student still interprets it clearly where 85% of them answered it correctly. Index equation is an equation that has a variable in power or an index. It can be vividly seen that the students are facing problem while answering rules of logarithm. Question 6 until Question 8 are sample of question which students have problem in answering it. They do not understand the concept of BODMAS (bracket, order, multiplication, addition and subtraction) and they could not apply it to the question. In the topic of index and logarithm, the students should make use of the rules of logarithm to simplify logarithm with same bases. Unfortunately, the students were unable to do so. This is proved by the question 10 which asked for solving logarithmic equation $\log 2 + \log 5 + \log x - \log 3 = 2$.

4 Conclusions

It can be concluded that while studying logarithm the students have problem in perception where they precepted it wrongly. The students only see the surface structure of the expression or the equation. They could not grasp the read concept and principle of logarithm. So, it is suggested that the process of teaching and learning should be organized in appropriate order and detail to fit in the student's cognitive schema. It is also recommended the study of neuroscience on conceptual learning of index and logarithms for future research.

References

- [1] E. Wood, "Understanding logarithms", *Teaching Mathematics and Its Applications: An International Journal of the IMA*, vol. 24, no. 4, pp. 167–178, 2005.
- [2] R. Kenney, *Proceedings of the 27th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 2005.
- [3] D. J. Watters, & J. J. Watters, "Student Understanding of pH", *Biochemistry and Molecular Biology Education*, vol. 34, no. 4, pp. 278–284, 2006.
- [4] K. Weber, "Students' understanding of exponential and logarithmic functions (at the undergraduate level). In D. Quinney (Ed.)", *Proceedings of the 2nd International Conference on the Teaching of Mathematics*, pp. 1-7, 2002.
- [5] H. R. A. Williams, "A conceptual framework for student understanding of logarithms", pp. 1–95, 2011.
- [6] K. Weber, "Developing students' understanding of exponents and logarithms", *Proceedings of the twenty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, vol. 2, pp. 1019-1027, 2002.
- [7] S. E. Kastberg, "Understanding mathematical concepts: The case of the logarithmic function", *PhD Thesis*, University of Georgia, 2002.
- [8] T. Berezovski, & R. Zazskis, "Logarithms: Snapshots from two tasks", *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, vol. 2, pp. 145-152, 2006.
- [9] C. B. Liang, & E. Wood, "Working with Logarithms: Students' Misconceptions and Errors", *The Mathematics Educator*, vol. 8, no. 2, pp. 53–70, 2005.
- [10] N. Aziah, "Pemahaman Pelajar Matrikulasi Tentang Logaritma", *PhD Thesis*, Universiti Utara Malaysia, 2014.
- [11] M. Dintarini, "Understanding Logarithm: What are the Difficulties That Students Have?", pp. 0–3, 2018.
- [12] H. Bukari, & A. Yakubu, "Using Constructivist Approach to Enhance Students Understanding of Logarithmic Functions: A Case Study of Kalpohin Senior High School, Tamale-Ghana", *International Journal of Engineering and Applied Sciences*, vol. 5, no. 3, 257250, 2018.