ANALYZING AND COMPARING CUSTOMER WAITING TIME IN ROAD TRANSPORT DEPARTMENT OFFICE USING QUEUING THEORY MODEL AND SIMULATION MODEL

Noor Hidayah Mohd Zaki[1*], Aina Nabila Ismanuddin[1], Siti Nur Amimah Mahusin[1]
Asmahanim Binti Haji Mohamad Yusuf [4]

[1]Faculty of Computer and Mathematical Sciences
[4]Academy of Language Studies
Universiti Teknologi MARA Negeri Sembilan,
Seremban Campus, 70300 Seremban,
Negeri Sembilan, Malaysia

[*]Corresponding author: hidayahzaki@uitm.edu.my

Abstract

Waiting for service or also known as queuing is a frequently occurring phenomenon. People queue for services at various place such as at the hospitals, banks, post offices and many more. However, long and unmanaged queues may cause dissatisfactions among the customers thus will affect the customers' evaluation of the services. Not only that, time will also be wasted since people need to wait for hours before being served. Other than that, every organization has their time limit in serving their customers. Due to this limitation, it leads to overcrowding and unmanageable queues at the premise. Ensuring this, queuing theory model has been applied for the selected situation in Road Transport Department located in Senawang, Negeri Sembilan. One of the main objectives was to analyze and compare the customer waiting time queuing model to simulation model. M/M/s queuing model was performed on the collected data and simulation with ARENA software was used to build and verify the result. It is concluded that the best method to measure the queuing situation is by using ARENA simulation model since the result is more accurate than Queuing Theory Model.

Keyword: Queue, M/M/s, Queuing Theory Model, Simulation Model, ARENA software

Introduction

Queuing theory is a mathematical study of waiting in lines that construct a model where the waiting times of the customers can be forecasted. Information needed in running the model is the arrival process, service mechanism, and queue characteristics. The first queuing theory research was made by A. K. Erlang in 1908 to solve the problem of queuing in customer service careline. Queuing theory has been widely used to improve service and line problems such as to control the pedestrian congestion delay, customers waiting in line at fast food restaurant in the University of Benin (Gumus et al., 2017), delay in health care (Vass & Szabo, 2015) and waiting line problems at ticket windows in railways station (Ituen-Umanah, 2017).

Simulation model is used to represent a real-world situation that enables people to know the behavior of a system and to explain the dynamic relationships of a system. In this model, although the real system is not done, an experiment can be made without intruding the real system. It is a versatile tool to evaluate complex measurement and to experiment with the system's behavior. Simulation method has been used to optimize the number of service counter in Post Office Bytca (Achimsk et al., 2019). In addition, ARENA software simulation

was used to establish an efficient plant layout for such dynamic systems in marble factory (Edis et al., 2011).
One of the major concerns in Road Transport Department office is when customers need to spend too much time to access the services. As the number of customers keeps increasing day by day, any delays in service may cause dramatic outcomes for the customer. Customer waiting for too long at the Road Transport Department office to access any services would be viewed as an indication of low quality and require enhancement. Therefore, the performance measures at the Road Transport Department office will be analyzed using Multi-Server Queuing Model and Simulation Model to determine the cost of waiting and service in order to achieve the optimum level of service. Simulation model will involve the use of ARENA software that explores new method without interrupting the current system. Moreover, ARENA software is able to define paths and routes for the simulation that will identify and analyze of the current situation.

Materials and Methods

The primary data was collected on 27th of September 2019. The observation done for four hours from 9.30 a.m. until 1.30 p.m. and the data was collected. The situation was observed and analysed. The data was recorded manually. The data obtained were the arrival time, waiting time, start service time, and departure time of every customer.

Calculation for Queuing Theory Model – M/M/6 Theorem

To analyse the situation, a mathematical model which is multiple queuing model with more than one channel in the queuing system. was developed. The arrival and service time were exponentially distributed therefore M/M/6 theorem was used. Assumptions made on the model were the arrival is independent, the average arrival rate (λ) will remain unchanged over the time and the arrivals come from infinitely big population were described as Poisson probability distribution. Besides, they are being served on First-in, First-out (FIFO) basis and the services time vary and independent but the average service rate (μ)was known. Then, the service time will follow the negative exponential probability distribution where it is assumed that:

$$\lambda < \mu$$

For M/M/6 Theorem analysis, the following variables were calculated:
Let,

λ = average arrival rate per hour,
μ = average service rate per hour,
S = number of servers

Since the arrival rate for each process is the same, the mean arrival rate (λ) was calculated only once. However, the average service time differs for every process. Therefore, the average service rate (μ) must be calculated based on each process.

Total number of customers within 9.30 am to 1.30 pm = 363 customers

The average number of customers per hour;

$$= \frac{363}{4}$$

$$= 90.75$$

$$\approx 91 \text{ customers}$$

Average number of arrivals per hour, λ;

$$\lambda = \frac{\text{Number of customers per hour}}{\text{Number of servers}}$$

$$= \frac{91}{6}$$

$$= 15.16$$

$$\therefore \lambda = 15 \text{ customers per hour}$$

Average service time per customer (minutes):

$$= \frac{\sum \text{Service time (minutes)}}{\sum \text{Number of customer}}$$

$$= \frac{921}{363}$$

$$= 2.53 \text{ minutes}$$

Average service rate per hour, μ:

$$= \frac{60 \min}{\text{Average service per customer}}$$

$$= \frac{60}{2.53}$$

$$= 23.71$$

$$\therefore \mu \approx 24 \text{ customers}$$

The utilization factor for the system (probability that the system is being used):

$$\rho = \frac{\lambda}{s\mu}$$

$$= \frac{15}{(6)(24)}$$

$$= 0.1042$$

$\therefore$ The percentage that the service has been used is 10.42%

The probability of no customers in the system:

$$P_0 = \frac{1}{\left[\left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{1}{1-\rho}\right)\right]}$$

$$P_0 = \frac{1}{\left[\left[1 + \frac{15}{24} + \left(\frac{1}{2!}\right)\left(\frac{15}{24}\right)^2 + \left(\frac{1}{3!}\right)\left(\frac{15}{24}\right)^3 + \left(\frac{1}{4!}\right)\left(\frac{15}{24}\right)^4 + \left(\frac{1}{5!}\right)\left(\frac{15}{24}\right)^5\right] + \left[\left(\frac{1}{6!}\right)\left(\frac{15}{24}\right)^6 \left(\frac{1}{1-0.1042}\right)\right]\right]}$$

$$= 0.5353$$

$\therefore$ There is an approximate of 53.53% that no customer in the system.

The average number of customers in the queue:

$$L_q = \frac{\rho\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\left(1-\rho\right)^2}$$

$$= \frac{\left(0.1042\right)\left(\frac{15}{24}\right)^6 \left(0.5353\right)}{6!\left(1-0.1042\right)^2}$$

$$= 0.0000058$$

$\therefore$ The average number of customers in the queue is 0.0000058 customer which equals to no waiting in the queue.

The average number of customers in the system:

$$L = L_q + \frac{\lambda}{\mu}$$

$$L = 0 + \frac{15}{24}$$

$$= 0.625$$

$$\approx 1 \text{ customer}$$

$\therefore$ The average numbers of customer being served is 1 customer per hour.

Average time a customer spends waiting in the system:

$$W = \frac{L}{\lambda}$$

$$W = \frac{1}{15}$$

$$= 0.0667 \text{ hour}$$
$$= 4.002 \text{ minutes}$$

$\therefore$ The average time spent by a customer in the system is 4 minutes

Average time a customer spends waiting in the queue:

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.0000058}{15}$$

$$= 0.00000039 \text{ hour}$$
$$= 0.0000232 \text{ minute}$$

$\therefore$ The average time a customer spent waiting in the queue is 0.0000232 minute

Simulation with ARENA Software and Analyzing the Data

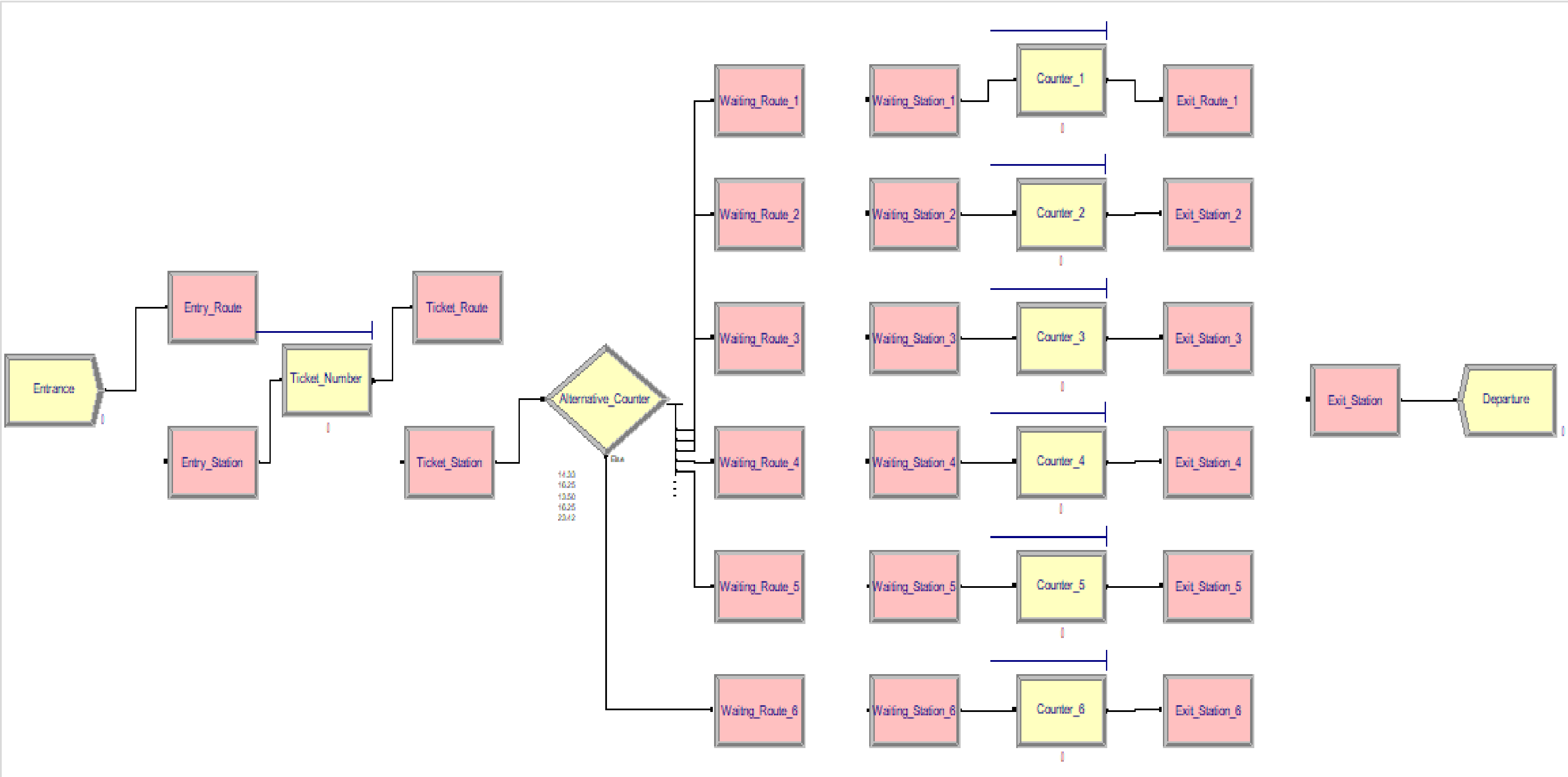

Figure 1 Flowchart of Simulation Model on ARENA software

Figure 1 above shows a flowchart of simulation model of the data at the Road Transport Department that had been recorded on ARENA software. The model has a create, process, decide, dispose module as a basic process which is denoted as yellow colour box. The pink colour box represented an advanced transfer that involves route module and station module. The flowchart shown is also a graphical representation of the real flow and full system in the Road Transport Department as a logical network of related activities. The collected data will undergo cleaning process before it is used in the simulation with ARENA by input analyzer. The input analyzer was used to align an expression with a statistical distribution as shown in **Table 1**. The analyzer was also used to observe the data, estimate the parameter value, and measure the quality of the data. Hypothesis testing was made in order to validate or verify the ARENA model used for the system.

Table 1 Summary for Distribution of Expression

Process	Distribution	Expression
Entrance (Arrival)	GAMMA	$-0.5 + GAMM(0.502, 2.29)$
Ticket Number	LOGNORMAL	$0.08 + LOGN(0.212, 0.07)$
Waiting Time at Counter 1	GAMMA	$0.5 + GAMM(0.981, 2.33)$
Waiting Time at Counter 2	BETA	$0.5 + 6 * BETA(1.26, 2.59)$
Waiting Time at Counter 3	WEIBULL	$0.5 + WEIB(2.84, 1.38)$
Waiting Time at Counter 4	GAMMA	$0.5 + GAMM(0.87, 2.39)$
Waiting Time at Counter 5	EXPONENTIAL	$0.5 + EXPO(1.58)$
Waiting Time at Counter 6	BETA	$0.5 + 7 * BETA(0.866, 2.29)$

Result and Discussion

Comparison of Multi-Server Queuing Theory Model

From the analysis, a comparison of average time customer spends waiting in the queue, W_q between M/M/4, M/M/5 and M/M/6 was made in order to analyze and get the best result to optimize the waiting time of the customer in the system. Once the best server has been obtained, the comparison for the best server needs to be done between Queuing Theory Model and ARENA SIMAN report.

Table 2 Comparison of Average Time Customer Spend Waiting in The Queue (W_q)of Multi-Server Queuing Theory Model

Server	Average Time Customer Spend Waiting in The Queue, W_q
M/M/4	0.0146 min
M/M/5	0.000623 min
M/M/6	0.0000232 min

Table 2 conveys the comparison of the average time customer spent waiting in the queue, W_q. The waiting time for the customer waiting in the queue for M/M/4 was 0.0146 minutes and for the M/M/5 server the customer spent waiting in the queue was 0.000623 minutes. It can be seen that the original system used by the Road Transport Department which–was M/M/6 had the lowest average time the customers spent waiting in the queue. Based on the results, the best server that can be used is M/M/6 server as it has the lowest average time of customer spends waiting in the queue and it can also increase the number of customers that can be served in the system.

Comparison of Queuing Theory Model - M/M/6 and ARENA SIMAN Report

Two information considered as the factors for this research were M/M/6 Queuing Theory Model and ARENA SIMAN Report. Utilization rate, ρ and the average time customer spends waiting in the system, Wq were the two factors being considered. The result was compared to get the error between the two model so that the verification of the result can be made.

Table 3 Comparison of Queuing Theory Model - M/M/6 and ARENA SIMAN Report

RESULT	M/M/6 Queuing Theory Model	ARENA SIMAN Report	ERROR
Average Time Customer Spends Waiting in The System, Wq (minute)	4.002 min	1.6795 min	2.3225 min
Utilization Rate, ρ	10.42%	61.29%	50.87%

The **Table 3** conveys that the utilization rate, ρ or the probability that the system is being used in percentage between M/M/6 queuing theory model and ARENA SIMAN report. The value ρ for the M/M/6 queuing theory model was lower than the ARENA SIMAN report. The error obtained from the comparison of both utilization rate was 50.87%.

W indicated the average time customer spends waiting in the system. The value for M/M/6 queuing theory model was greater than the ARENA SIMAN report. The error obtained from the comparison of both average time customer spent waiting in the system was 2.32 minutes.

Conclusion

In this paper, Multi-Server Queuing Theory Model was successfully applied and the simulation model which represents the real-life situation was developed using ARENA software to verify the result. Calculation of the queuing theory model result has proven that the current system which uses M/M/6 server is the optimal system to be used. M/M/6 has the lowest average time customer spends waiting in the queue compared to M/M/4 and M/M/5. The number of counter opens in a day cannot exceed 6 to maintain the cost of hiring staff. The results of this study showed that the utilization rate for M/M/6 queuing model was unbalanced with ARENA simulation result since the error result for service counter were too huge. In contrast, the average time a customer spent in system for both results were nearly the same. It is concluded that the best method to measure the queuing situation is by using ARENA simulation model since the result is more accurate than Queuing Theory Model. Hence, we recommend a further study to cover the situation in Road Transport Department Senawang for a longer period of data collection. Other than that, other further research may use an additional of method which is Fuzzy Model to compare with the Queuing Theory Model and Simulation Model in order to get the best optimization result for the system used in the Road Transport Department.

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Conflict of interests

The authors hereby proclaim that no conflict of interests happens with any organization or financial body in supporting this research.

References

Achimsk, K., Dutkova, S., & Hostakova, D. (2019). *Simulation of Queuing System of Post Office*. *00*(40), 1037–1044. https://doi.org/10.1016/j.trpro.2019.07.145

Edis, R. S., Kahraman, B., Araz, Ö. U., & Özfirat, M. K. (2011). A facility layout problem in a marble factory via simulation. *Mathematical and Computational Applications*, *16*(1), 97–104. https://doi.org/10.3390/mca16010097

Gumus, S., Bubou, G. M., & Oladeinde, M. H. (2017). Application of queuing theory to a fast food outfit: a study of blue meadows restaurant. *Independent Journal of Management & Production*, *8*(2), 441. https://doi.org/10.14807/ijmp.v8i2.576

Ituen-Umanah, & Udoh, W. (2017). Queuing Theory Application at Ticket Windows in Railway Stations (A Study of the Lagos Terminus, Iddo, Lagos State, Nigeria). *SSRN Electronic Journal*, *2*(1), 1–5. https://doi.org/10.2139/ssrn.3012477

Vass, H., & Szabo, Z. K. (2015). Application of Queuing Model to Patient Flow in Emergency Department. Case Study. *Procedia Economics and Finance*, *32*(15), 479–487. https://doi.org/10.1016/s2212-5671(15)01421-5.