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PRICING EUROPEAN OPTION PRICE IN JUMP-DIFFUSION MODEL

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Abstract
This research presents a numerical method for pricing European options. The method is based on the jump-diffusion process. The Merton's jump-diffusion model has become a popular model among researchers. The problem of pricing options with Black-Scholes framework remains a contemporary research topic. The Merton model extends the Black-Scholes model making iteasy to produce an analytical solution for a variety of option pricing problems. According to Peter Car, jump-diffusion has become a popular model being used by the researchers because it is better able to fit smile volatility. There exists a consistent theoretical framework enabling experimentations with adapting the stock hedge or hedging with option.In essence, the Merton model can be applied directly, given a slight reinterpretation of the parameters of the model. The reinterpretation requires that we substitute the stock index value, for the stock price in the Merton's model. We also substitute the dividend rate on stock index, which we presume to equal risk-free rate. With these substitutions, we can apply the Merton's model to price the options.

Keywords: Jump-diffusion model, European options, Black-Scholes Model

1. Introduction

Option is a financial contract which gives its holder the right to buy or sell the underlying asset at a predetermined price. Options can be divided into two groups, a call option which is a contract that gives the right to its holder (buyer) to buy and put option gives to its holder the right to sell a pre-specified underlying asset at a predetermined price (strike price, K) for a specific time period (maturity date, T). The holder is not obligated to buy or sell the underlying according to the terms of the contract. The holder may wish to end up his position by selling the option.

There are two fundamental kinds of options which is American and European option. An American option allowed the owner to exercise at any time before expiration however the owner of a European option can exercise only at expiration. In finance, the value of an option consists of two components, its intrinsic value and its time value. Intrinsic value is the difference between the exercise price of the option (strike price, K) and the current value of the underlying instrument (sport price, S). If the option does not have positive monetary value, it is referred to as out-the-money which it will expire worthless as an option not to lose money by exercising, an option will never have a value less than zero.

The option may increase in value due to volatility in the underlying asset. Numerically, this value depends on the time until the expiration date and volatility of the underlying instrument's price. The time value of an option is always positive and decline exponentially with time, reaching zero at the expiration date. Time value is zero at expiration where the option value is simply its intrinsic value. Prior to expiration, the change in time value with time is non-linear, being a function of the option price.

In other words, volatility intervened to be the most important variable in finance, in the frame of asset pricing, derivatives, corporate finance, futures and other portfolios. Hence, a number of studies continuously have been done to examine the power of volatility in option price fluctuation. According to Dimitris (2010), Brenner and Galai (1989, 1993) were the first proposed the response of volatility to the growing and decreasing of the instruments in the market. It completed the market by allowing direct hedging of volatility risk which exposes the impact to the underlying asset price.

Black-Scholes (1973) appeared early in introducing a model in option pricing. The work began with the idea of geometric Brownian motion, started by assuming the random movement of asset price in the market. Black-Scholes model operated under several assumptions included taking a known and constant interest rate through time and happened to be the first mechanism to be able to produce a closed-form call-put option price for the market. However, Black-Scholes model of option value only depended on the price of the stock (Navas, 2003) and time. Other variables are known and assume to be constant.

In a way of overcoming the heavy-tailed in Poisson distribution introduced by Black-Scholes, jump-diffusion models adapt naturally in high skewness and leptokurtosis levels which appeared commonly in financial time series. As also stated by (Linetsky, 2008), jump diffusion is also compatible of evaluating large and sudden changes in the state variable. In other words, within a particular time interval, jump will permit and able to evaluate strong movement of underlying instruments.

2. Literature Review

Options have existed for decades, though only since late 1970s it regained the attentions of the investors for its security, protecting the investors from stock market turmoil. Black-Scholes (1973) embark the option market with the first path-breaking closed-form solution of option pricing. Without hesitation, new innovations are based on the framework of Black-Scholes model have been developed, resented from its incapability to capture the real problem occurred in option market.

Merton (1978) was the first scholar derived to an option price from the feature of Black-Scholes in jump-diffusion model (Kou, 2008). The model presented to be more general when the underlying stock returns are the mixture of continuous and jump processes. Since then, jump-diffusion model have been one of the attractive mechanism to price option both exotic and vanilla option. Chunfa Wang et al (2011) introduced a model of double exponential jump diffusion with regime switching. Numerically, the option prices being calculated by using Fast Fourier transform (FFT). The prices were shown to have satisfactory accuracy when compared with switching Black-Scholes model.

Volatility is known to be the primary mechanism in option pricing. Hence, In 2010, Dimitris et al took into calculation the mean reverting logarithms with diffusion for better approximation for VIX (volatility index) to determine option price. The numerical result presented claimed to have closed form models for pricing futures and options on spot VIX. Though, the application at the stage of financial managerial support is unclear as the impacts of volatility to the firm are widespread and complex.

Liming et al (2008) proposed a simpler, fast and accurate scheme, applied high-order time discretization on partial integrodifferential equation (PIDE) that able to integrate diffusion implicitly and jump explicitly.

Peter Carr and Anita Mayo (2007) developed more efficient method to estimate the correlation jump integrals. These integrals were usually evaluated using fast Fourier methods which is computationally expansive due to the calculation at every mesh of time frame. Carl et al (2005) explored on pricing American option in jump diffusion model by extending the work of Chiarella et al (1999), expanding the Fourier-Hermite series where the size of the jump is lognormally distributed.

Here in this study, we presented a modest approach of pricing European option in jump-diffusion model. The result will then be compared to Black-Scholes model to present more adaptable numerical result in option price.

3. Methodology

3.1 Black-Scholes-Merton (BSM) Model

Fisher Black and Myron Scholes (1973) spark the option market with computed closed-form solution of option pricing . Initiated from the Brownian motion, the movement of the stock is said to fit with the model and give the value S using the following formula (Brandimarte, 2002)

$$dS = \mu S dt + \sigma S dz(\tau) \quad [1]$$

In this formula, S is the initial known of asset price and it is exponentially growing with respect to time. μ is nonnegative variable, lognormally distributed and independent. $z(\tau)$ is a standard Gauss-Wiener process, normally distributed with zero mean and variance dt .A geometric Brownian motion will yield a positive call option and the asset price function can be written as follows:

$$\ln\left(\frac{S(\tau)}{S}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)\tau + \sigma z(\tau) \quad [2]$$

The used of lognormal distribution [2] is to overcome the negative values associated in normal distribution and allow larger asset prices cause by sudden events (David Bonnemort and etc, 2006). Recall the original portfolio of Black-Scholes [1] and the application of lognormal by Merton [2], and integrated with Ito's formula yield the BSM formula. By Ito we have:

$$dF = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S_t + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S_t^2 \right)dt + \frac{\partial f}{\partial S}\sigma S dW \qquad [3]$$

The construction of [3] at all time consist of securities and bonds of $\Pi_t = F_t$ as $t \geq T$. Hence, if $\Pi = \theta_t^0 dB + \theta_t^1 dS$ then $d\Pi = (\theta_t^0 r_t B_t + \theta_t^1 \mu S)dt + \theta_t^1 \sigma S dW$ [4]

for $d\Pi_t = dF_t$, where θ_t^0 represents units of the riskless bond and θ_t^1 units of the underlying. Comparing [2] and [4], both equations can be solved to get the BSM formula:

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 f}{\partial S^2} - r_t f + r_t \frac{\partial f}{\partial S} S = 0 \qquad [5]$$

Equation [5] claims that the underlyings must satisfy formula to overcome the unrealistic assumption in [1] (Bemis, 2006).

3.2 Jump-Diffusion Model

Jump-diffusion model is an integration of Brownian motion and Poisson process where the Brownian giving the character of diffusion and Poisson hold the properties of the jump part. The Brownian motion appear early in option pricing as the Black-Scholes used it to form the path-breaking formula. On the other hand, Poisson define the corresponding equation to be constructed to form the Jump-diffusion model.

$$N_t = \sum_{n \geq 1} 1_t \geq T_n \qquad [6]$$

The above equation is called the Poisson process taking into the formula the sequence of independent exponential random variable $\{\tau_i\}_{i \geq 1}$, parameter λ, cumulative distribution function $P[\tau_i \geq y] = e^{-\lambda y}$ and $T_n = \sum_{i=1}^{n} \tau_i$. This process emphasize the jumps of size one (1) only (Ekaterina Voltchkove, Peter Tankov).

Hence, in one dimension the jump diffusion process gave the formula of:

$$dX_t = \mu dt + \sigma dB_t + dJ_t \qquad [7]$$

This model contained $\mu = r - q - \sigma^2 / 2 + \lambda(1 - E[e^z])$, $\{B_t, t \geq 0\}$ is the Brownian motion, $\{J_t, t \geq 0\}$ the Jump process such that $J_t = \sum_{n=1}^{N_t} Z_n$ and $\lambda > 0$ is a compound of Poisson process. N_t is called the Poisson process [6], holding the intensity of λ and $\{Z_t\}$ which are independent and identically distributed. This jump-diffusion process model also assumed that the Brownian motion, the Poisson process and the jump magnitudes are all independent. In 1976, Merton's used the jump-diffusion process to form his own model where the jump magnitude distribution is normal with m as the mean and s as the standard deviation.

$$p(z) = \frac{1}{\sqrt{2\pi s^2}} \exp\left(-\frac{(z-m)^2}{2s^2} \right) \qquad [8]$$

This is the probability function and the drift parameter is $\mu = r - q - \sigma^2 / 2 + \lambda(1 - \exp(m + s^2 / 2))])$ (Liming Feng, Vadim Linetsky, 2008).

3.3 Option Pricing in Jump-Diffusion Model

Starting from Merton's pioneering work (1976), jump-diffusion model have become one of the essential tool in equity, derivatives market and increasingly being used by finance community. The increases of interest in this model supported by the following reasons (Liming Feng, Vadim Linetsky, 2008):

i. This jump-diffusion model is enhancing the number of jump to the diffusion dynamics with respective variables which allows to quantify the strong movement of stock price over a short period of time.
ii. This model is very adaptive to any changes, even for large and sudden fluctuation in financial market.
iii. Jump-diffusion models naturally depicted high skewness and leptokurtosis levels that being observed in financial time series.

Here, we will observe the work of constructing the model to corporate with the option price by S. G. Kou (1999). In order to estimate the price the European options in jump-diffusion model, first we have to compute the expectation of the discounted final payoff of the option. In detail, we will have the price of a call option at time 0 $(\psi_c(0))$ given by:

$$(\psi_c(0)) = E^*(e^{-rT}\psi_c(T))$$

$$= E^*\left(e^{-rT}\left(S(0)\exp\left\{\left(r-\frac{\sigma^2}{2}-\lambda\xi\right)T+\sigma\sqrt{T}Z\right\}\prod_{j=1}^{N(T)}V_j - K\right)^+\right) \quad [9]$$

where $\psi_c(T) = (S(T)-K)^+$ and Z is a standard normal variable.

In simple word, we say

$$E^*\left(e^{-rT}S(T)\right) = S(0) \quad [10]$$

Particularly, European put-call option is given by:

$$\psi_c(0) = \sum_{n=1}^{\infty}\sum_{j=1}^{n} e^{-\lambda T}\frac{(\lambda T)^n}{n!}\frac{2^j}{2^{2n-1}}\binom{2n-j-1}{n-1}$$

$$\bullet\{S(0)e^{-\lambda\xi T+nk}\frac{1}{2}\left(\frac{1}{(1-\eta)^j}+\frac{1}{(1+\eta)^j}\right)\phi(a_+) - e^{-rT}K\phi(a_-)$$

$$+\frac{1}{2}e^{-rT}e^{-h/\eta}e^{\sigma^2T/(2\eta^2)}K\sum_{i=0}^{j-1}\left(\frac{1}{(1-\eta)^{j-i}}-1\right)\left(\frac{\sigma\sqrt{T}}{\eta}\right)^i\frac{1}{\sqrt{2\pi}}Hh_i(c_-)$$

$$+\frac{1}{2}e^{-rT}e^{h/\eta}e^{\sigma^2T/(2\eta^2)}K\sum_{i=0}^{j-1}\left(1-\frac{1}{(1-\eta)^{j-i}}\right)\left(\frac{\sigma\sqrt{T}}{\eta}\right)^i\frac{1}{\sqrt{2\pi}}Hh_i(c_+)\}$$

$$+e^{-\lambda T}\{S(0)e^{-\lambda\xi T}\phi(b_+) - Ke^{-rT}\phi(b_-)\} \quad [11]$$

The formula above comes together with five (5) important variables that include:

$$a_\pm = \frac{\log(S(0)/K)+\left(r\pm\frac{\sigma^2}{2}-\lambda\xi\right)T+nk}{\eta\sqrt{T}},$$

$$b_\pm = \frac{\log(S(0)/K)+\left(r\pm\frac{\sigma^2}{2}-\lambda\xi\right)T}{\eta\sqrt{T}},\quad c_\pm = \frac{\sigma\sqrt{T}}{\eta}\pm\frac{h}{\sigma\sqrt{T}},$$

$$h = \log(K/S(0))+\lambda\xi T-\left(r-\frac{\sigma^2}{2}-\lambda\xi\right)T-nk,\quad \xi = \frac{e^k}{1-\eta^2}-1$$

At this stage, it is worth to mention two (2) cases of jump-diffusion model of option pricing, the first the case is when the jump sizes are getting smaller and the other case occur when there are no jumps at all.

From equation [10], for $k=0$ if $\eta\to 0$ and so $\xi = \frac{1}{1-\eta^2}-1$, which fortunately the equation converges to Black-Scholes formula. This is apply to the first case scenario. The second case on the other hand occur when $\lambda = 0$.

In this study, after a simplification, it is easy to say that the equation used is just as simple as:

$$\text{Put-call} = Ke^{-rT} - S(0) \quad [12]$$

Hence, the formula above will give the price of put and call of European option.

4. Result and Analysis

In valuation of the option price in both methods, there are several fundamental variables used in the model. The numerical result obtained came from the relationship of current stock price (S), strike rice (K), maturity time (T), time to maturity (t), volatility (σ) and interest rate (r).

In this section, we demonstate the result of both Black-Scholes and jump-disffusion model. Even though the valuation in Black-Scholes seems to be pointless, the formula however easier to be evaluated and will be a good introductory exercise in option price. blsprice is a call-put option pricing function of Black-Scholes. The syntax and the arguments on Black-Scholes model in MATLAB is given below:

```
[call,put]=blsprice(Price,Strike,Rate,Time,Volatility,Yield)
```

Where yield in the function is optional descripted for annualized, continuously compounded yield of the underlying asset over the life of the option with default value zero.

Despite the extended mobility of jump-diffusion model, the function still required Black-Scholes function in its syntax. Consider numbers of similar variables as in Black-Scholes, jumps, gamma and maxiter are variables needed in jump-diffusion function respective to number of jumps, percent of total volatility explained by jumps and maximum number of iterations to be used.

```
function [ ] = MertonJumpEuro(CallPut, AssetP, Strike, RiskFree, Time, Vol, Jumps, Gamma,
MaxIter)

Delta = sqrt((Gamma * (Vol ^ 2)) / Jumps);
A = sqrt(Vol ^ 2 - Jumps * Delta ^ 2);
Value = 0;

for i = 0:MaxIter
    VV = sqrt(A ^ 2 + Delta ^ 2 * (i / Time));
    Value = Value + (exp(-Jumps * Time) * (Jumps * Time) ^ i / factorial(i)) *
BlackScholesEuro(CallPut, AssetP, Strike, RiskFree, Time, VV);
end

MertonJumpEuro = Value
```

4.1 Valuing Option Jump-Diffusion Model

Table 4.1.1: Values of European option price using jump-diffusion method with interest rate is 50%

Result: S=95, K=100, Jumps=100, r =0.5, gamma=0.6					
Time /vol	0.4	0.5	0.6	0.7	0.8
0.1	2.9780	4.1376	5.3166	6.5060	7.7008
0.2	5.0937	6.7789	8.4719	10.1667	11.8598
0.3	6.7994	8.8743	10.9490	13.0187	15.0803
0.4	8.2783	10.6737	13.0621	15.4389	17.8009
0.5	9.6072	12.2791	14.9373	17.5774	20.1956
0.6	10.7282	13.7439	16.6406	19.5124	22.3548
0.7	11.9595	15.0961	18.2063	21.2847	24.3258
0.8	12.8596	16.1547	19.4175	22.6416	25.8210
0.9	12.1488	15.1981	18.2135	21.1885	24.1170
1.0	7.9025	9.8493	11.7720	13.6660	15.5272

Table 4.1.2: Values of European option price using jump-diffusion method with interest rate is 70%

Result: S=95, K=100, Jumps=100, r =0.7, gamma=0.6					
Time /vol	0.4	0.5	0.6	0.7	0.8
0.1	3.0449	4.2086	5.3901	6.5808	7.7764
0.2	5.2434	6.9318	8.6258	10.3204	12.0125
0.3	7.0333	9.1088	11.1819	13.2487	15.3065
0.4	8.5964	10.9888	13.3720	15.7424	18.0973
0.5	10.0092	12.6734	15.3221	17.9517	20.5589
0.6	11.3123	14.2160	17.0983	19.9550	22.7820
0.7	12.5267	15.6443	18.7348	21.7930	24.8139
0.8	13.4998	16.7697	20.0072	23.2061	26.3606
0.9	12.7796	15.8008	18.7886	21.7365	24.6385
1.0	8.3282	10.2540	12.1564	14.0308	15.8729

In a way to show that these option prices are marketable to be traded, we presented different prices evaluated in Black-Scholes, Monte Carlo and Monte Carlo with variance gamma method.

Table 4.1.3: Values of European option price using four (4) model in volatility range from 10% to 50%.

Result: S=100, K=105, r =0.1, gamma=0.6				
vol	Black-Scholes	Monte Carlo	Monte Carlo with variance Gamma	Jump Diffusion
0.1	2.2422	2.2802	[3.0242, 3.2095]	2.1983
0.2	4.8155	4.8527	[5.3611, 5.6683]	4.0903
0.3	7.3888	7.2940	[7.6845, 8.1074]	5.7047
0.4	9.9553	9.9397	[10.0923, 10.6244]	7.0638
0.5	12.5116	12.2978	[12.8631, 13.4967]	4.4771

In general, table 4.1.1 and 4.1.3 depicted prices in jump-diffusion with range of time and volatility. Prices formed in bell shaped, from low to moderate and eventually decreasing towards the maturity time. In the sense that the options at out of the money stage. In the last table, prices presented in jump-diffusion method are the most economical price compared to other models.

5. Conclusion

In this research we proposed a jump-diffusion method to evaluate price of European options instead of conventional way to price in Black-Scholes method. In general, we know that options give the owners the right to buy or sell some stock for a fixed price at a fixed time. Options prices are largely driven by volatility, and volatility is largely driven by news. Thus, news moves stock prices and the more news come in, the more prices move. The more prices move, the valuable is a given option.

Typically speaking, the more news that is anticipated to arrive, the higher the price at which the option trades on the market. In fact, besides its intrinsic value, the main thing that we need to know to value an option is the amount of news that will arrive by the time the option matures. Option traders refer to this premium over intrinsic value as volatility value. Their job is to access this volatility value of options. Here we know that volatility plays an important role in determining the option price. Investors should be clear that option do not put them as the shareholders of the organization. To be a shareholder, the investors must also buy a stock at or within maturity date.

This study applied jump-diffusion model, one of the earliest model in option pricing. Table 4.1 and table 4.2 demonstrate the movement of option price as the volatility and time changes. In the last table, option price from jump-diffusion being compared with other models, and basically worth to see that the price offered is lower than others. In other words, this study has met the objectives in develop an efficient or economical approach for pricing European option.

It can be said that this study is simple for expertise who have been in this area long enough to understand the chaos in stock market. On the hand, for new entrant researchers it may lead to deficiency and yet the whole work brings new facets of knowledge that waiting to be explored.

For a brief recommendation, as mentioned that the prices are driven by volatility and fluctuated roughly in time which it is driven by news. This is where speculation in stock market can bring unbearable loss. In our country, options are not popular instruments to be traded in the essence they are lack of Islamic sentiment in their trading. However, recently we can be proud by the transitioning Islamic banking adapting Islamic principles in conventional banking. It works and response positively by people. Hopefully, we will receive the same good response for option (khiyar). The main reason is, it will provide more secure investment to both parties, writer and buyer from loss.

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