

**UNIVERSITI TEKNOLOGI MARA**

**POSITIVE PERIODIC SOLUTIONS OF  
CERTAIN SINGULAR  
NON-AUTONOMOUS DIFFERENCE  
AND DIFFERENTIAL EQUATIONS**

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Thesis submitted in fulfillment  
of the requirements for the degree of  
**Master of Science**

**Faculty of Computer & Mathematical Sciences**

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## AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.


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## ABSTRACT

This thesis is concerned with the study of singular non-autonomous first order difference equation for a single equation and  $n$ -dimensional systems with a positive parameter. The study also involves singular non-autonomous first order differential equations for  $n$ -dimensional systems with delay and a positive parameter. Sufficient conditions for the existence and multiplicity of positive periodic solutions for singular first order functional differential and difference equations under various assumptions are presented. First, we employ Krasnoselskii fixed point theorem and obtain sufficient conditions for the existence and multiplicity of positive periodic solutions to a scalar singular first order difference equation with a positive parameter. Next, we investigate the existence and multiplicity of positive periodic solutions for singular first order non-autonomous systems of difference equations with a positive parameter by applying the Krasnoselskii fixed point theorem. Finally, we apply a fixed point index theorem to study the existence, multiplicity and nonexistence of positive periodic solutions with a positive parameter to nonlinear singular systems of first order functional differential equations.

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