Performance Analysis of Iterative Turbo Decoding Stopping Criteria with Quadrature Amplitude Modulation

Mohamad Yusuf Mat Nasir, Roslina Mohamad, Murizah Kassim, Nooritawati Mat Tahir

***Abstract*—Iterative turbo decoding stopping criteria are known to be used in early termination of the turbo decoding iteration without sacrificing its performance. However, most of these stopping criteria were tested using low modulation techniques. In this paper, several stopping criteria were tested using higher modulation techniques, such as quadrature amplitude modulation (QAM). The sign-change-ratio (SCR), hard-decision-aided (HDA), and cross-entropy (CE) stopping criteria were simulated with 4-QAM, 8-QAM, and 16-QAM in additive white Gaussian noise. From the simulation results, HDA and SCR are capable of terminating early at various stages of noise while maintaining the bit error rate performance. However, CE fails to terminate early in high-noise channels and only terminates early when the output of the turbo decoder is in a low-noise channel.**

***Index Terms*— Iterative decoding, turbo codes, quadrature amplitude modulation, stopping criterion**

I. Introduction

Back in 1993, a paper entitled "Near Shannon Limit Error-correcting Coding and Decoding: Turbo-codes" was published. It introduced turbo codes to the world[1]. This introduction brought a major improvement to communication systems [2], such as excellent bit error rate (BER) performance because of the iterative decoding, concatenated coding, and maximum a posteriori (MAP) decoders [1]. However, these improvements required much delay and computation complexity for decoding process [3]. These major problems were further improved by many researchers by adding a stopping criterion to the decoder [4]–[7], such as sign-change-ratio (SCR), hard-decision-aided (HDA), and cross-entropy (CE) [4], [6]. Most of the research on the iterative turbo decoding stopping criteria is limited to testing in low modulation techniques, such as binary phase-shift keying (BPSK) [1]–[3], [6], [7]. For example, research in [4] tested CE, HDA and SCR stopping criteria using BPSK modulation. Another research [6] also tested the enhancement of CE stopping using the BPSK modulation.

The researcher in [8] presented the performance analysis on the sign difference ratio (SDR) stopping criterion in three different methods of Max-Log MAP algorithm where each method had different numbers of mathematical operations. Method 1 required seven mathematical operations, method 2 required three mathematical operations, and method 3 used two mathematical operations. These three methods are then tested using three different modulations: BPSK, quadrature phase-shift keying (QPSK), and 16- quadrature amplitude modulation (QAM). The research in [9] discussed the performance analysis of HDA stopping in BPSK modulation by using half iteration. A robustness test and performance analysis of the enhancement of CE stopping criterion were done in [6]. Research in [10] analysed the performance analysis of SDR in additive white Gaussian noise (AWGN).

L. H. Abderramane tested CE stopping criterion in AWGN and Rayleigh channels [11]. The turbo codes used two sets of interleaver: the CE interleaver and the Dithered golden interleaver. Yuejun Wei et al. [12] applied HDA, cyclic redundancy check (CRC), and parity-check stopping (PCS) with Max-log-MAP and log-MAP decoding. The researcher in [13] used a CE minimisation method for turbo decoding.

Most of the research on the iterative turbo decoding stopping criteria are limited to testing in low modulation techniques, such as BPSK [8]. Hence, the lack of information about the performance of the stopping criteria in the higher modulation has led to the purpose of this paper. The rest of this paper is organised as follows. In Section II, the simulation parameters used in the research are elaborated. In Section III, the results of the CE, SCR and HDA stopping criteria with 4, 8 and 16-QAM are analysed and compared. Finally, the conclusion is drawn in Section IV.

II. Simulation Parameters

Table I shows all the parameters used in the simulation. The research chose the well-known stopping criteria such as CE, SCR and HDA. The stopping criteria algorithms for CE, SCR and HDA can be referred in [4]. The thresholds for CE and SCR were set to 0.0001 and 0.012, respectively. One million random binary data with three different modulations (4-QAM, 8-QAM, and 16-QAM) were applied in the simulation. Meanwhile, for the frame size, 1000, 2000 and 10000 were used for each data transmission. The turbo code generator and

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code rate were [31,17] and 1/3, respectively. The codes were simulated under the AWGN channel and the maximum iteration number, $I_{\max}$ was set to eight.

TABLE I
SIMULATION PARAMETERS

Parameters	Name/ Value
Stopping Criteria	CE, SCR, HDA
Modulation	4-QAM, 8-QAM, 16-QAM
Frame size	1000 (1K), 10000 (10K)
Code Generator	[31,17]
Code rate	1/3
Channel model	AWGN
Maximum iteration number	8

III. RESULTS AND DISCUSSION

In this section, the performance of turbo codes with three different stopping criteria, modulations and frame sizes are compared. The performance of each stopping criterion is analysed based on the BER and average iteration number (AIN). The formula for saving iteration percentage (α) is derived in (1).

$$\alpha = \frac{\left| I_n - \mathrm{I}_{\max} \right|}{I_{\max}} \times 100 \quad (1)$$

Where I_n is the AIN at n of the energy per bit to noise power spectral density ratio (Eb/No) and n is the Eb/No level.

A. Performance of CE Stopping Criterion

The AIN of turbo codes with CE stopping criterion for 4-QAM, 8-QAM, and 16-QAM modulations are shown in Figures 1, 2, and 3, respectively. Meanwhile, the BER performance of turbo codes with CE stopping criterion for 4-QAM, 8-QAM and 16-QAM modulations are shown in Figures 4, 5 and 6, respectively. From Fig. 1, the AIN for 1K frame size and 4-QAM start to reduce at – 0.2 dB and for 10k frame size at 0 dB. At 3 dB Eb/No, both 1K and 10K can terminate early at the second iteration, which can save 72% AIN from the fixed stopping criterion.

Meanwhile, for Fig. 2, it can be observed that the iteration number for 8-QAM with 1K frame size starts to reduce at 1 dB and for 10K frame size at 1.2 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at 2.1 and 2.3 iteration, respectively, which can save around 71-74% from a fixed stopping criterion. From Fig. 3, the AIN performance for frame 1K, 16-QAM turbo code with CE stopping starts to reduce at 1.2 dB, meanwhile, for a 10K frame, it starts at 2 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at 2.4 and 3 iteration, respectively, which can save around 63-70% from a fixed stopping criterion.

For BER performances of 4-QAM, as shown in Fig. 4, it can be observed that the BER for CE and fixed stopping for 1K frame size are the same. For 10K frame size, there is a slight difference in the BER start at 0 dB. For 8-QAM, the BER performance for CE and fixed stopping criteria for 1K and 10K frame sizes are almost the same as depicted in Fig. 5. The same performance is also achieved by 16-QAM turbo codes, as shown in Fig. 6.

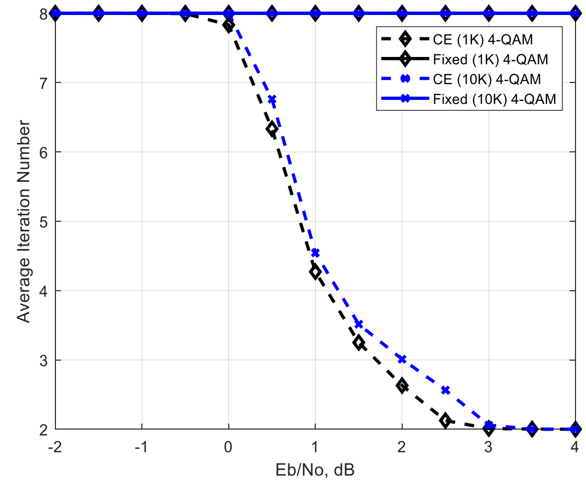

Fig. 1. AIN performance for 4-QAM turbo codes with CE and fixed stopping criteria.

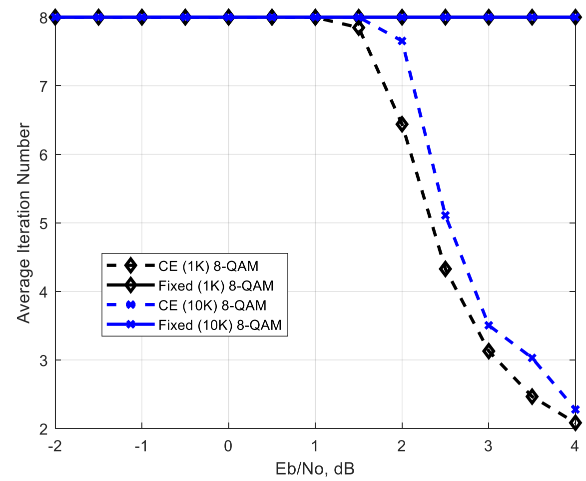

Fig. 2. AIN performance for 8-QAM turbo codes with CE and fixed stopping criteria.

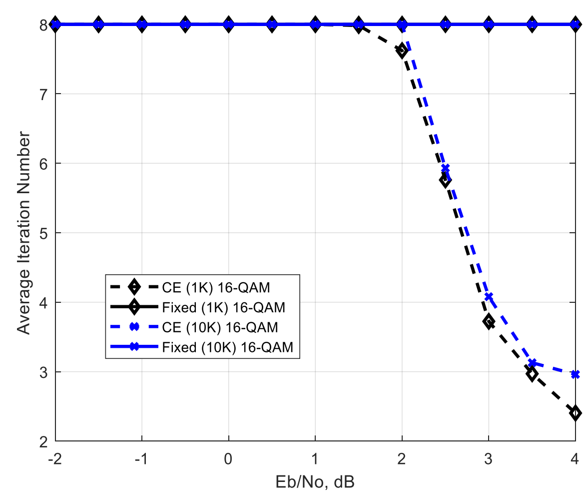

Fig. 3. AIN performance for 16-QAM turbo codes with CE and fixed stopping criteria.

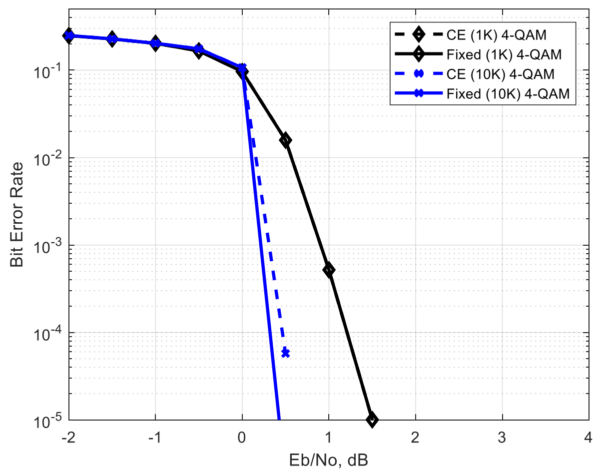

Fig. 4. BER performance for 4-QAM turbo codes with CE and fixed stopping criteria.

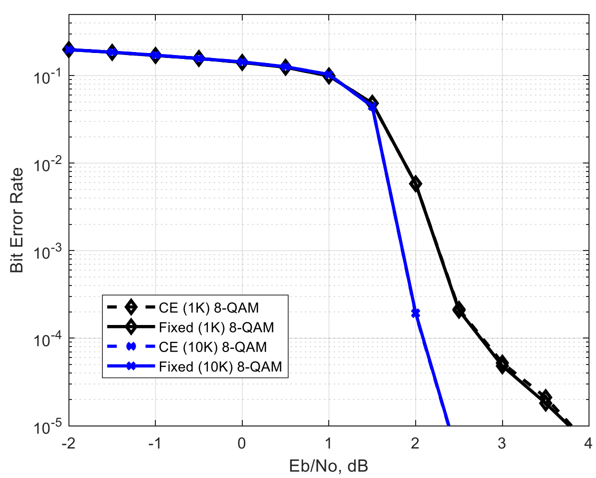

Fig. 5. BER performance for 8-QAM turbo codes with CE and fixed stopping criteria.

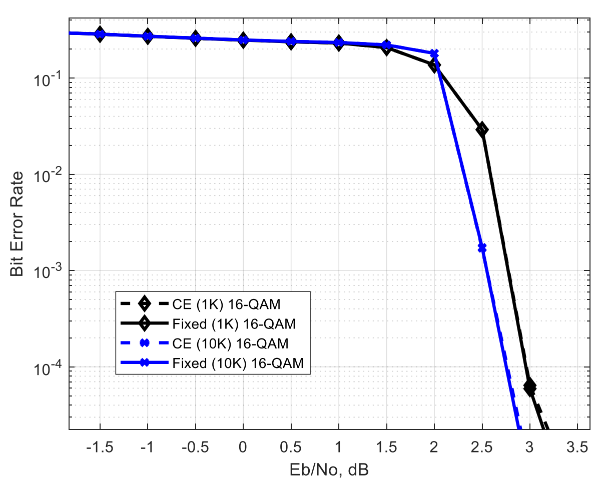

Fig. 6. BER performance for 16-QAM turbo codes with CE and fixed stopping criteria.

B. *Performance of SCR Stopping Criterion*

The AIN of turbo codes with SCR stopping criterion for 4-QAM, 8-QAM and 16-QAM modulations are shown in Figures 7, 8 and 9, respectively. Meanwhile, the BER performance of turbo codes with CE stopping criterion for 4-QAM, 8-QAM and 16-QAM modulations are shown in Figures 10, 11 and 12, respectively. From Fig. 7, the SCR for 4-QAM with 1K and 10K frame sizes is capable of terminating early at 3 AIN, -2dB. The AIN for both frame sizes starts to increase and achieves maximum AIN at -1.2 dB. It can be observed that the iteration number for 1K frame size starts to reduce at –0.2 dB. Meanwhile, for 10K frame size, it starts to reduce at 0 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at the second iteration, which can save around 72% from the fixed stopping criterion.

Meanwhile, for Fig. 8, the SCR for 8-QAM with 1K and 10K frame sizes is capable of terminating early at 3 AIN, -2dB. The AIN for both frame sizes starts to increase and achieves maximum AIN at 0.2 dB. It can be observed that the iteration number for 1K frame size starts to reduce at 1 dB. Meanwhile, for a 10K frame size, it starts to reduce at 1.2 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at the second iteration, which can save around 72% from the fixed stopping criterion. From Fig. 9, the SCR for 16-QAM with 1K and 10K frame sizes is capable of terminating early at 3 AIN, -2dB. The AIN for both frame sizes starts to increase and achieves maximum AIN at 0.2 dB. It can be observed that the iteration number for 1K frame size starts to reduce at 1.2 dB, meanwhile, for a 10K frame size, it starts to reduce at 2 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at the third iteration, which can save around 63% from the fixed stopping criterion.

For BER performances of 4-QAM as shown in Fig. 10, it can be observed that the BER for SCR and fixed stopping for 1K frame size are the same. For 10K frame size, there is a slight difference in BER starting at 0 dB and increasing until 0.6 dB at BER = 10^{-2}. For 8-QAM, the BER performances for SCR and the fixed stopping criteria for 1K have a slight difference. For the 10K frame size, there is a slight difference in the BER starting at 1.2 dB, increasing until 0.2 dB at BER = 10^{-2}, as depicted in Fig. 11. The same performance is also achieved by 16-QAM turbo codes. Meanwhile, it can be observed that the BER for SCR and fixed stopping for 10K frame size are almost the same. For 1K frame size, there is a slight difference in BER starting at 2.2 dB and increasing until 0.3 dB at BER = 10^{-2}, as shown in Fig. 12.

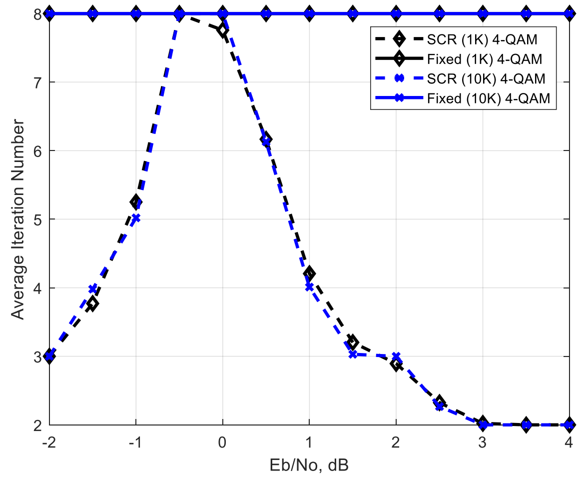

Fig. 7. AIN performance for 4-QAM turbo codes with SCR and fixed stopping criteria.

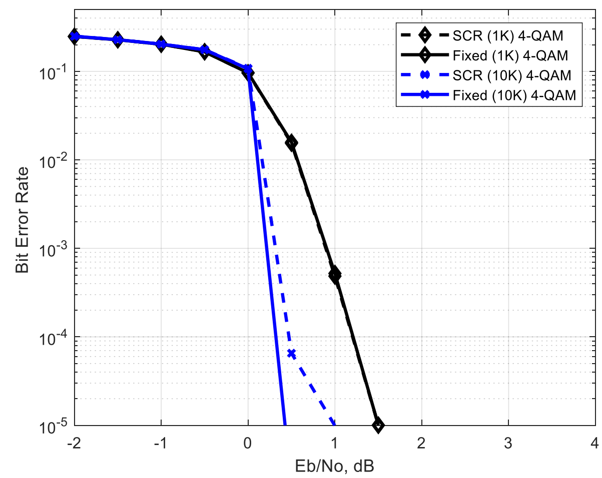

Fig. 10. BER performance for 4-QAM turbo codes with SCR and fixed stopping criteria.

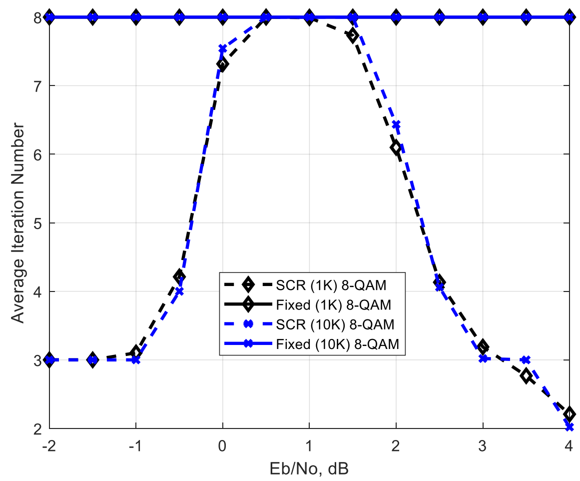

Fig. 8. AIN performance for 8-QAM turbo codes with SCR and fixed stopping criteria.

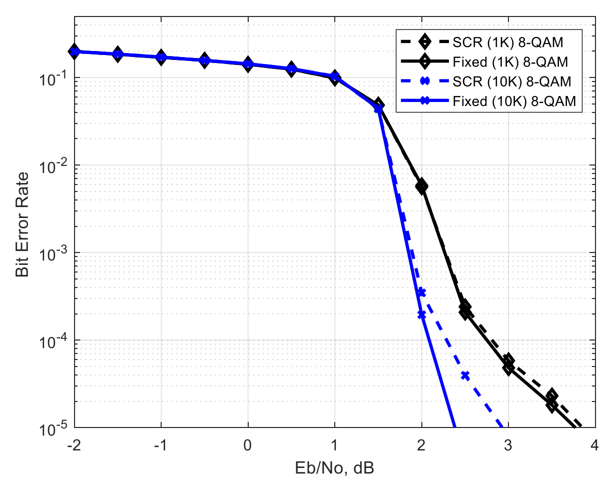

Fig. 11. BER performance for 8-QAM turbo codes with SCR and fixed stopping criteria.

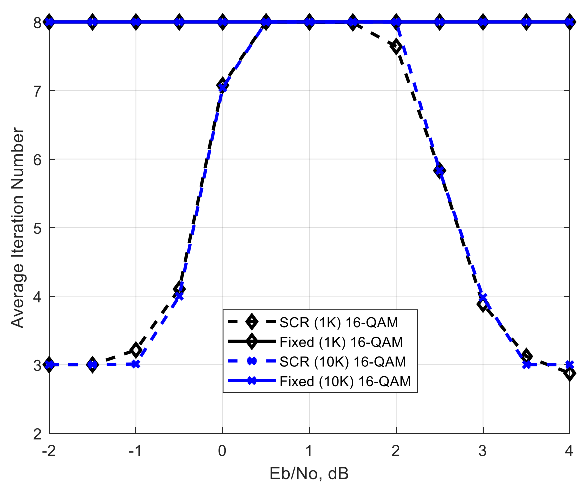

Fig. 9. AIN performance for 16-QAM turbo codes with SCR and fixed stopping criteria.

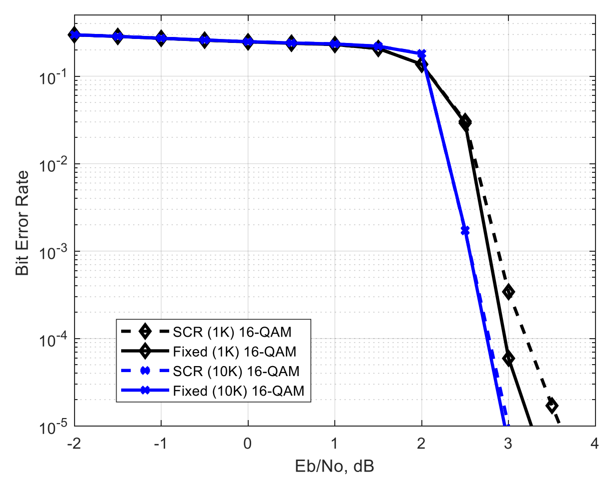

Fig. 12. BER performance for 16-QAM turbo codes with SCR and fixed stopping criteria.

C. Performance of HDA Stopping Criterion

The AIN performances of turbo codes with HDA stopping criterion for 4-QAM, 8-QAM, and 16-QAM modulations are shown in Figures 13, 14, and 15, respectively. The BER performances of turbo codes with HDA stopping criterion for 4-QAM, 8-QAM, and 16-QAM modulations are shown in Figures 16, 17 and 18, respectively. From Fig. 13, at -2 dB, the HDA for 4-QAM with 1K and 10K frame sizes is capable of terminating early at 3.3 and 4 AIN, respectively. The AIN for 1K and 10K starts to increase and achieves maximum AIN at -0.2 dB and -1 dB, respectively. It can be observed that the iteration number for 1K frame size starts to reduce at –0.2 dB while for a 10K frame size it starts to reduce at 0 dB. At 4 dB, both 1K and 10K can terminate early at the second iteration, which can save around 72% from the fixed stopping criterion.

Meanwhile for Fig. 14, at 2 dB, the HDA for 8-QAM with 1K and 10K frame sizes is capable of terminating early at 3 and 3.2 AIN, respectively. The AIN for both frame sizes starts to increase and achieves a maximum AIN at 0 dB. It can be observed that the iteration number for 1K frame size starts to reduce at 1 dB while for the 10K frame size it starts to reduce at 1.2 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at 2.2 and 3 AIN, which can save around 69% and 63% from the fixed stopping criterion, respectively. From Fig. 15, at 2 dB, the HDA for 16-QAM with 1K and 10K frame sizes is capable of terminating early at 2.8 and 3.2 AIN, respectively. The AIN for both frame sizes start to increase and achieves maximum AIN at 0 dB. It can be observed that the iteration number for 1K frame size starts to reduce at 1.2 dB while for the 10K frame size it starts to reduce at 2 dB. At 4 dB Eb/No, both 1K and 10K can terminate early at 3 and 3.1 AIN, which can save around 63% and 61% from the fixed stopping criterion, respectively.

For BER performance of 4-QAM, as shown in Fig. 16, it can be observed that the BER for HDA and fixed stopping for 1K frame size are the same. For 10K frame size, there is small BER degradation starting at BER = 10^{-1} and increasing until 0.2 dB at BER = 10^{-4}. For 8-QAM, the BER performance for HDA and the fixed stopping criteria for 1K and 10K have similar performance as depicted in Fig. 17. The same performance is also achieved by 16-QAM turbo codes. Meanwhile, it can be observed that for the BER for SCR and fixed stopping for 1K frame size, there is a slight difference in BER starting at 3.4, as depicted in Fig. 18.

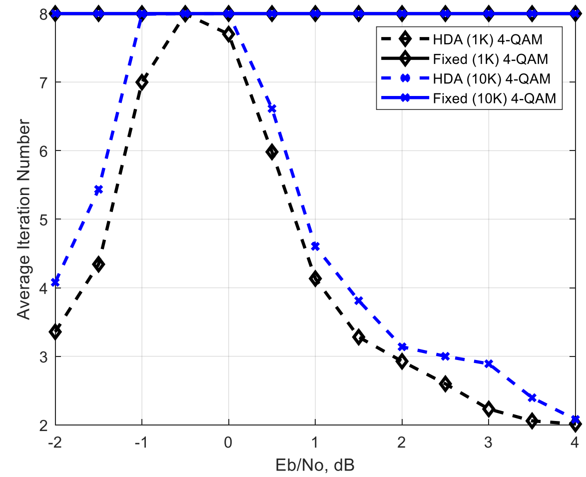

Fig. 13. AIN performance for 4-QAM turbo codes with HDA and fixed stopping criteria.

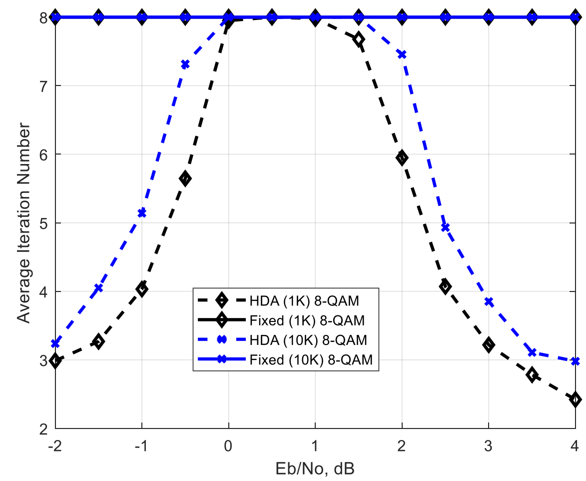

Fig. 14. AIN performance for 8-QAM turbo codes with HDA and fixed stopping criteria.

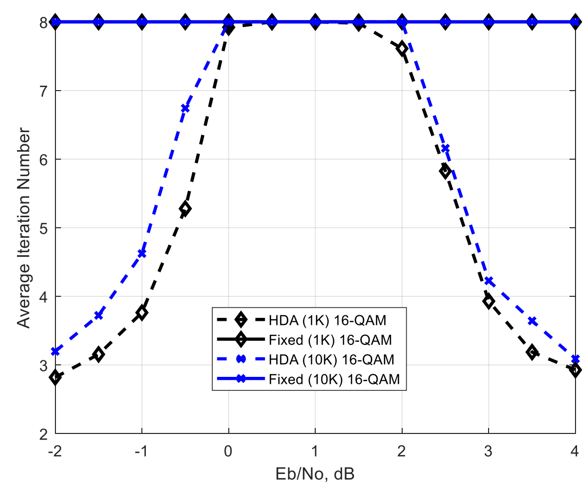

Fig. 15. AIN performance for 16-QAM turbo codes with HDA and fixed stopping criteria.

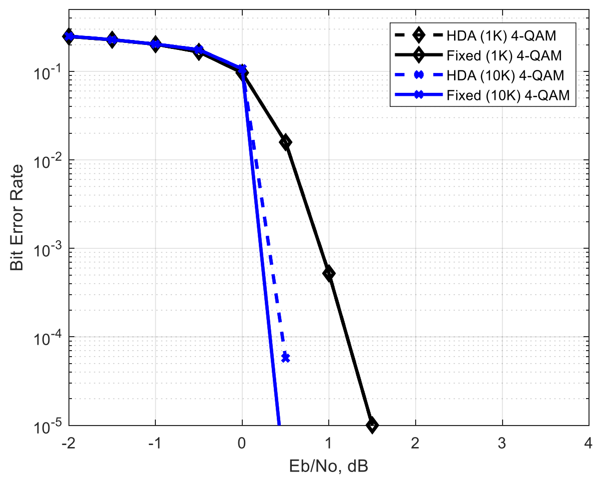

Fig. 16. BER performance for 4-QAM turbo codes with HDA and fixed stopping criteria.

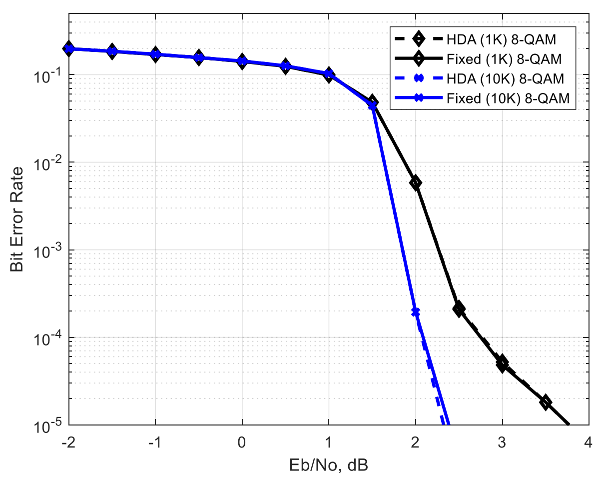

Fig. 17. BER performance for 8-QAM turbo codes with HDA and fixed stopping criteria.

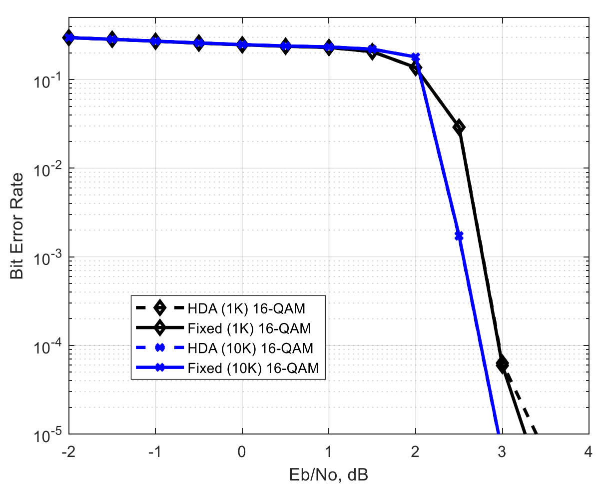

Fig. 18. BER performance for 16-QAM turbo codes with HDA and fixed stopping criteria.

D. Overall Performance of Iterative Turbo Decoding Stopping Criteria with QAM

From the results in Sections III A to C, the BER performances of 4-QAM, 8 QAM and 16 QAM for 1K and 10K frame sizes for CE and HDA stopping criteria are maintained successfully. Both stopping criteria only face a very small degradation which is less than 0.2 dB at high Eb/No region. This proves that CE and HDA stopping algorithm and its thresholds are capable to detect the correct convergence output from turbo decoder and make the correct early termination of the iterative turbo decoder.

Meanwhile for SCR, the stopping algorithm and its threshold result an early termination for iterative turbo decoding. However, this early termination causes the BER degradation that approaching to 0.6 dB at high Eb/No region. This indicates that SCR detects the false convergence output and makes the turbo decoder stops early even though the output is not converged yet. This false detection/false alarm cases results the error in received data and increase the BER degradation.

IV. Conclusion

From the results obtained, it can be concluded that CE is not suitable for use with higher-order modulation at low Eb/No as it cannot terminate early. However, in high EbNo, CE performs with all the modulations and gives a significant reduction in AIN while maintaining the BER performance. Meanwhile, for SCR, this stopping criterion is capable of terminating early at low and high Eb/No for all frame sizes. The SCR is also capable of maintaining the BER performance for small frame sizes while it has a small degradation in BER at high Eb/No for large frame sizes. As with SCR, HDA is also capable of terminating early at low and high Eb/No for all frame sizes. The HDA is also capable of maintaining the BER performance for small and large frame sizes for various Eb/No.

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