A FUZZY SET APPROACH ON STUDENTS' ANSWER SCRIPTS EVALUATION

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Abstract: In 1965, Zadeh [6] proposed the theory of fuzzy sets. A fuzzy sets approach has been developed to solve problems in which descriptions of activities and observations are imprecise, vague and uncertain. In mathematics, fuzzy sets have triggered new research topics in connection with categorical theory, topology, algebra, analysis, integrals and evaluation. In this paper, the researchers use fuzzy sets approach to evaluate students' answer scripts in mathematics course. Based on the work of Biswas (1985) [2], Chen & Lee (1999) [3] presented a new method of students' evaluation compared to the traditional marking method. The researchers will try to improvise some of the drawbacks found in Chen & Lee's research and propose some other alternatives in evaluating students' answer scripts. Chen & Lee utilized a fixed value for each satisfaction levels, not using fuzzy numbers and gave fuzzy marks merely based on the instructor's opinion. In this paper, the researchers use normalized values to represent some of extreme cases of satisfaction levels and utilize fuzzy numbers to generate more consistent fuzzy marks. After the instructors mark the scripts by using the traditional method, the satisfaction levels of each question will be identified by using fuzzy numbers. Then, the degree of satisfactions of each question will be calculated. The fuzzy marks will be generated to produce the total score. Finally, the fuzzy grade will be obtained. The result that based on the fuzzy sets approach could provide more and better information which portrays the student performance of each question.

Keywords: Fuzzy sets, Fuzzy Mark, Fuzzy grade sheet, Satisfaction level, Degree of satisfaction

INTRODUCTION

A grading system is very important in evaluating the educational process in order to determine the student's performance in learning. A proper grading system could motivate or encourage students to learn, consequently, improving the quality of the educational process. Some forms of traditional grading systems to use z-scores, to establish a fixed proportion of grades and to use a normal distribution (Law, 1996 [4], Weon & Kim, 2001 [5], Allsopp, 2002 [1]). However, traditional grading systems could sometimes create a problem for instructors because the score given does not reflect the real performance of the students. Therefore, a few studies have been conducted to narrow such problems in order to come up with a better evaluation method.

In 1965, Zadeh [6] proposed the theory of fuzzy sets. Fuzzy sets theory has been applied to various other fields such as in expert system, decision making, operations research pattern recognition, behavioral and social sciences, etc. It also has been developed to solve problems in which descriptions of activities and observations are imprecise, vague and uncertain. (Law, 1996) [4]. In mathematics, fuzzy sets have triggered new research topics in connection with categorical theory, topology, algebra, analysis, integrals and evaluation. Chen and Lee (1999) [3] extended Biswas's (1995) [2] work by proposing two new methods for evaluating students' answer scripts using fuzzy sets. The proposed methods aim to overcome the drawbacks in Biswas' due to the fact that instructors do not need to perform the complicated matching operations and they can evaluate students' answer scripts in a fairer manner. However, the methods introduced by Chen and Lee could be further improved.

Thus, this paper attempts to overcome the drawbacks identified in the technique as carried out by Chen and Lee. The drawbacks and the alternative ways suggested to overcome them are explained as follows:

In their work, Chen and Lee have a fixed value for each satisfaction level. However, in our case, we use normalized values to represent the degree of satisfaction for lower extreme cases (i.e grade E) and for upper extreme cases (i.e. grade A) while keeping the degree of satisfaction of A-, B+, B, B-, C+, C, C-, D+, and D.

They did not utilize the advantage of using fuzzy numbers in their evaluation method. In this paper, we propose to use fuzzy numbers as this technique can provide equations for each grade and finally produce a consistent result.

They also mentioned that the higher the degree of satisfaction the more the fuzzy mark satisfies the instructor's opinion. The drawback is that the fuzzy marks given are merely based on the instructor's opinion. In our case, however we use fuzzy numbers to generate a more consistent fuzzy mark.

MATERIALS AND METHODS

Generally, a fuzzy set is an extension of a classical set. The explanation of the concept of fuzzy set is as follows: Let $U = \{ u_1, u_2, \ldots, u_n \}$, and let A be a fuzzy set of U, then the fuzzy set A can be represented as $A = \{ (u_1, f_A(u_1)), (u_2, f_A(u_2)), \ldots, (u_n, f_A(u_n)) \}$ where f_A is the membership function of the fuzzy set A, $f_A: U \rightarrow [0,1]$, $f(u_i)$ indicates the degree of membership of u_i in A. If the universe of discourse U is an infinite set, then the fuzzy set A can be expressed as $A = \int_X f_A(u_i)/u_i, \quad u_i \in U$. A membership function is a curve that defines how each point in the input space is mapped to a degree of membership (or membership value) between 0 and 1. The input space is sometimes referred to as the universe of discourse.

Chen and Lee's methods for students' answer scripts evaluation

Assume that there are eleven satisfaction levels to evaluate the students' answer to questions in a test or examination. The range of marks and the degree of the eleven satisfaction levels are shown in Table 1.

Table 1: Satisfaction Level And Their Corresponding Degrees Of Satisfaction

Satisfaction level, *(X)*	Degrees of satisfaction	Degrees of satisfaction, *T (X)*
Extremely good (EG)	100% (i.e., 1.00)	1.00
Very very good (VVG)	91% – 99% (i.e., 0.91 – 0.99)	0.99
Very good (VG)	81% – 90% (i.e., 0.81 – 0.90)	0.90
Good (G)	71% – 80% (i.e., 0.71 - 0.80)	0.80
More or less good (MG)	61% – 70% (i.e., 0.61 – 0.70)	0.70
Fair (F)	51% – 60% (i.e., 0.51 – 0.60)	0.60
More or less bad (MB)	41% – 50% (i.e., 0.41 – 0.50)	0.50
Bad (B)	25% – 40% (i.e., 0.25 – 0.40)	0.40
Very bad (VB)	10% – 24% (i.e., 0.10 – 0.24)	0.24
Very very bad (VVB)	1% – 9% (i.e., 0.01 – 0.09)	0.09
Extremely bad (EB)	0% (i.e., 0)	0

where $T(X)$ is a mapping function which illustrates a satisfaction level to the maximum degree of satisfaction of corresponding satisfaction level, where $T{:}X \rightarrow [0,1]$.

Table 2 shows the extended fuzzy grade sheet with thirteen columns and n rows. The second to the twelfth columns shows the fuzzy mark awarded to the answer to the corresponding question. The fuzzy mark is represented as a fuzzy set in the universe of discourse X, X ={extremely good (EG), very very good (VVG), ..., very very bad (VVB), extremely bad (EB)}.

Table 2: Chen And Lee's Extended Fuzzy Grade Sheet

Question	Satisfaction levels											Degree of
No.	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	Satisfaction
Q.1	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	D(Q.1)
Q.2												D(Q.2)
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
Q.*n*												D(Q.*n*)
										Total mark =		

A student's answer script evaluation is now presented as follows:

Step 1: Assume that the fuzzy mark of the question Q.*i* of a student's answer scripts evaluated by an instructor is shown in Table 2, $y_i \in [0,1]$ and $1 \leq i \leq 11$. From Table 1, we can see that *T*(EG)=1, *T* (VVG) = 0.99, *T* (VG) = 0.90, ..., and *T* (EB) =0. D (Q.*i*) refers to the degree of satisfaction of the student's answer to question *i*.

$$D(Q.i) = \frac{y_1 * T(\text{EG}) + y_2 * T(\text{VVG}) + \ldots + y_{11} * T(\text{EB})}{y_1 + y_2 + \ldots + y_{11}} \qquad (1)$$

where D(Q.*i*) $\in [0,1]$. The larger the value of D(Q.*i*), the higher the degree of satisfaction that the question Q.*i* of the student's answer script satisfies the instructor's opinion.

Consider the example shown in Table 3. From Table 1, the degree of satisfaction D(Q.1) of the student's answer to question Q.1 can be evaluated as follows:

$$D(Q.1) = \frac{0.9*0.99 + 0.8*0.90 + 0.5*0.80}{0.9 + 0.8 + 0.5} = 0.9141$$

It indicates that the degree of satisfaction of the student's answer to question 1 is 0.9141.

Table 3: An Example Of Chen & Lee's Extended Fuzzy Grade Sheet

Question	Satisfaction levels											Degree of
No.	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	Satisfaction
Q.1	0	0.9	0.8	0.5	0	0	0	0	0	0	0	0.9141
Q.2												
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
Q.*n*												
										Total mark =		

Step 2: Consider a candidate's answer script to a paper of 100 marks. Assume that in total there were *n* questions to be answered. Let s_1, s_2, ..., s_n represent the marks allocated to Q.1, Q.2, ..., Q.n, respectively where $\sum_{i=1}^{n} s_i = 100$, $0 \leq s_i \leq 100$, and $1 \leq i \leq n$. If the evaluated degree of satisfaction of the question Q.1, Q.2, ..., and Q.*n* are D(Q.1), D(Q.2), ..., and D(Q.*n*), respectively, then the total mark is evaluated as follows:

$$s_1 * D(Q.1) + s_2 * D(Q.2) + \ldots + s_n * D(Q.n) \qquad (2)$$

An Improvised Method

In this section, we will continue using the method implemented by Chen & Lee. We will also propose linguistic values for each satisfaction level based on some modifications of grading system from York University (2004) as shown in Table 4. This modification is needed in order to incorporate with the existing Universiti Pendidikan Sultan Idris (UPSI) grading system. The standardized grading system as practiced in UPSI is basically divided into eleven satisfaction levels. The range of marks and the degree of satisfaction for each level are shown in the Table 4.

Table 4: UPSI Modified Satisfaction Level, Range Of Marks And Degrees Of Satisfaction

Satisfaction level, *(X)*	Range of marks	Degrees of satisfaction, *T (X)*
Exceptional (A)	96 – 100	1.00
	80 – 95	normalized value
Excellent (A-)	75 – 79	0.79
Very good (B+)	70 – 74	0.74
Fairly good (B)	65 – 69	0.69
Marginally good (B-)	60 – 64	0.64
Competent (C+)	55 – 59	0.59
Fairly competent (C)	50 – 54	0.54
Marginally competent (C-)	45 – 49	0.49
Passing (D+)	40 – 44	0.44
Barely passing (D)	35 – 39	0.39
Failing (E)	6 – 34	normalized value
0 – 5	0	

Normalized value refers to a value in a range of [0,1]. Assume that, the student gets 83 marks, which is in the range of 80-95, then the degree of satisfaction is 0.83. If the student gets 73 marks, the range of marks is 70-74, then the degree of satisfaction will be a fixed value of 0.74.

A membership function needs to be defined as it transforms a mark grade into a letter grade, which is commonly used to reflect the students' performance as shown in Figure 1.

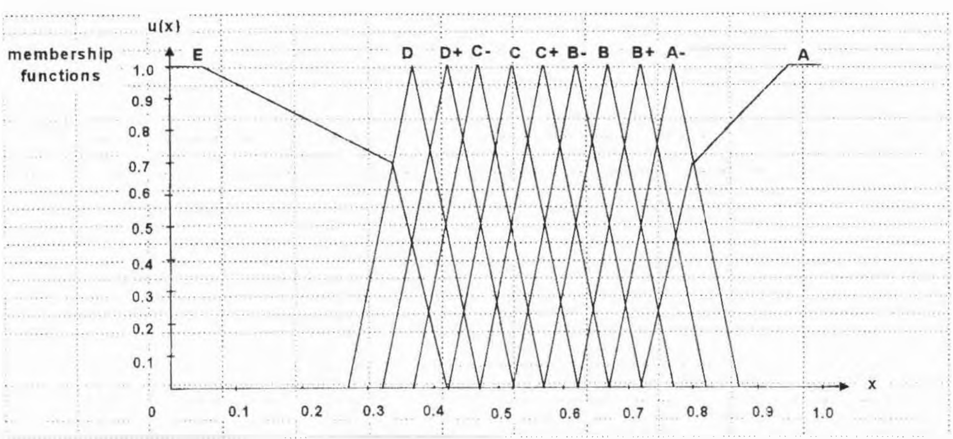

Figure 1: Membership Functions For Satisfaction Level Of The UPSI Grading System

Let T be a mapping function which illustrates a satisfaction level of each grade, where $T: U \rightarrow [0,1]$. A fuzzy mapping function T, is used to combine the membership functions together and to form a comprehensive grade.

This improvised method is using fuzzy numbers which we put values more objectively to generate fuzzy marks compared to Chen & Lee's method which values merely based on their intuition.

The improvised fuzzy grade sheet as shown in an example of Table 5 is a matrix-type structure containing seventeen columns and seven rows, which is the total number of questions in a test or examination.

Table 5: An Example Of An Improvised Fuzzy Grade Sheet

Question No.	Full Mark	Mark obtained	Normalized score	Satisfaction levels											Degree of	Fuzzy Mark
				E	D	D+	C-	C	C+	B-	B	B+	A-	A	Satisfaction	
Q.1	15	14	0.9333	0	0	0	0	0	0	0	0	0	0	1	0.9333	14
Q.2	11	6	0.5455	0	0	0	0.2455	0.7455	0.7545	0.2545	0	0	0	0	0.5655	6.22
Q.3	12	5	0.4167	0.0292	0.5333	0.9667	0.4667	0	0	0	0	0	0	0	0.4383	5.26
Q.4	25	22	0.8800	0	0	0	0	0	0	0	0	0	0	0.86	0.88	22
Q.5	10	5	0.50	0	0	0.2	0.7	0.8	0.3	0	0	0	0	0	0.52	5.2
Q.6	11	2	0.1818	0.8636	0	0	0	0	0	0	0	0	0	0	0.1818	2
Q.7	16	6	0.3750	0.3938	0.95	0.55	0.05	0	0	0	0	0	0	0	0.4067	6.5075
	100	60	0.6	0	0	0	0	0.2	0.7	0.8	0.3	0	0	0		61.1875
		B				Total fuzzy mark (assuming that no half mark is given in the total score) =										61
															Fuzzy grade =B-	
															(marginally good)	

An example to illustrate the student's answer script evaluation process is set out below

Example: Consider a candidate's scores to a paper of 100 marks. Assume that in total there are 7 questions to be answered. Let 15, 11, 12, 25, 10, 11, and 16 represent the marks allocated to Q.1, Q.2, ..., and Q.7, respectively.

Step1 From Table 4 and **(1)**:

$$D(Q.2) = \frac{0.2455*T(C-)+0.7455*T(C)+0.7545*T(C+)+0.2545*T(B-)}{0.2455+0.7455+0.7545+0.2545}$$

$$= \frac{0.2455*0.49+0.7455*0.54+0.7545*0.59+0.2545*0.64}{0.2455+0.7455+0.7545+0.2545}$$

$$= 0.5655$$

The calculation of other values such as D(Q.1), D(Q.3),..., and D(Q.7) would be the same.

Step 2 By applying **(2)**: $m_1 * D(Q.1) + m_2 * D(Q.2) + \ldots + m_n * D(Q.n)$

The total fuzzy mark of the student can be calculated as follows:

= 15*D(Q.1) + 11*D(Q.2) + 12*D(Q.3) + 25*D(Q.4) + 10*D(Q.5) + 11*D(Q.6) + 16*D(Q.7)
= 15* 0.9333 + 11*0.5655 + 12*0.4383 + 25*0.88 + 10*0.52 + 11*0.1818 + 16*0.4067
= 14 + 6.22 + 5.26 + 22 + 5.2 + 2 + 6.5075
= 61.1875
= 61 (assuming that no half mark is given in the total score)
= B - (marginally good)

RESULTS AND DISCUSSION

After setting up the improvised fuzzy grade sheet and filling in certain values, we discover some differences between this improvised method and the previous research done by Chen and Lee.

For example, in Table 5, for question 2 (Q.2), when the mark obtained is 4 out of 11, we can compute normalized score as 4/11 which is 0.5455. Such a computation has to be made as fuzzy sets deal with the range of [0,1] only. By setting up the equations for each grade in the satisfaction levels, the value of 0.5455 generates 0.2455, 0.7455, 0.7545, and 0.2545 degrees corresponding to grade C-, C, C+ and B-, respectively. It indicates that the satisfaction level of the student's scores with respect to the Q.2 is described as 0.2455 degree corresponding to grade C-, 0.7455 degree corresponding to grade C, 0.7545 degree corresponding to grade C+, and 0.2545 degree corresponding to grade B-. These values give more and better information in describing students' performance in Q.2. The result also shows that the performance of the student can be categorized in the range of C+ grade as the value of this grade (i.e., 0.7545) is higher compared to the other range of grades as mentioned before

From Table 5, we can see that this student gets a total fuzzy mark of 61 which is B- grade (marginally good). The degrees of satisfaction of grade C, C+, B-, and B are 0.2, 0.7, 0.8, and 0.3, respectively. These figures can explain that the student gets 0.2 degree corresponding to grade C, 0.7 degree corresponding to grade C+, 0.8 degree corresponding to grade B- and 0.3 degree corresponding to B grade. For the overall mark, the student is entitled to a B- grade as a final result because of the B- grade degree (i.e. 0.8) is higher compared to the other values. So that, the final result of this particular student by using fuzzy set approach is 61 as compared to traditional approach mark (total of mark obtained) which is only 60. As conclusion, the results that based on this fuzzy sets approach could provide more and better information of the student performance regarding each question compared to the traditional approach.

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